OPTIMAL CONTROL BY ENERGETIC CRITERIUM OF DRIVING SYSTEMS ACCELERATION WITH STATIC TORQUE FUNCTION OF SPEED

- SOLUTION THROUGH DIGITAL SIMULATION -

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Abstract: In the paper we consider an electric drive with a static load torque function of speed, in the hypothesis of constant inertia moment and of proportionality between the electromagnetic torque and the load current. Using the variational calculus, the optimality condition is determined which ensures the least energy losses caused by the load current through a Joule effect in the acceleration processes. By the digital simulation of nonlinear differential equation (optimality condition) we obtain the extremal trajectory and extremal control for the electric drive system.©

Keywords: electric drive, optimal control.

1. INTRODUCTION

In the case of drives that work in continuous type service (S1) appears the necessity of achieving starting and braking processes and in the case of those that work in uninterrupted type service with periodical change of speed (S8) appears the necessity of achieving speed variation. To evaluate these processes of acceleration and deceleration, the minimum of energy losses may be adopted as a quality index, and the solving of this optimisation problem can be obtained by using the classical variational calculus or Euler – Lagrange algorithm and numerical computer.

2. MATHEMATICAL MODEL

Considering an electrical drive with static load torque having a constant component, a speed proportional component and a squared speed proportional component

$$M_S = M_0 + k_1\omega + k_2\omega^2 \qquad (1)$$

that, in the hypothesis of neglecting the electromagnetic inertia as compared to the mechanical inertia, and a constant moment of inertia, this driving system will be described by the general motion equation (Boteanu N. and Degeratu P, 2000), (Degeratu P. and Boteanu N., 2000)

$$M = M_S + J\,d\omega/dt \qquad (2)$$

and by the dependence between speed and acceleration (Degeratu P. and Boteanu N., 2000)

$$\omega = \int \dot{\omega} dt . \qquad (3)$$

To extend the interpretations and the conclusions as well as to restraint the value intervals, relative coordinates will be used (Degeratu P. and Boteanu N., 2000), (Degeratu P. and Săvulescu N.C., 1997). By this way, considering as a reference for time, the mechanical time constant

$$T = \frac{J\omega_N}{M_N} \qquad (4)$$

and for current, torque and speed their nominal values, relative values are obtained

$$\tau = \frac{t}{T}, \quad i = \frac{i}{I_N}, \quad \mu = \frac{M}{M_N}, \quad \nu = \frac{\omega}{\omega_N}, \quad \mu_s = \frac{M_S}{M_N}, \mu_0 = \frac{M_0}{M_N}, k_1 = \frac{k_1\omega_N}{M_N}, k_2 = \frac{k_2\omega_N^2}{M_N} \tag{5}$$

resulting for the relative acceleration following relation

$$\dot{\nu} = \frac{\dot{\omega}}{\omega_N / T} \tag{6}$$

In the hypothesis of proportionality between the electromagnetic torque and the load current, the equations (1), (2) and (3) in the relative coordinates have the forms (Degeratu P. and Săvulescu N.C., 1997)

$$\mu_S = \mu_0 + k_1\nu + k_2\nu^2, \qquad \mu_0 + k_1 + k_2 = 0, \tag{7}$$

$$i = \mu = \mu_S + \dot{\nu} = \mu_0 + k_1\nu + k_2\nu^2 + \dot{\nu}, \tag{8}$$

$$\nu = \int \dot{\nu} d\tau \tag{9}$$

with the initial and terminal conditions

$$\tau = \tau_1, \quad \nu(\tau_1) = \nu_1, \quad \tau = \tau_2, \quad \nu(\tau_2) = \nu_2. \tag{10}$$

that conducts the construction of the structural block diagram depicted in fig. 1.

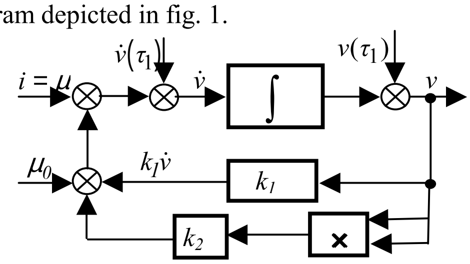

Fig.1 Structural block diagram of electric drive

The admissible controls set and the admissible trajectories set are considered limited and open sets.

