ON THE ESTIMATION OF THE REGION OF THE ACCEPTED TOLERANCE

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Abstract: The vector of the characteristic parameters describes each shape, and the similarity between two shapes is estimated based on the reference distance. If an unknown shape is inside the region of the accepted tolerance associated to a model, that unknown shape is identified with the model. Many definitions of the reference distance are presented and their influence on the shape recognition is analyzed. The paper recommends the maximum weighted distance and a reference distance based on the standard deviation of the analyzed imprints of each prototype.

Keywords: shape, model, description, distance, tolerance, similarity, identification, recognition.

1. INTRODUCTION

The shape recognition in robotics is often based on many known shapes (models, prototypes) that are firstly explored and their descriptors (characteristic parameters) are memorized. The identification of an unknown shape with one prototype is based on the similarity between this shape and the prototype. The similarity is usually estimated by the value of a computed distance between the two analyzed shapes (Belaïd A. and Belaïd Y., 1992; Dougherty, 1988; Purcaru, 1999; Purcaru, 2001; Purcaru, 2003c); the choice of the proper distance and its reference value is a very difficult problem in the shape recognition.

For a quick recognition, each unknown shape must be explored only once with a sensory system. So, the shape identification must be assured regardless the shape position and orientation in the sensory space. This is the reason for the shape description only by some parameters invariant or quasi-invariant to rotations and translations of the shape in the sensory space.

After V explorations in the same conditions, V imprints result for each model. Each imprint is described by D characteristic parameters (descriptors). Let us consider that the first s descriptors are invariant, and the others (D-s) descriptors are quasi-invariant to shape localization in the sensory space. The precision of the model description increases when V (the number of explorations) increases too. The characteristic parameter vector of the i model is $m_i = [m_{i,1} m_{i,2} \dots m_{i,D}]^T$, and for the k imprint of this model is $m_i^k = [m_{i,1}^k m_{i,2}^k \dots, m_{i,D}^k]^T$.

The vector of the accepted tolerance associated to the i model is $\varepsilon_i = [\varepsilon_{i,1} \varepsilon_{i,2} \dots \varepsilon_{i,D}]^T$ and its components must represent the maximum accepted deviations of the descriptor values for the i model. The definition of the components $\varepsilon_{i,j}$ affects the decision of the recognition process.

In the parameter space, the i model is represented by the point $M_i(m_{i,1}, m_{i,2}, \dots, m_{i,D})$, that corresponds to m_i. An unknown shape is described by the vector of the same D characteristic parameters, $x = [x_1 x_2 \dots x_D]^T$, obtained by processing only one imprint. The point $X(x_1, x_2, ..., x_D)$ in the parameter space represents the x shape.

In the D-dimensional parameter space, there is a region of the accepted tolerance, $R^i_{t,q}$. An unknown shape x is identified with the i model if the associated point $X(x_1, x_2, ..., x_D)$ is inside $R^i_{t,q}$.

When the d_q distance is used for estimate the shape similarity and there is a reference distance ($d^i_{q,ref}$) associated to the i model, the region of the accepted tolerance associated to i model is (Purcaru, 2002)

$$R^i_{t,q} = \{x \mid d_q(x, m_i) \le d^i_{q,ref}\}. \quad (1)$$

The unknown shape can be identified with only one prototype (certain recognition), can present the same similarity with minimum two prototypes (ambiguous recognition) or cannot be identified with a prototype (rejection of x).

2. DISTANCES AND REGIONS OF THE ACCEPTED TOLERANCE FOR SHAPE IDENTIFICATION

Various distances (Belaïd A. and Belaïd Y., 1992; Dougherty, 1988; Purcaru, 1997; Purcaru, 2001; Purcaru, 2003a; Purcaru, 2003b) enable the estimation of the shape similarity:

d_1 - Hamming distance,

d_2 - Euclidean distance,

d_n - generalized distance,

d_∞ - maximum distance,

d^w_∞ - maximum weighted distance.

For each distance d_q, $q = 1, 2, n, w, \infty$, it is important to define a reference value $d_{q,ref}$ for the shape recognition.

