NONLINEAR SYSTEMS IDENTIFICATION USING THE VOLTERRA MODEL

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Abstract: Nonlinear adaptive filtering techniques are widely used for the nonlinearities identification in manny applications. This paper investigates the performances of the Volterra estimator by considering a nonlinear system identification application. The Volterra estimator parameters are compared with those of a linear estimator. For the nonlinear estimator, based on a second order RLS Volterra filter, a new implementation is proposed. The experimental results show that the proposed Volterra identifier has better performances than the linear one. *Different degrees of nonlinearity for the nonlinear system are considered*.
Keywords: Recursive least squares algorithm, nonlinear estimator, linear estimator, Volterra filter.

1. INTRODUCTION

Detection, representation and identification of nonlinearities in different telecommunication systems represent important tasks in many applications and had a major contribution to the development of the main nonlinear modeling techniques. The current trend in the telecommunication systems design is the identification and compensation of unwanted nonlinearities. It was demonstrated that unwanted nonlinearities in the system will have a determinant effect on his performance (Tsimbinos, 1995). There are various ways of reducing the effects of undesired nonlinearities (Stenger, 2000 a,b; Küch, 2002). The use of nonlinear models considered in this paper to characterize and compensate harmful nonlinearities offer a possible solution. The Volterra series have been widely applied as nonlinear system modeling technique with considerable success. However, at present, none general method exists to calculate the Volterra kernels for nonlinear systems, although they can be calculated for systems whose order is known and finite. When the nonlinear system order is unknown, adaptive methods and algorithms are widely used for the Volterra kernel estimation. The accuracy of the Volterra kernels will determine the accuracy of the system model and the accuracy of the inverse system used for compensation. The speed of kernel estimation process is also important. A fast kernel estimation method may allow the user to construct a higher order model that give an even better system representation.
This paper investigates the performances of the Volterra estimator by considering a nonlinear system identification application. The Volterra estimator parameters are compared with those of a linear estimator. The nonlinear estimator is based on a second order RLS Volterra filter.
A new implementation of the second order RLS Volterra filter based on the extended input vector and on the extended filter coefficients vector is also proposed. Due to the linearity of the input-output relation of the Volterra model with respect to filter coefficients, the implementation of the RLS algorithm was realized as an extension of the RLS algorithm for linear filters.

2. THE VOLTERRA MODEL

This section will discuss some important aspects of the second order Volterra model. For a discrete-time and causal nonlinear system with memory, with input x[n] and output y[n], the Volterra series expansion is given by (Schetzen, 1980) :

$$y[n]=\sum_{r=1}^{N}\sum_{k_1=0}^{M-1}\Lambda\sum_{k_r=0}^{M-1}h_r[k_1,\Lambda\ k_r]x[n-k_1]\Lambda\ x[n-k_r] \qquad (1)$$

where N represent the model nonlinearity degree.
Choosing $N=2$, the input-output relationship of the second order Volterra filter (FV_2) can be expressed as:

$$\begin{aligned} y[n]=h_0+\sum_{k_1=0}^{M-1}h_1[k_1]x[n-k_1]+ \\ \sum_{k_1=0}^{M-1}\sum_{k_2=0}^{M-1}h_2[k_1,k_2]x[n-k_1]x[n-k_2] \end{aligned} \qquad (2)$$

The input-output relation can also be written in terms of nonlinear operators as indicated in relation (3).

$$y[n]=H_0[x[n]]+H_1[x[n]]+H_2[x[n]] \qquad (3)$$

In the above representations, the functions $h_i, i=\overline{0,2}$ represent the kernels associated to the nonlinear operators $H_i[x[n]]$.

The nonlinear model described by the relations (2) and (3) is called a second order Volterra model. Note that the above representations has the same memory for all nonlinearity orders. In the most general case the relation (1) may used different memory for each nonlinearity order. A further simplification can be made to relation (1) by considering symmetric Volterra kernels.