3. OPTIMIZATION CRITERION

The valuation of acceleration and deceleration process will be done considering the quality index of energy losses minimization caused by the load current through Joule effect, during those processes, expressed by the integral (Degeratu P. and Săvulescu N.C., 1997)

$$\Delta w = \int_{\tau_1}^{\tau_2} \Delta p \, d\tau = \int_{\tau_1}^{\tau_2} \rho \, i^2 d\tau \tag{11}$$

In this case, taking into account the general motion equation (8), the optimization functional-criterion has the expression (Degeratu P. and Săvulescu N.C., 1997), (Petrov Iu. P., 1961).

$$J[\nu(\tau)] = \int_{\tau_1}^{\tau_2} i^2 d\tau = \int_{\tau_1}^{\tau_2} (\mu_S + \dot{\nu})^2 \, d\tau . \tag{12}$$

4. FORMULATION OF OPTIMIZATION PROBLEM

The optimization problem consist in determining the admissible optimal control function $i^*(\tau)$ or $\mu^*(\tau)$, witch is able to transfer the system from the initial conditions (τ_1, $\nu(\tau_1)$) to the terminal conditions (τ_2, $\nu(\tau_2)$), on an the admissible extremal trajectory $\nu^*(\tau)$, ensuring the minimum of the optimality criterion (Degeratu P. and Boteanu N., 2000).

$$J[\nu(\tau)] = \int_{\tau_1}^{\tau_2} (\mu_s + \dot{\nu})^2 \, d\tau = min \tag{13}$$

for a fixed value of speed variation, expressed by the integral

$$\Delta\nu = \nu_2 - \nu_1 = \int_{\tau_1}^{\tau_2} \dot{\nu}(\tau) d\tau, \tag{14}$$

variation that is realized in a given interval of time

$$\tau_2 - \tau_1 = \int_{\tau_1}^{\tau_2} 1 \, d\tau , \tag{15}$$

and satisfying the restrictions

$$|i(\tau)| < i_{max}, \qquad |\mu(\tau)| < \mu_{max}, \qquad |\nu(\tau)| < \nu_{max}, \qquad |\dot{\nu}(\tau)| < \dot{\nu}_{max}. \tag{16}$$

In conformity with the principle of reciprocity (Petrov Iu. P., 1961), the given formulation is equivalent with the formulation through which every isoperimetric condition (14) and (15) can become optimization criterion ($\nu_2 - \nu_1$ = max, τ_2-τ_1 = min) or a linear combination of them. So, it results a linear – quadratic optimization problem of isoperimetric extremum (Lavrentiev M.A. and Liusternik L.A, 1955). To solve the issued problem, the primal problem of conditional extremum will be reduced to a dual problem of unconditional extremum by a Lagrange adjoin function based on Lagrange multiplier λ_0 (Lavrentiev M.A. and Liusternik L.A, 1955), (Petrov Iu. P., 1961).

$$L = (\mu_S + \dot{\nu})^2 + \lambda_0\dot{\nu} = (\mu_0 + k_1\nu + k_2\nu^2 + \dot{\nu})^2 + \lambda_0\dot{\nu} \tag{17}$$

and by determining the unconditioned extremum with the functional

$$J[\nu(\tau)] = \int_{\tau_1}^{\tau_2} L(\tau) d\tau = min \tag{18}$$

on the same extremals as those of the primal problem (Lavrentiev M.A. and Liusternik L.A, 1955), (Petrov Iu. P., 1961).

5. NECESSARY CONDITION OF EXTREMUM

The necessary condition of extremum is expressed by the Euler – Lagrange equation (Lavrentiev M.A. and Liusternik L.A, 1955)

$$\frac{\partial L}{\partial v} - \frac{d}{d\tau}\frac{\partial L}{\partial \dot{v}} = 0 \qquad (19)$$

that leads to the nonlinear differential equation of the second order

$$\ddot{v} - 2k_2^2 v^3 - 3k_1 k_2 v^2 - \left(k_1^2 + 2k_2\mu_0\right) v = k_1\mu_0 \quad (20)$$

Because the condition of extremum expressed by the differential equation (20) does not contains Lagrange multiplier λ_0, it corresponds to that which might result in the case that Euler – Lagrange equation would be applied directly for the functional of the criterion (13).

6. OPTIMAL SOLUTION THROUGH DIGITAL SIMULATION

If the load static torque has a constant component, a speed proportional component and a squared speed proportional component

$$\mu_0 \neq 0, \quad k_1 \neq 0, \quad k_2 \neq 0, \quad \mu_0 + k_1 + k_2 = 1 \quad (21)$$

the static torque has the expression

$$\mu_s = \mu_0 + k_1 v + k_2 v^2 , \qquad (22)$$

and the necessary condition of optimality expressed by the nonlinear differential equation (20) implies some difficulties for an analytical solution. For this reason, we use the solving through numerical simulation with the aid of the computer on the basis of the structural block diagram (fig. 2) as a optimisation problem, either bilocal or with arbitrary and finite terminal moment (Popescu M.C., 2002).