2.1 First Definition of the Reference Distance

If the distance between x and m_i is

$$d_q(x, m_i) = \min_{k=\overline{1,V}} d_q(x, m^k_i), q = 1, 2, n, \infty, \quad (2)$$

the reference distance can be (Purcaru, 1997; Purcaru, 2001)

$$d^i_{q,ref} = \max_{k=\overline{1,V}} \left[\min_{\substack{p=\overline{1,V} \\ p \ne k}} d_q(m^k_i, m^p_i) \right]. \quad (3)$$

Let us consider $q = 2$, $V = 7$ and two geometrical parameters (p_1 and p_2) for the shape description. In these conditions, the region of the accepted tolerance ($R^i_{t,2}$) is represented by all circular surfaces in the (p_1, p_2) parameter plane, with radius $d^i_{2,ref}$, centered on $M^k_i, k = \overline{1,7}$ points that correspond to $m^k_i, k = \overline{1,7}$ imprints of the i model. These seven imprints have two distributions in the parameter plane, represented in Table 1. The points $M^1_i, M^2_i, ..., M^5_i$ have the same positions in both situations. For first imprint distribution,

$$d^i_{2,ref} = \overline{M^4_i M^5_i} = \sqrt{\sum_{j=1}^{2} \left(m^4_{i,j} - m^5_{i,j}\right)^2} = 21.08, \quad (4)$$

and for second imprint distribution,

$$d^i_{2,ref} = \overline{M^4_i M^6_i} = \sqrt{\sum_{j=1}^{2} \left(m^4_{i,j} - m^6_{i,j}\right)^2} = 10.55. \quad (5)$$

So, the different positions of M^6_i and M^7_i impose a considerable change of $d^i_{2,ref}$.

Table 1. Two distributions of the imprints in the parameter plane

	First distribution		Second distribution	
	p_1	p_2	p_1	p_2
M^1_i	31.5	55.5	31.5	55.5
M^2_i	35.5	51.5	35.5	51.5
M^3_i	31.5	43.5	31.5	43.5
M^4_i	16.0	35.5	16.0	35.5
M^5_i	24.0	16.0	24.0	16.0
M^6_i	20.0	51.5	25.0	41.0
M^7_i	24.0	51.5	33.0	22.0

In the (p_1, p_2) parameter plane, seven unknown shapes have the positions specified in Table 2.

Table 2. Positions of seven unknown shapes in the parameter plane

	X_1	X_2	X_3	X_4	X_5	X_6	X_7
p_1	27.5	43.5	48.0	43.5	16.0	3.5	3.5
p_2	67.5	63.5	47.5	31.5	3.5	27.5	47.5

For each unknown shape, the distance $d_2(x, m_i)$ is computed and compared with $d^i_{2,ref}$. The unknown shapes are all identified with the m_i model, for first distribution of the model imprints (Table 1). None of the same unknown shapes is identified with m_i, for second distribution of the model imprints (Table 1).

In conclusion, the size of $R^i_{t,2}$ depends on the distribution of all model imprints in the parameter plane.

2.2 Second Definition of the Reference Distance

When each model is described by its characteristic feature vector m_i, with

$$m_{i,j} = \frac{1}{2}\left(\min_{k=\overline{1,V}} m^k_{i,j} + \max_{k=\overline{1,V}} m^k_{i,j}\right),\ j=\overline{1,D}, \quad (6)$$

the second definition of the reference distance is proposed in (Purcaru, 1999; Purcaru, 2001; Purcaru 2002):

$$d^i_{q,ref} = d_q(e_i, m_i),\ q = 1,2,n,\infty, \quad (7)$$

where $e_i = [e_{i,1} e_{i,2} ... e_{i,D}]^T$ and

$$e_{i,j} = \min_{k=\overline{1,V}} m^k_{i,j}. \quad (8)$$

In the (p_1, p_2) parameter plane, the regions $R^i_{t,q}, q = 1,2,n,\infty$, are all centered on M_i point:

- $R^i_{t,1}$ is a rhomb with $2d^i_{1,ref}$ each diagonal,
- $R^i_{t,2}$ is a circle with $d^i_{2,ref}$ the radius,
- $R^i_{t,\infty}$ is a square with $2d^i_{\infty,ref}$ the side.

When the imprints of the i prototype present very different spreads for two or more characteristic parameter value domains, the classical distances determine regions $R^i_{t,q}, q = 1,2,n,\infty$ too large, and shapes very different to the i prototype are falsely identified with this.