The kernel $h_i[k_1,\Lambda\ k_i]$ is symmetric if the indices can be interchanged without affecting its value. The second order Volterra kernel is a *(M×M)* matrix. If we consider symmetric kernels of memory M, the second order Volterra kernel requires the determination of *M(M+1)/2* coefficients as indicated in relation (4).

$$y_2[n]=\sum_{k_1=0}^{M-1}\sum_{k_2=k_1}^{M-1}h_2[k_1,k_2]x[n-k_1]x[n-k_2] \qquad (4)$$

The kernel estimation accuracy becomes the major problem in the practical applications. It was shown that the Volterra operators are homogeneous and generally not orthogonal. As a consequence of this last characteristic the Volterra kernels can not be measured using the cross correlation techniques and the values of the Volterra kernels will depend on the order of the Volterra representation being used. If the order of the Volterra model is changed the Volterra kernels will change and they must be recalculated (Schetzen, 1980; Budura and Nafornita 2002; Budura and Botoca, 2002a). However, for an input having a symmetric amplitude density function, such as the Gaussian noise, the odd order Volterra functionals are orthogonal to the even order Volterra functionals. It follows that for this type of input, a 2nd order Volterra model, with zero DC component, is an orthogonal model (Budura and Botoca, 2005). This leads to direct Volterra kernel measurement by the cross-correlation methods.

In practical situations when confronted with system which parameters are time varying, adaptive methods for kernels estimation are widely used. Due to the linearity of the input-output relation according to the kernels, or filter coefficients, the application of adaptive algorithms for the Volterra filters implementation is quite simple. The nonlinearity is reflected only by multiple products between the delayed versions of the input signal.

The nonlinear estimator proposed in the article is based on a second order Volterra filter (FV_2) describbed by relation (2)-(4) and considering $h_0=0$. The filter structure is indicated in Fig.1.

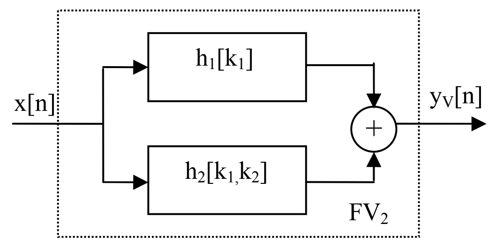

Fig. 1. The second order Volterra filter structure

Next we will introduce the input vectors corresponding to different orders kernels. The first order input vector is defined as follows:

$$X_1=[x[n]x[n-1]\Lambda\ x[n-M+1]] \qquad (5)$$

„The second order input vector" can be expressed by:

$$X_2=X_1^{'}*X_1 \qquad (6)$$

When we deal with symmetric Volterra kernels only *M(M+1)/2* elements of the *MxM* matrix indicated in relation (6) are used in the input output relation (2). Based on this observation we introduced the extended input vector for the second order Volterra filter as indicated in relation (7):

$$\begin{aligned} X=\Big[x[n]\Lambda\ x[n-M+1]x^2[n]x[n]x[n-M+1] \\ \Lambda\ x^2[n-M+1]\Big] \end{aligned} \qquad (7)$$

and the extended filter coefficients vector according to (8).

$$H=[h_0\Lambda\ h_{M-1}h_{00}h_{01}\Lambda\ h_{M-2M-1}h_{M-1M-1}] \qquad (8)$$

3. RLS VOLTERRA ESTIMATOR

Adaptive methods and algorithms are widely used for the purpose of kernel estimation. A Volterra filter of fixed order and fixed memory adapts to the unknown nonlinear system using one of the various adaptive algorithms. The use of adaptive techniques for Volterra kernel estimation has been well published. Most of the previous work considers 2nd order Volterra filters and some consider the 3rd order case.