Numerical simulation solving as a bilocal optimization problem, consist in the introducing into the block diagram (fig. 2) the triplet of values that are characteristic to the load static torque (μ_0, k_1 and k_2), fixing the initial condition of acceleration and the variation interval of speed ($v(\tau_1)$, $v(\tau_2)$), after that, is determined, the evolution in time of extremal trajectory (speed $v^*(\tau)$, acceleration $\dot{v}^*(\tau)$, and shock $\ddot{v}^*(\tau)$), static load torque $\mu_s(\tau)$ and optimal control function (electromagnetic torque of motor $\mu^*(\tau)$) and load current $i^*(\tau)$) is determined. In this case, a certain interval of time (τ_2- τ_1) and a certain minimal value of the optimality criterion will result, corresponding to the initial value of acceleration $\dot{v}(\tau_1)$.

Simulation solving with arbitrary and finite terminal moment consists in determining of the time interval (dependent of the initial acceleration) necessary to the evolution of the system which ensure absolute minimum of the optimisation criterium. For this reason, for simulation the following steps wild be followed:

- First step: we introduce into the block diagram (fig. 2) the triplet of values, that on characteristic to the static torque (μ_0, k_1 and k_2) and we fix the speed variation interval, that is the initial value v_1 and the terminal value v_2;
- Second step: introducing different values of the initial acceleration $\dot{v}(\tau_1)$, into a certain value interval, through integration up to the terminal value of speed, we obtain the graphs of relations

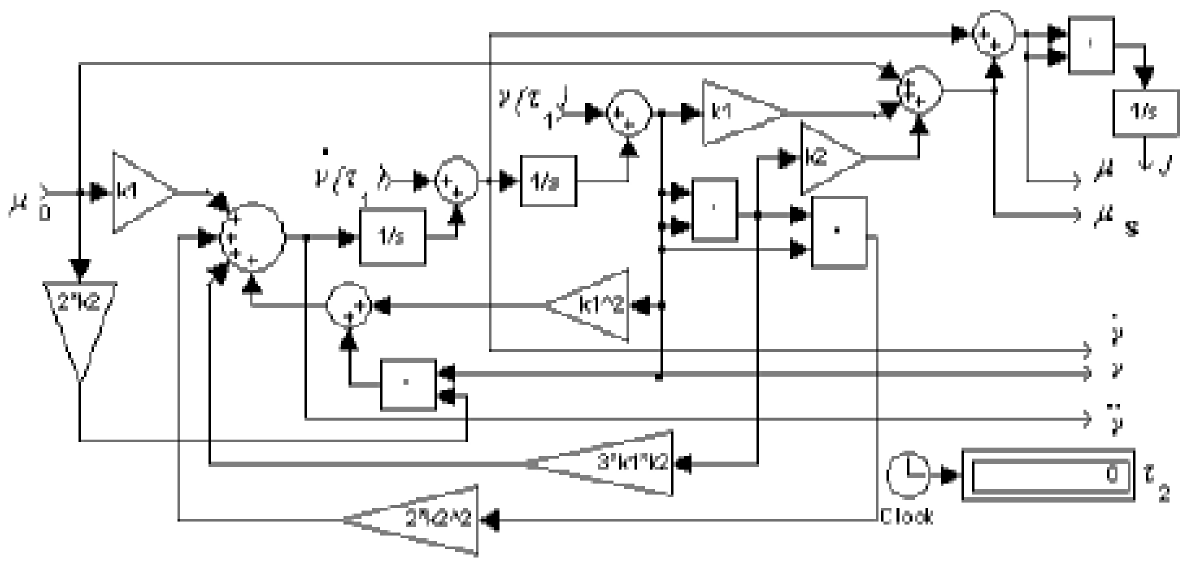

Fig. 2 Block diagram of simulation in Matlab - Simulink

(dependences) between the minimum optimization criterion J_{min} and the initial acceleration $\dot{v}(\tau_1)$ (fig. 3, fig. 5)

$$Jmin = f\left[\dot{v}\left(\tau_1\right)\right], \tag{23}$$

and between the terminal moment τ_2 and the initial acceleration $\dot{v}(\tau_1)$ (fig. 4, fig. 5);