2.3 Reference Distances Associated to Maximum Weighted Distance

The maximum weighted distance (d^w_∞) can be used for shape recognition and different results are obtained. This distance (between an unknown shape and the i model) is defined in (Purcaru, 1999; Purcaru, 2001; Purcaru, 2002):

$$d^w_\infty(x, m_i) = \max_{j=\overline{1,D}} w_{i,j} \left| x_j - m_{i,j} \right|, \quad (9)$$

where

$$w_{i,j} = \frac{\min_{j=\overline{1,D}} \varepsilon_{i,j}}{\varepsilon_{i,j}} = \frac{\varepsilon_{i,\min}}{\varepsilon_{i,j}} \quad (10)$$

is the coefficient for weighting.

The proposed reference distance is

$$d^i_{w,ref} = \varepsilon_{i,\min}. \quad (11)$$

- First possible definition of $\varepsilon_{i,j}$ is

$$\varepsilon_{i,j} = \frac{1}{2}\left(\max_{k=\overline{1,V}} m^k_{i,j} - \min_{k=\overline{1,V}} m^k_{i,j}\right). \quad (12)$$

If $D = 2$, the region of the accepted tolerance ($R^i_{t,w}$) is a rectangular region centered on M_i point in the (p_1, p_2) parameter plane. The sides of the rectangle are the value spreads of the parameters p_1 and p_2. The size of this region depends only on the positions of the points associated with imprints characterized by minimum and/or maximum values of one or more geometrical parameters. So, $R^i_{t,w}$ is the same region for both imprint distributions presented in Table 1. For this example, none of the unknown shapes $X_1, X_2, ..., X_7$ is identified with m_i, because all the associated points are not inside $R^i_{t,w}$.

$R^i_{t,w}$ is always inside $R^i_{t,q}, q = 1,2,n,\infty$, obtained when the reference distance is defined with (7). If there are any parameters with very different spreads of the values, the area of $R^i_{t,w}$ is much smaller than that of $R^i_{t,q}, q = 1,2,n,\infty$.

- Another definition for $\varepsilon_{i,j}$ is based on the standard deviation of the imprints analyzed for a model (Purcaru, 2003a; Purcaru, 2003b). The standard deviation (root-mean-square deviation) of the finite number of data is computed for each parameter of the i model (Purcaru, 2004):

$$\sigma_{i,j} = \sqrt{\frac{1}{V-1}\sum_{k=1}^{V}\left(m^k_{i,j} - m_{i,j}\right)^2},\ j=\overline{1,f}. \quad (13)$$

The area under the Gaussian probability curve, between the limits $-\infty$ and $+\infty$, represents the entire number of observations (Helfrick, 1990). Following the Gaussian distribution, for one parameter and for normally dispersed data,

a) 95.46% of all possible values lie between the limits of -2σ and $+2\sigma$ from the arithmetic mean, and
b) 99.72% of all possible values lie between the limits of -3σ and $+3\sigma$ from the arithmetic mean.

In shape identification, the components of the accepted tolerance vector, associated to the i model, can be

$$\varepsilon_{i,j} = 2\sigma_{i,j},\ j=\overline{1,D}, \quad (14)$$

or

$$\varepsilon_{i,j} = 3\sigma_{i,j},\ j=\overline{1,D}. \quad (15)$$

The reference distance $d^i_{w,ref} = \varepsilon_{i,\min}$ is now based on standard deviations. So, only the unknown shapes which satisfy the condition (Purcaru, 2003d)

$$\left|x_i - m_{i,j}\right| \le \varepsilon_{i,j},\ \ j = \overline{1,D} \qquad (16)$$

are identified with the i model.

3. EXPERIMENTAL RESULTS

The estimation of the shape similarity in robotics, using different distances, was verified for many 2D-shapes explored with a tactile matrix sensor. Their touch was simulated with a program that also computes some parameters of the resulted binary imprints. This program, presented in (Purcaru, 2000), simulates the generation of the binary imprint of a shape touched with a matrix sensor that contains 16×16 square tactile cells with l_t the side. The analyzed shape can be a geometrical one, a shape previously learned or a shape created by the user. For such a shape can be established the size, position and orientation in the sensory plane; so, almost all possible binary imprints can be generated and processed. The resolution of the shape identification depends on the size of the sensory cell.

The shapes analyzed in this paper are the following:

- three squares (S_1, S_2, S_3), with $6 \cdot l_t$, $7 \cdot l_t$ and $8 \cdot l_t$ respectively the side;
- a circle (S_4), with $4.5 \cdot l_t$ the radius;
- a pentagon (S_5), with $6.5 \cdot l_t$ the side.

