# MODELING AND SIMULATION OF HYDRAULIC SYSTEMS USING BOND GRAPH METHOD

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Abstract: In this paper one present a short description of bond graph method and its application in hydraulic domain. The system modeling using bond graph method is one of the best choices when dealing with systems containing different physical subsystems (mechanical, electrical, hydraulical). One of the great advantages of this method compared to other graphical methods is that it is applicable both to linear and nonlinear systems.

Keywords: Bond graphs, Modeling, Simulation, Hydraulic systems

## 1. COMPUTER AIDED MODELING

The first program created to simulate bond graph models was ENPORT. ENPORT models are represented using a combination of bond graph and block diagram elements that interact using power bonds and signal connectors. For each component can be defined the constitutive equations.
The next program designed for simulation was TUTSIM, developed at Twente University, in the 1970's. It was initially developed for block diagrams and modified to accept causal bond graphs.
The same research group later developed 20-SIM. It is an advanced modeling and simulation package for Windows. This program uses icons for component representation, bond graph, block diagrams as well as differential equations. System model processing is performed using computational causality analysis.
Another program designed for modeling and simulation of dynamic systems with continuous elements is MS1. Models developed in MS1 are solved using ACSL (Advanced Continuous Simulation Language), Matlab-SIMULINK. CAMP-G is a model generating tool that interfaces with programs like Matlab, Simulink, ACSL and others.
A very important language for modeling and simulation is Dymola (Dynamic Modeling Language). It builds hierarchically structured models, but is unable to execute a model, therefore it provides for interfaces for simulation languages such as ACSL, Simulink, or C.
A new simulation tool is BondSim (Damic, 2002). An advantage of this modeling environment is the fact that it supports many levels of decomposition depending on the complexity of engineering system. Thus the model can be created as a multi-level structure.
BondSim's general concept of modeling and simulation is presented in figure 1.

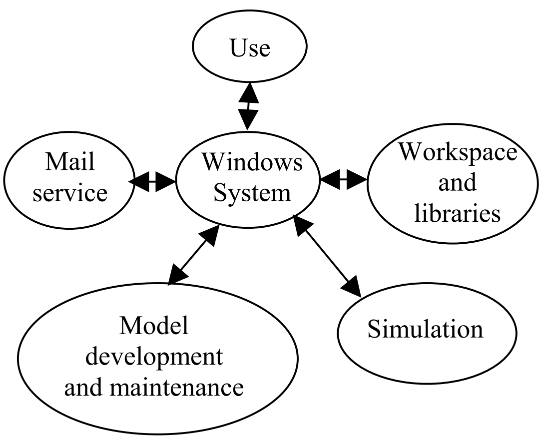
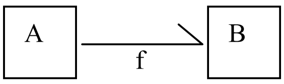



Fig. 1. Visual modeling and simulation environment.

Modeling in BondSim consists of creating objects in the computer memory that are represented on screen as bond graphs.
Models development is done entirely visually, without coding, using the support of the Windows system.
The components models are real objects that serve as interfaces to the document that contains the model.
A component can be saved, loaded, copied, deleted, imported, exported, sent or received by e-mail. After a bond graph model has been developed, its mathematical representation is built in the form of differential algebraic equations that are solved using numerical routines.

## 2. INTRODUCTION TO BOND GRAPHS

The concept of bond graph was first introduced by H. M. Paynter in 1961, and developed by D. Karnopp, R. Rosenberg, Thoma and others in their books. Bond graph is a modeling language that offers a unified approach to the modeling and analysis of dynamic systems, including electrical, mechanical, hydraulic and pneumatic, thermodynamic, chemical, etc. Bond graph models describe the dynamic behavior of physical systems and they are based on the principle of conservation of power.
Bond graph method illustrates the energetic transfer in a system using bond lines. Bond lines are represented by a half arrow.




The orientation of the arrow shows the direction in which power flows.
Every bond has two power variables, the effort *e*, and the flow *f*. Thus, power can be expressed as the product of the effort and flow variables, $P=ef$.
In a bond graph there are two other variables, the generalized momentum *p*, and the generalized displacement *q*, called the energy variables.
In table 1 are presented the power variables and their significations in several energetic domains.
Table 2 presents the generalized variables and the energetic domains in which these variables are often used.