A simple and commonly used algorithm uses an LMS adaptation criterion (Mathews, 1991; Budura and Botoca, 2002b; Budura and Botoca 2004). Adaptive Volterra filters based on the LMS adaptation algorithm are computational simple but suffer from slow and input signal dependent convergence behavior and hence are not useful in many applications. The aim of this section is to discuss the efficient implementation of the RLS adaptive algorithm on a second order Volterra filter.

As in the linear case the adaptive nonlinear system minimizes the following cost function at each time:

$$J(n)=\sum_{k=0}^{n}\lambda^{n-k}\left(y(k)-H^{T}(n)X(k)\right)^{2} \tag{9}$$

where $H(n)$ and $X(n)$ are the coefficients and the input signal vectors, respectively, as defined in (8) and (7), λ is a factor that controls the memory span of the adaptive filter and $y(k)$ represents the desired output. The solution of equation (8) can be obtained recursively using the RLS algorithm.

Based on the second order Volterra filter presented in Section 2 we have implemented the RLS nonlinear estimator. The filter coefficients are adapted according to the following steps:

I. Initialization:

-define the filter memory(length for H(n) and X(n));

$H(0)=[0\ 0\ \Lambda\ \ 0]$;

$C_{XX}(0)=\delta * I$;

where δ is a small positive constant;

II. Operations: for $n=\overline{1,\text{nr. of iterations}}$

1. Create the extended input vector:

$X(n)$;

2. Compute the error:

$e(n/n-1)=d(n)-H(n-1)*X^{'}(n)$;

3. Compute the scalar:

$\mu(n)=X(n)*C_{XX}(n-1)*X^{'}(n)$;

4. Compute the matrix:

$G(n)=\left(C_{XX}(n-1)*H^{'}(n-1)\right)/(\lambda+\mu)$;

5. Updates the filter vector:

$H(n)=H(n-1)+e(n/n-1)*G^{'}(n)$;

6. Updates the matrix C_{XX} :

$C_{XX}(n)=\lambda^{-1}*(C_{XX}(n-1)-G(n)*X(n)*C_{XX}(n-1))$;

In the relations above C_{XX} denotes the inverse autocorrelation matrix of the extended input signal. Inversion was done according to the matrix inversion lemma (Morthensen, 1987).

4. EXPERIMENTS AND RESULTS

The RLS Volterra estimator performances were studied and compared with the linear estimator performances in a typical nonlinear system identification application presented in figure 2.

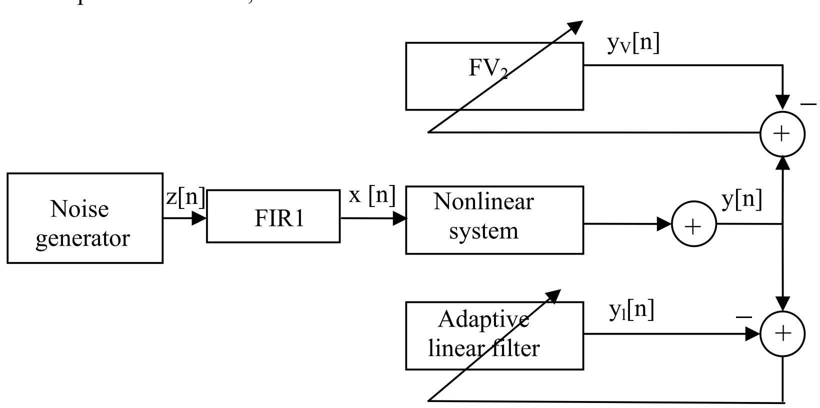

Fig. 2. The nonlinear system identification

The nonlinear system with memory being identified is represented in figure 3 and consists of a linear filter(FIR2) with impulse response given by:

$$h[n] = 2^{-(n+1)} \quad 0 \le n \le M_1 - 1 \tag{10}$$

followed by a nonlinear system without memory which input-output relation is:

$$y[n] = u[n] + bu^2[n] \tag{11}$$

The linear filter memory in relation (10) is $M_1=10$. The coeficient b permits to change the nonlinearity degree.
The input signal x[n] is generated by colouring the white Gaussian sequence z[n], with the autoregresive filter (FIR1) described by:

$$x[n] = x[n-1] - 0{,}9x[n] + 0{,}5z[n] \tag{12}$$

Experiments have been done regarding the identification of a second order nonlinear system with different degrees of nonlinearity.