Table 3. The value spread of each characteristic parameter

Shape	i	Value spread of the parameter NA ($j=1$)	CE ($j=2$)	P ($j=3$)	FF ($j=4$)
S_1 ($6 \cdot l_t$)	1	35-40	16-20	20-23.31	10.99-13.59
S_2 ($7 \cdot l_t$)	2	45-51	17-24	24-26.14	11.76-14.54
S_3 ($8 \cdot l_t$)	3	60-77	20-28	26.63-31.31	11.42-15.09
S_4 ($4.5 \cdot l_t$)	4	62-69	23-24	26.73-27.31	10.18-11.84
S_5 ($6.5 \cdot l_t$)	5	69-76	25-27	28.73-31.56	11.96-13.51

Each shape is described by 4 geometrical parameters:

- number of the activated sensory cells from the binary imprint, NA;
- number of the activated sensory cells from the binary outline, CE;
- Freeman perimeter of the binary outline, P;
- form factor, FF.

20 different locations were established for the selected five geometrical shapes: 4 positions and 5 orientations, for each position. The value spread of each characteristic parameter is specified in Table 3.

Because the most probable value of each parameter is the arithmetic mean of the number of readings taken, the following components for the characteristic parameter vector are recommended (Purcaru, 2003a; Purcaru, 2003b):

$$m_{i,j} = \frac{\sum_{k=1}^{V} m_{i,j}^{k}}{V},\ \ j = \overline{1,D}. \qquad (17)$$

The resulted values of the components of the characteristic parameter vectors are presented in Table 4.

Table 4. The characteristic parameter vectors

Shape	i	$m_{i,1}$	$m_{i,2}$	$m_{i,3}$	$m_{i,4}$
Square ($6 \cdot l_t$)	1	36.8	17.9	21.17	12.20
Square ($7 \cdot l_t$)	2	48.5	21.2	25.17	13.03
Square ($8 \cdot l_t$)	3	64.53	24.97	29.26	13.28
Circle ($4.5 \cdot l_t$)	4	64.35	23.35	26.93	11.28
Pentagon ($6.5 \cdot l_t$)	5	73.37	26.27	30.46	12.65

Table 5. The domains of the accepted values for each parameter when $\varepsilon_{i,j} = 2\sigma_{i,j}$

Shape	i	$DAV_{i,1}^{(2)}$	$DAV_{i,2}^{(2)}$	$DAV_{i,3}^{(2)}$	$DAV_{i,4}^{(2)}$
S_1 ($6 \cdot l_t$)	1	30.22-43.38	14.71-21.09	19.44-22.9	10.64-13.76
S_2 ($7 \cdot l_t$)	2	45.29-52.10	16.87-25.53	23.13-27.21	11.23-14.83
S_3 ($8 \cdot l_t$)	3	59.28-69.77	19.61-30.32	26.55-31.96	11.55-15.22
S_4 ($4.5 \cdot l_t$)	4	61.58-67.12	21.11-25.59	26.37-27.50	10.76-11.79
S_5 ($6.5 \cdot l_t$)	5	70.11-76.62	25.23-27.3	29.34-31.59	11.86-13.45

The regions of the accepted tolerance for each model are represented by the domains of the accepted values for each parameter ($DAV_{i,j}^{(2)}, DAV_{i,j}^{(3)}$), specified in Tables 5 and 6:

- $DAV_{i,j}^{(2)}$, $i = \overline{1,5}$, $j = \overline{1,4}$ if $\varepsilon_{i,j} = 2\sigma_{i,j}$;
- $DAV_{i,j}^{(3)}$, $i = \overline{1,5}$, $j = \overline{1,4}$ if $\varepsilon_{i,j} = 3\sigma_{i,j}$.

Table 6. The domains of the accepted values for each parameter when $\varepsilon_{i,j} = 3\sigma_{i,j}$

Shape	i	$DAV^{(3)}_{i,1}$	$DAV^{(3)}_{i,2}$	$DAV^{(3)}_{i,3}$	$DAV^{(3)}_{i,4}$
S_1 $(6 \cdot l_t)$	1	26.94-46.66	13.11-22.68	18.58-23.76	9.86-14.54
S_2 $(7 \cdot l_t)$	2	43.59-53.81	14.70-27.69	22.12-28.23	10.34-15.73
S_3 $(8 \cdot l_t)$	3	56.66-72.40	16.93-33.00	25.20-33.31	10.38-16.19
S_4 $(4.5 \cdot l_t)$	4	60.19-68.51	19.99-26.71	26.08-27.78	10.51-12.05
S_5 $(6.5 \cdot l_t)$	5	68.48-78.25	24.71-27.82	28.78-32.15	11.46-13.84