$$\tau_2 = f\left[\dot{v}\left(\tau_1\right)\right] \tag{24}$$

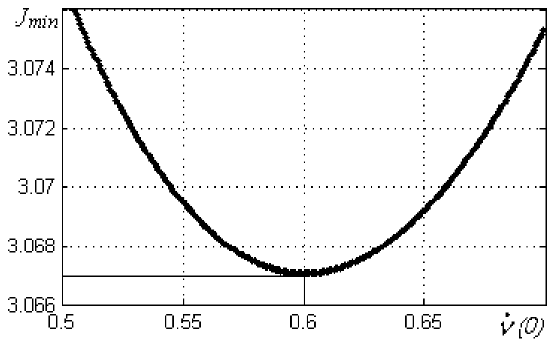

Fig. 3. Minimum value of optimization criterion as function of initial acceleration during acceleration *(μ_0=0.6, k_1=0.2, k_2=0.2, ν_1=0 şi ν_2=1)*

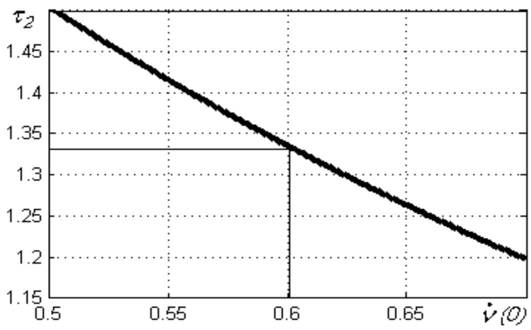

Fig. 4. Terminal moment as function of initial acceleration during acceleration with *(μ_0=0.6, k_1=0.2, k_2=0.2, ν_1=0 şi ν_2=1)*

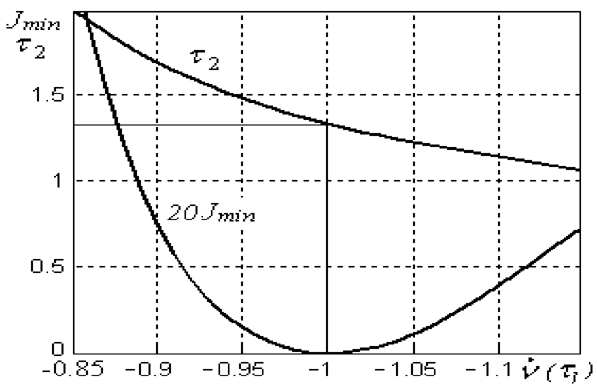

Fig. 5. Graphic of relations $J_{min} = f[\dot{v}(\tau_1)]$ and $\tau_2 = f[\dot{v}(\tau_1)]$ during deceleration (μ_0=0.6, k_1=0.2, k_2=0.2, ν_1=1 şi ν_2=0)

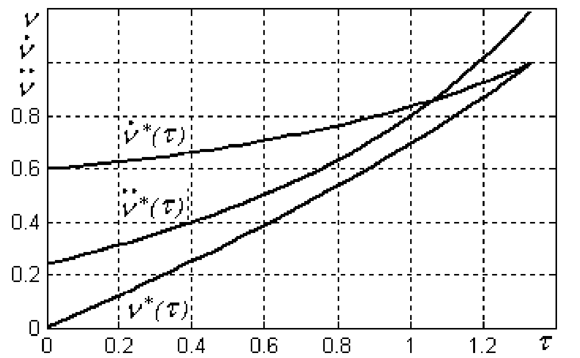

Fig. 6 Evolution of extremal trajectory (speed, acceleration and shock) during acceleration (μ_0=0.6, k_1=0.2, k_2=0.2 şi $\dot{v}$ (0)=0.6)

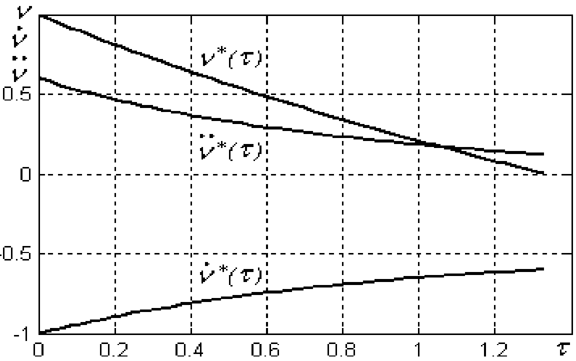

Fig.7 Evolution of static torque and optimal control (load current, motor torque and dynamic torque) during acceleration (μ_0=0.8, k_1=0.2, k_2=0.2 şi $\dot{v}$ (0)=-1)