Table 1

| Domain | Effort (e) | Flow (f) |
|---|---|---|
| Electrical | Voltage | Current |
| Mechanical translation | Force | Velocity |
| Mechanical rotation | Torque | Angular velocity |
| Hydraulic | Pressure | Volume flow rate |
| Thermal | Temperature | Heat flow |

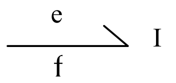
Table 2

| Domain | Momentum (p) | Displacement (q) |
|---|---|---|
| Electrical | Flux linkage | Charge |
| Mechanical translation | Momentum | Displacement |
| Mechanical rotation | Angular momentum | Angle |
| Hydraulic | Pressure momentum | Volume |
| Thermal | - | Heat energy |

The method uses the effort-flow analogy to describe physical processes that are graphically represented using elementary components. These elements have one or more ports. The ports are places where interactions with other processes take place. There are two types of ports, power ports and control ports. Ports characterized by power transfer are power ports. Ports characterized by negligible power transfer, but with a content full of information are control ports. These ports realize power and information transfer between components.
Causality is a fundamental concept in modeling. It presents the dependencies between elements. A bond graph model represents the way in which the power is exchanged between the elements of the system and it permits also the association of “cause” and “effect” notions to power variables. Causality is graphically represented by a short stroke, called causal stroke, placed perpendicular to the bond at one of its ends indicating the direction of the effort variable. Causal stroke assignment is independent of the power flow direction. This leads to the description of bond graphs in terms of state-space equations. After the assignation of causality, it can be easily drawn the block diagrams which are often used to denote input-output relations.
Causality assignment can be made more efficient by the following procedure (Thoma, 1990):

1. Select a source causality
2. Extend causal implications throughout the bondgraph
3. If any source causality remains free *goto* 1, else *goto* 4
4. Assign a desirable causality
5. Extend causal implications
6. If any desirable causality remains free *goto* 4 else *goto* 7
7. Assign a free causality
8. Extend causal implications
9. If any causality remains free, *goto* 7, else causal assignment complete.

One of the great advantages of bond graph is that it deals with only nine elements. Bond graph’s elements are represented by letters describing the type of the element. These elements are:
I – inertial elements, which are one-port elements that store energy, for p-type variable.
Bond graph symbol for an I – element is:



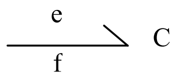

The constitutive relation of the process in linear case is given by:

$$p = I \cdot f \tag{1}$$

In nonlinear case, the constitutive relation is:

$$\varphi_I(p, f) = 0 \tag{2}$$

C – capacitive elements, which are one-port, storing elements, for q-type variable.
Bond graph symbol for an C – element is:



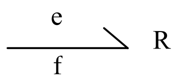

The constitutive relation of the process in linear case is:

$$q = C \cdot e \tag{3}$$

The constitutive relation can be nonlinear:

$$\varphi_C(q, e) = 0 \tag{4}$$

R – resistive elements, which are also one-port elements that dissipate energy.
Bond graph symbol for an R – element is:



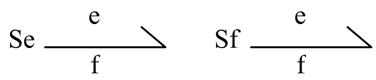

The constitutive relation of the process in linear case is:

$$e = R \cdot f \tag{5}$$

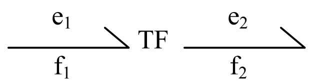
In nonlinear case, the constitutive relation is:

$$\varphi_R(e, f) = 0 \tag{6}$$

It can be seen that the orientation of the half-arrow is pointing towards the elements because the I, C, R - elements are passive and they can only process the received power.
Se, Sf - power source elements, called idealized effort source, and respectively idealized flux source, which are one-port elements. These elements are passive because they supply power to the system. The fundamental property defining a source is that the effort variable (for Se) or flow variable (for Sf) supplied by the source is supposed to be independent of the complementary variable flow (for Se) or effort (for Sf) which depends of the system's characteristics.
Bond graph symbols for Se and Sf elements are:




TF – elements, called transformers, which are two-ports elements that conserve power ($e_1f_1=e_2f_2$). For a transformer, both efforts are proportional, and both flows are proportional.
Bond graph symbol for TF – elements is:




The constitutive relations of this element are:

$$\begin{cases} e_1 = m e_2 \\ f_2 = m f_1 \end{cases} \tag{7}$$

where *m* is called the transformer modulus.
GY – elements, called gyrators, which are also two-ports elements that conserve power. For gyrators the effort in one bond is proportional to the flow in the other.
Bond graph symbol for GY – elements is:


The constitutive relations for gyrators are:

$$\begin{cases} e_1 = r f_2 \\ e_2 = r f_1 \end{cases} \tag{8}$$

J0 - elements, which are multi-port elements. Junction 0 represents a node at which all efforts of the connecting bonds are equal and the algebraic sum of the flows is equal to zero.

$$\begin{cases} e_1 = e_2 = \ldots = e_n \\ \sum_i a_i f_i = 0 \end{cases} \tag{9}$$

where $a_i$=+1 if the half-arrow is pointing towards junction and $a_i$=-1 if not.
J1 – elements, which are also multi-port elements that serve to interconnect elements of bond graph models. Junction 1 is a node at which all flows are equal and the algebraic sum of the efforts is zero.

$$\begin{cases} f_1 = f_2 = \ldots = f_n \\ \sum_i a_i e_i = 0 \end{cases} \tag{10}$$

where $a_i$=+1 if the half-arrow is pointing towards junction and $a_i$=-1 if not.

## 3. MODELING OF HYDRAULIC SYSTEMS

The classical procedure used for the construction of bond graph model of a hydraulic circuit is (Karnopp, 1975):

1. For each distinct pressure it is introduced a 0-junction.
2. It is introduced a 1-junction for each element Se, Sf, I, C, or R, between two 0-junction, and bond lines are drawn to connect these junctions.
3. There are assigned the half-arrows indicating power directions.
4. All reference pressures (usually atmospheric pressure) are identified and the 0-junctions that characterize these junctions are eliminated together with their bonds.
5. The bond graph is simplified.

### *3.1. Hydraulic I elements*

For a volume of section $A(x)$, and length $l$, the kinetic co-energy is given by (Dauphin-Tanguy, 2000):

$$E_I^*(q) = \frac{1}{2}\rho q^2 \int_0^l \frac{dx}{A(x)} \tag{11}$$

Defining the parameter:

$$I = \rho \int_0^l \frac{dx}{A(x)} \tag{12}$$

one obtain the classical expression of kinetic co-energy in hydraulic domain:

$$E_I^*(q) = \frac{1}{2} I q^2 \tag{13}$$

The storage of kinetic energy is supposed to be reversible,

$$\frac{dE_I^*}{dt}(q) = \frac{dE_I^*}{dq} \cdot \frac{dq}{dt} = \Gamma \cdot \frac{dq}{dt} \tag{14}$$

where $\Gamma$ is the generalized momentum in hydraulic domain.
The characteristic relation of this storage phenomenon of kinetic energy has the expression:

$$\Gamma = \frac{dE_I^*}{dq} = \rho \int_0^l \frac{dx}{A(x)} \cdot q = I \cdot q \tag{15}$$

The kinetic energy $E_I(\Gamma)$ in hydraulic domain is equal with the Legendre transformation of kinetic co-energy $E_I^*(q)$:

$$E_I(\Gamma) = \frac{1}{2}\frac{\Gamma^2}{I} \tag{16}$$

Bond graph representation of this storage phenomenon of kinetic energy is given below:

$$\xrightharpoonup[q]{\frac{d\Gamma}{dt}} \; I : \rho \int_0^l \frac{dx}{A(x)}$$

### *3.1. Hydraulic C elements*

Being considered a volume $V$ of fluid comprised in a tank of section $A$, the potential energy due to gravity has the expression:

$$E_C = \int_0^V \rho g \frac{V_1}{A} dV_1 \tag{17}$$

or

$$E_C(V) = \frac{1}{2}\frac{\rho g}{A} V^2 \tag{18}$$

The parameter that characterize the storage of potential energy due to gravity is:

$$C = \frac{A}{\rho g} \tag{19}$$

Thus, the expression of potential energy in hydraulic domain is given by:

$$E_C(V) = \frac{1}{2}\frac{V^2}{C} \tag{20}$$

The power exchanged with the environment of this storage phenomenon is written:

$$\frac{dE_C}{dt} = \frac{dE_C}{dV} \cdot \frac{dV}{dt} = p \frac{dV}{dt} \tag{21}$$

with $p$ being the effort variable.
The characteristic relation between the generalized displacement $V$ and the effort variable $p$ is:

$$p = \frac{dE_C}{dt} = \frac{\rho g}{A} V \tag{22}$$

Bond graph representation is given below:

$$\xrightharpoonup[\frac{dV}{dt}]{p} \; C : \frac{A}{\rho g}$$

*3.3. Hydraulic R elements*

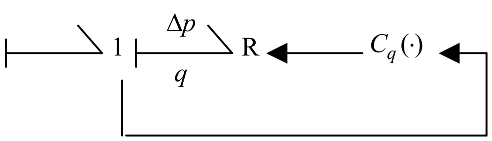
In hydraulic domain, the volume flow rate may be written as a nonlinear function of pressure difference and flow coefficient. The volume flow rate through an orifice is:

$$q = AC_q \sqrt{\frac{2}{\rho}|p_1 - p_2|} \cdot sign(p_1 - p_2) \qquad (23)$$

where: $A$ is the nominal section of the orifice, $C_q$ is the flow coefficient, and $p_1$, $p_2$ are the pressures at the ends of orifice.
This flow coefficient is a function of Reynolds number, which has the expression:

$$R_e = \frac{qd_h}{A\nu} \qquad (24)$$

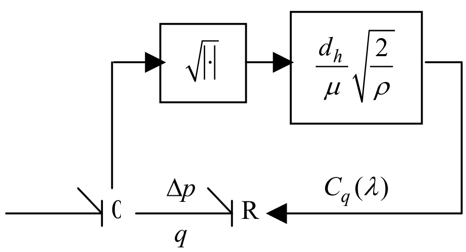
where $d_h$ is the hydraulic diameter.
This law of flow through an orifice makes a connection between effort and flow variables, and it corresponds to a macroscopic phenomenon of energy dissipation.
The bond graph representation appears as a causal R-element and modulated if the interaction between the hydraulic and mechanic domains is neglected. The variable of modulation is $C_q$ in the case of a constant orifice, or the product of passing section and $C_q$ if the case of an orifice with variable section.




This causal bond graph implies flux causality for R-element.
The flow coefficient may be considered as a function of flow number:

$$\lambda = \sqrt{\frac{2}{\rho}|p_1 - p_2|} \qquad (25)$$

which yields to the representation of dissipation phenomenon using the following causal bond graph:




This causal bond graph implies effort causality for R-element.

# 4. SIMULATION RESULTS

Let us consider the system in Fig. 2 consisting of a hydraulic part composed of a pressure source $\Delta p_1 = p(t)$, a pipeline and a piston located in a hydraulic cylinder in which is generated a pressure $\Delta p_2 = p_c(t)$, and a mechanical part situated on the other side of the piston composed of a spring with stiffness $k_e$ and a body of mass $m$. An external force $F$ acts on the body.
The pipeline has a hydraulic resistance $R_h$, and a hydraulic inductance $L_h$.
The simulation of system's behavior is done using BondSim.
The evolution of output variables is graphically represented in Fig. 4 and Fig. 5.

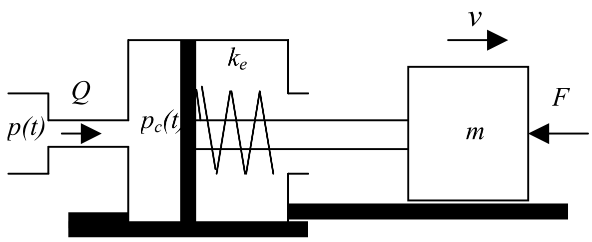



Fig. 2. A mechanic and hydraulic combined system.

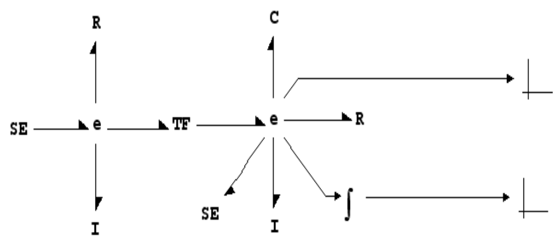



Fig. 3. BondSim representation.