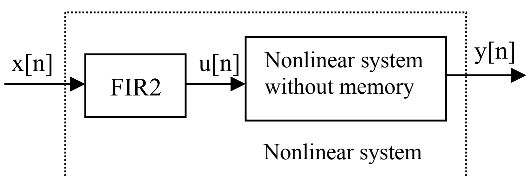

Fig. 3. The nonlinear system structure

The performance of the RLS adaptive Volterra filter was appreciated by comparing the error of the nonlinear identifier with the error of a linear identifier.
The same memory M=10 has been chosen for the linear and for the second order Volterra kernel according to the relation (2). For the linear identifier we fixed the same memory. The factor λ was chosen equal to 0,995.
The simulations have been done in the MATLAB software.
The following degrees of nonlinearity are considered:
-Nonlinear system identified: b=0,01;
The error using the RLS Volterra identifier is indicated in Fig.4.

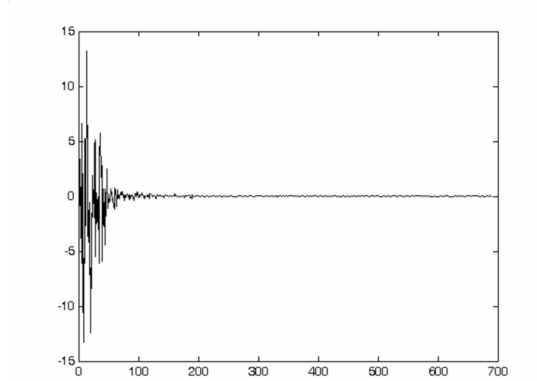

Fig.4. The error of the RLS Volterra identifier $b = 0.01$

The error using the linear adaptive filter as identifier is presented in Fig.5.

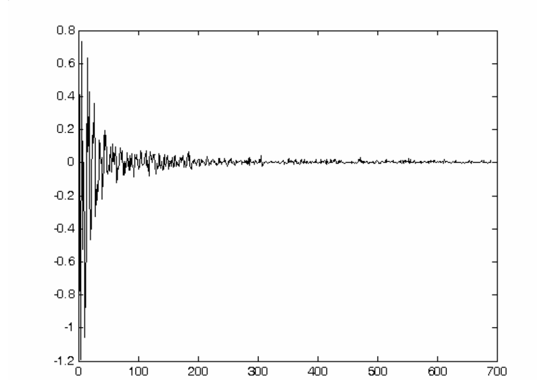

Fig.5. The error of the linear identifier $b = 0{,}01$

-Nonlinear system identified: b=0,1;
The error using the RLS Volterra identifier is indicated in Fig.6 and the error using the linear identifier is depicted in Fig.7.