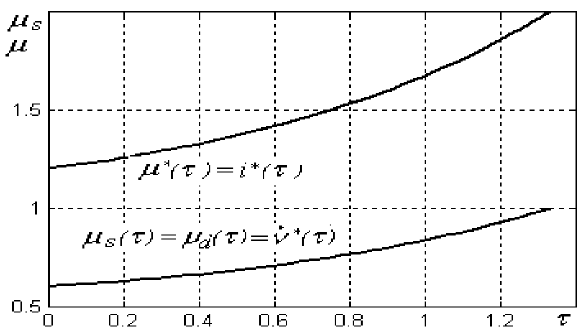

Fig. 8 Evolution of extremal trajectory (speed, acceleration and shock) during deceleration (μ_0=0.6, k_1=0.2, k_2=0.2, $\dot{v}$ (0)=0.6)

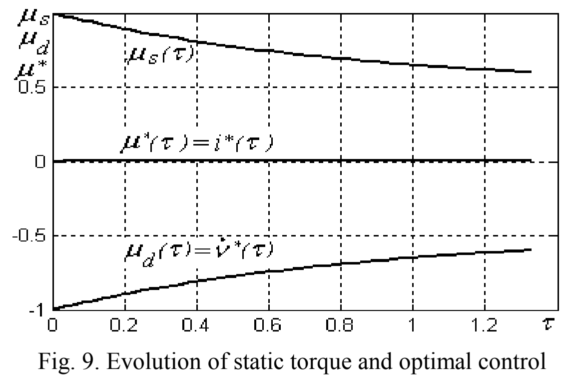

Fig. 9. Evolution of static torque and optimal control (load current, motor torque and dynamic torque) during deceleration (μ_0=0.6, k_1=0.2, k_2=0.2 şi $\dot{v}$ (0)=-1)

- Third step: from this graphs (fig. 3, fig. 4, fig. 5), we determine the optimum value of the initial acceleration $\dot{v}^*(\tau_1)$, and the optimum value of the terminal moment τ_2^*, which ensures the absolute minimum (least minimum) of quality index, for acceleration (fig. 3, fig. 4) and deceleration (fig. 5), respectively;
- Fourth step: introducing the optimal value of initial acceleration $\dot{v}^*(\tau_1)$ in block diagram (fig. 2), through digital simulation (integration), we determine the time function for evolution of extremal trajectory (speed $v^*(\tau)$, acceleration $\dot{v}(\tau)$ and shock $\ddot{v}^*(\tau)$ (fig. 6, fig. 8), static load torque $\mu_s(\tau)$ and the evolutions of optimal control $\mu^*(\tau)$ and load current $i^*(\tau)$, respectively (fig. 7, fig. 9). From the results of this simulation, it is observed that optimal acceleration (dynamic torque) is equal to momentary static load torque value (fig. 7, fig. 9)

$$\dot{v}^*(\tau) = \mu_d(\tau) = \pm\left(\mu_0 + k_1 v + k_2 v^2\right) \quad , \quad (25)$$

having as initial and terminal values

$$\begin{aligned} \dot{v}^*(\tau_1) &= \mu_d(\tau_1) = \pm\left(\mu_0 + k_1 v_1 + k_2 v_1^2\right) , \\ \dot{v}^*(\tau_2) &= \mu_d(\tau_2) = \pm\left(\mu_0 + k_1 v_2 + k_2 v_2^2\right) , \end{aligned} \quad (26)$$

and that the optimal control function is equal to the double of the static load torque

$$\begin{aligned} i^*(\tau) &= \mu^*(\tau) = \mu_s(\tau) + \mu_d(\tau) = \\ &= 2\left(\mu_0 + k_1 v_2 + k_2 v_2^2\right) \end{aligned} \quad (27)$$

during the acceleration period (fig. 7) and it is null during the deceleration period (fig. 9)

$$i^*(\tau) = \mu^*(\tau) = 0 = \mu_s(\tau) + \mu_d(\tau) \quad (28)$$

7. CONCLUSIONS

The block diagram for simulation is general and it can be adapted for different expressions of static torque depending on the speed. The results expressed through obtained extremal trajectory and the optimal control function can be used both for the design and for the optimal control of the electric drive systems with the static torque depending on speed that work in the a continuous type service (S1) or in the uninterrupted type service with the periodical change of speed (S8).

These results, through the energy saving that is achieved in the acceleration and deceleration processes, lead to the increase of quality and efficiency of the electric drives systems.

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