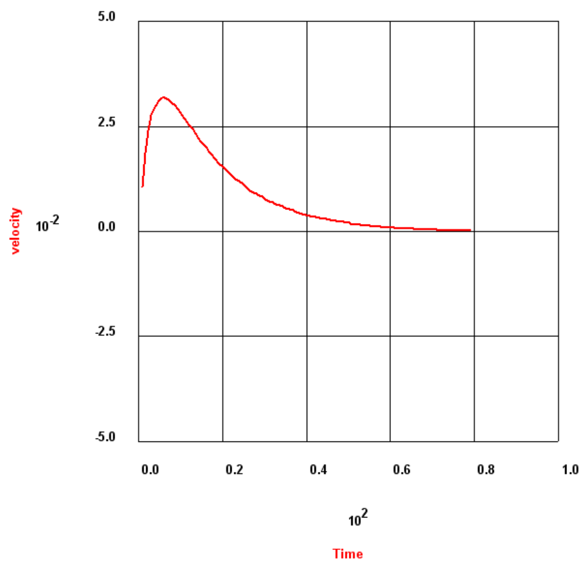



Fig. 4. Mass velocity.

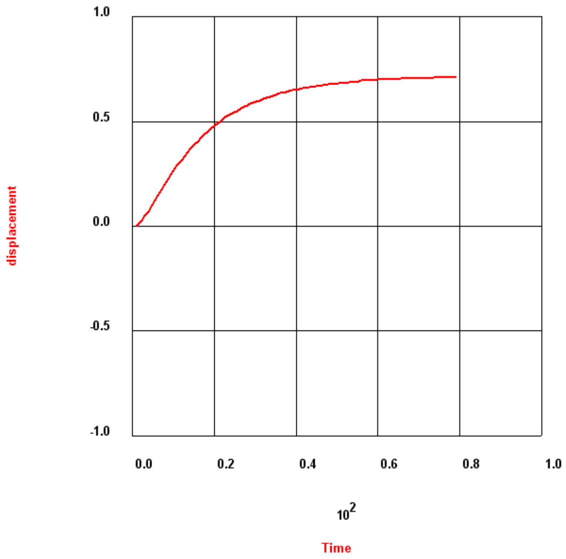



Fig. 5. Mass displacement.

We have considered the following values for systems parameters (Pastravanu, 2001). For hydraulic part: $\Delta p_I$=9.2 · $10^6$ N/m$^2$, $L_h$=2.8648 · $10^6$ kg/m$^4$ and $R_h$=1.3751·$10^7$ kg/sm$^2$. For the mechanic part: $k_e$=2.4 · $10^3$ N/m, $m$=$10^5$ kg, $\gamma$ = $10^4$ N · s/m, $F$=4.5 · $10^5$ N. The conversion from the variables of hydraulic power to the variables of mechanic power is done using a transformer with the transformer modulus equal with 0.0491. The simulation period is 80 seconds.

## 5. CONCLUDING REMARKS

The great advantage of using BondSim for simulating physical systems is that it supports many levels of decomposition depending on the complexity of engineering system. The construction of bond graph model is done very easily creating objects in the computer memory that are represented on screen. After the development of bond graph model it is built its mathematical representation in the form of differential algebraic equations that are solved using BDF (Backward Difference Formula) methods.

## REFERENCES

Bobasu E. (2002). *Hydraulic Systems. Modelling and Control,* (in Romanian). Universitaria, Craiova.

Damic V., Montgomery J. (2002). *Mechatronics by Bond Graphs*. Springer, Germany.

Dauphin-Tanguy G. et al. (2000). *Les Bond Graphs*. Hermes Science, Paris.

Karnopp D., Rosenberg R. (1975). *System Dynamics: A Unified Approach.* John Wiley & Sons, New York.

Pastravanu O., Ibanescu, R. (2001). *Bond-graph language in modeling and simulation of physical-technical systems* (in Romanian). Gh. Asachi, Iasi.

Thoma J. (1990). *Simulation by bond-graphs. Introduction to a Graphical Method.* Springer Verlag, New York.