Different situations can appear in the shape recognition based on the maximum weighted distance.

a) Two shapes are considered *different* if one or more parameters have distinct domains $DAV^{(3)}_{i,j}$. Certain recognition is obtained and only 0.28% of all possible imprints of an unknown shape, identical with a prototype, are not correctly identified in these situations. For example the circle with $4.5 \cdot l_t$ the radius and the pentagon with $6.5 \cdot l_t$ the side can be considered different shapes because $DAV^{(3)}_{4,1} \neq DAV^{(3)}_{5,1}$ and $DAV^{(3)}_{4,3} \neq DAV^{(3)}_{5,3}$.

b) Two shapes (i and k) are considered *little similar* if their domains $DAV^{(3)}_{i,j}$ are not distinct for any characteristic parameter and at least one parameter (q) has distinct associated domains $DAV^{(2)}_{i,q}$ and $DAV^{(2)}_{j,q}$. For example the squares, with $6 \cdot l_t$ and $7 \cdot l_t$ respectively the side, that have $DAV^{(2)}_{1,q} \neq DAV^{(2)}_{2,q}$ for $q = 1$ and $q = 3$. The recognition is certain, but 4.54% of all possible imprints of an unknown shape, identical with a prototype, are not correctly identified.

c) Two shapes are *similar* if only the domains of the accepted values for $\varepsilon_{i,j} = \sigma_{i,j}$ are distinct for at least one parameter. In this situation the recognition is also certain, but 31.72% of all possible imprints of an unknown shape, identical with a prototype m_i, are outside the region of the accepted tolerance. For example the circle with $4.5 \cdot l_t$ the radius and the square with $8 \cdot l_t$ the side are similar. The domains of their accepted values $DAV^{(1)}_{i,j}$, $i = 3,4$, $j = \overline{1,4}$ for $\varepsilon_{i,j} = \sigma_{i,j}$ are specified in Table 7.

Table 7. The domains of the accepted values for S_3 and S_4, when $\varepsilon_{i,j} = \sigma_{i,j}$

Shape	i	$DAV^{(1)}_{i,1}$	$DAV^{(1)}_{i,2}$	$DAV^{(1)}_{i,3}$	$DAV^{(1)}_{i,4}$
S_3 $(8 \cdot l_t)$	3	61.91-67.15	22.30-27.64	27.91-30.61	12.31-14.25
S_4 $(4.5 \cdot l_t)$	4	62.96-65.74	22.23-24.47	26.65-27.21	11.02-11.54

d) Two shapes are *very similar* if they have not distinct domains of the accepted values for $\varepsilon_{i,j} = 3\sigma_{i,j}$, $\varepsilon_{i,j} = 2\sigma_{i,j}$ or $\varepsilon_{i,j} = \sigma_{i,j}$, for any characteristic parameter.

An improvement of the shape discrimination in tactile recognition can be obtained if more parameters describe each shape and the active surface of the sensory cell decreases.

4. CONCLUSIONS

By comparing the results obtained using a classical distance or the maximum weighted distance, and various reference distances in the shape recognition, the following conclusions result:

- A proper reference distance is very difficult to find because a value too small sometimes determines the rejection of the imprints that correspond to shapes identical with some learned prototypes, and the shape discrimination capability decreases if the value of this distance is too large.
- When a prototype presents very different spreads for two or more parameter value domains, a classical distance determines a region of the accepted tolerance too large and thus, shapes different from the i prototype are falsely identified with this.
- The maximum weighted distance is proper for estimating the similarity between an unknown shape and a prototype if most imprints of the i prototype are known, one or more parameters have value domains distinct and very near and if there are parameters with very different spreads of the value domains.
- If the unknown shape x is an unknown imprint of the i prototype, x is not identified with m_i if the recognition is based on the maximum

weighted distance and the definition (12) of the reference distance.

- If the components $\varepsilon_{i,j}$ of the accepted tolerance vector are based on the standard deviations and $\varepsilon_{i,j} = 3\sigma_{i,j}$, only 0.28% of all possible imprints of a prototype are outside the associated region of the accepted tolerance; so, in the shape identification, the number of the false decisions decreases.
- The shape discrimination capability increases if more parameters assure the shape description.

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