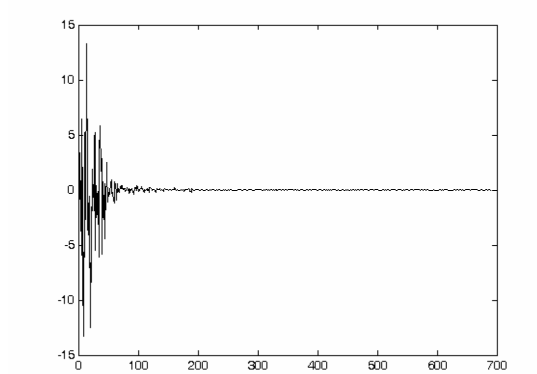

Fig.6. The error of the RLS Volterra identifier $b = 0.1$

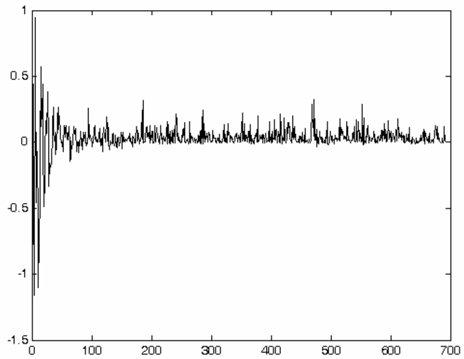

Fig.7. The error of the RLS linear identifier $b = 0.1$

-Nonlinear system identified: b=1;
The error signal using the RLS Volterra identifier is indicated in Fig.8 and the error using the linear identifier is depicted in Fig.9.

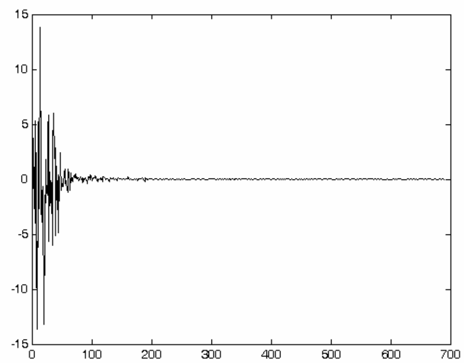

Fig.8. The error of the RLS Volterra identifier $b = 1$

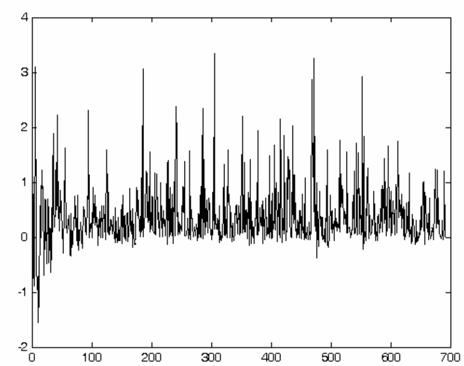

Fig. 9. The error of the RLS linear identifier $b = 1$

The mean and the dispersion of the error signals were calculated too, in order to characterize the performances of both identifiers. The corresponding values are indicated in Table 1. As it can be seen the nonlinear identifier performances (m_{eV}, σ_{eV}) are very good for different nonlinearity degrees, while the linear identifier performances (m_{el}, σ_{el},) are unsatisfactory when the nonlinearity increases.

Table 1 The identifiers parameters

b	m_{el}	σ_{el}	m_{eV}	σ_{eV}
0,01	$35 \cdot 10^{-4}$	$65 \cdot 10^{-4}$	$6,3 \cdot 10^{-5}$	$40 \cdot 10^{-4}$
0,1	0,03	0,0618	$6,5 \cdot 10^{-5}$	$41 \cdot 10^{-4}$
1	0,3531	0,6175	$7,5 \cdot 10^{-5}$	0,0042

5. CONCLUSIONS

The identification and compensation of unwanted nonlinearities are required in manny practical application in order to improve the system performances. Usualy this are week nonlinearities and can be well represented with the Volterra model. This paper investigates the performances of the Volterra estimator by considering a nonlinear system identification application. The nonlinear identifier is based on the RLS Volterra filter. A new implementation of the second order RLS Volterra filter based on the extended input vector and on the extended filter coefficients vector is also proposed. Due to the linearity of the input-output relation of the Volterra model with respect to filter coefficients, the implementation of the RLS algorithm was realized as an extension of the RLS algorithm for linear filters.
For simplicity, there were considered only second order nonlinearities, but the proposed technique can be extented to higher order nonlinearities.
The nonlinear adaptive filter performances were evaluated in a typical system identification application and compared with the performances of a linear identifier. The experimental results showed that the RLS Volterra filter performed better than the linear filter which performances were inacceptable when the nonlinearity degree was increased.
The costs of these performances are paid by the computational complexity required by the nonlinear adaptive Volterra filter implementation .

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