STABILITY ANALYSIS OF VEHICLE FOLLOWING CONTROL SYSTEMS

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Abstract: A simple car-following model replace human drivers and their low-predictable reaction time with respect to traffic problems (reaction time 0.25 – 1.25 sec needs an inter-vehicle spacing around 30 *m* or more at 60 *km/hour*). A way to solve the problem is to organise the traffic into platoons, that is groups of vehicles consisting of a leader and a number of followers "tightly" spaced, all moving in longitudinal direction. The stability of the system in the parameter space is analysed using a specialized software DDE-BIFTOOL v. 2.00.

Keywords: stability, cruise control, time delay system

1. INTRODUCTION

Traffic flow is a comprehensive stochastic process of interactions between drivers, vehicles and the geometric conditions of the roadway. To be able to design and operate transportation systems in the most efficient way one must determine the relationship between the traffic flow, traffic density and traffic speed.

The development of traffic flow models usually require the determination of the following elements:
-the general equation of traffic flow, where flow is the product of speed and density
-the equation of conservation of vehicles (Lighthill and Whitham, 1955)
-the relationship between speed and density or between the flow-density

Traffic congestion has growing rapidly in many transportation systems in recent years and has become a rather acute problem. An appropriate combination of control and communication technologies placed on the vehicle to form a platoon of vehicles travelling at high speed can lead to significant increases in capacity and safety without requiring more land for new highways. One idea it was the use of automatic control to replace human drivers and their low-predictable reaction time with respect to traffic problems (reaction time 0.25 – 1.25 sec needs an inter-vehicle spacing around 30 *m* or more at 60 *km/hour*). A way to solve the problem is to organise the traffic into platoons, that is groups of vehicles consisting of a leader and a number of followers "tightly" spaced, all moving in longitudinal direction.

Traffic models may be classified as follows:
- scale of the independent variables: continuous, discrete, semi-discrete.

- continuous models describe how the traffic systems state changes continuously over time in response to continuous stimuli.
- discrete models assume that state changes occur discontinuously over time at discrete time instants.

Besides time, also other independent variables can be described by either continuous or discrete variables (position, velocity).
- representation of the processes: deterministic, stochastic.

- deterministic models have no random variables implying that variables in the model are defined by exact relationships.
- stochastic models incorporate processes that include random variables.

- operational: analytical, simulation. Models can be operationalized either as analytical solutions of sets of equations or as a simulation model.

- scale of application: networks, stretches, links, and intersections. It indicates the area of application of the model.

- level of detail: macroscopic, mesoscopic, microscopic.

- macroscopic traffic flow models maps traffic flow as a continuous unity of "fluidized" vehicles. No vehicle in the traffic flow is identifiable.

The traffic flow is characterized by macroscopic state values like density, volume and mean velocity, which is associated with each other by the flux relation. Following this approach a traffic flow is treated as a continuous fluid flow in fluid dynamics.
- microscopic traffic flow models that maps traffic flow as a set of individual vehicles. Each vehicle is identifiable and is modeled. The behavior of a vehicle depends on its own drive and on influences of its environment. The basic microscopic modeling is called *Follow the-Leader* modeling, in which the motion of a vehicle depends on the distance and the velocity difference to a leading vehicle.
- mesoscopic traffic modeling is a consistent link between microscopic and macroscopic modeling. The link is reached by a transition from a microscopic to a macroscopic modeling. Mesoscopic modeling bases on a distance density relation establishing a link between the distance of two sequenced vehicles in a microscopic modeling and the density in a macroscopic modeling.

In this paper we discuss the stability problem of vehicle following systems within throttle and brake control. First, we propose a reduced vehicle model used in a platoon configuration. Second we study individual vehicle stability and give a delay-dependent sufficient condition. Finally, simulations show that the platoon control under-satisfying the conditions above can maintain a constant spacing and avoid the slinky-effects.

The paper contains four sections besides the introduction. Section 2 describe the platoon structure and requirements and develops a reduced vehicle model to throttle and brake control. Section 3 comprises a detailed stability analysis including the control law and some aspects of frequency sweeping tests based on Tsypkin's criterion. Section 4 is dedicated to the simulation experimental results. The conclusion in section 5 closes the paper.

2. VEHICLE MODEL

In principle, there are two solutions to the proposed multi-objective control problem, depending on whether or not one includes any communication of the lead vehicle information to each vehicle in the platoon.

In the first structure, one needs to assume that there exists some large space between platoons through interactive vehicle-to-vehicle communication. In such a structure, each vehicle has access to the state information (relative position, velocity and acceleration) of the preceding vehicle and of the relative lead vehicle.

In the second scheme, known as autonomous intelligent cruise control, the controller has access only to the preceding vehicle. Several assumption are to be made: the lead vehicle performs a maneuver in finite time before reaching a steady state, and prior to a maneuver, all the vehicles move at the same steady speed.

Both control strategies avoid slinky effects. In the sequel, we shall focus on the second control technique: autonomous intelligent vehicle control.
For our analysis one consider a microscopic *Follow the-Leader* modeling which consist of a platoon of n vehicles "tightly" spaced, all moving in longitudinal direction. In fig. 1 is presented the well known configuration of a platoon consisting of n vehicles

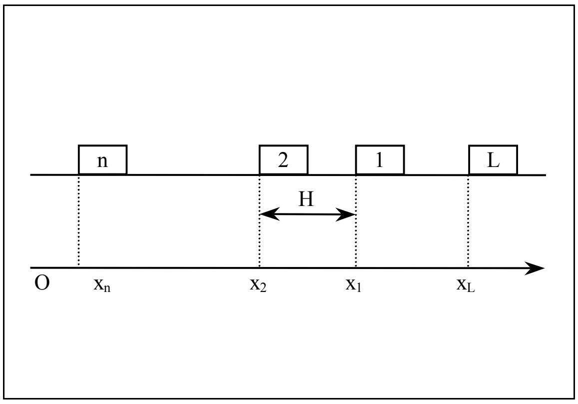

Fig. 1. Platoon configuration of n vehicles

One denotes $x_i(t)$ the position of the *i*-th vehicle with respect to some well-defined reference point O on the roadside. The main goal is to maintain a distance H between subsequent vehicles (i-1) and i. Denote $\varepsilon_i(t)$ the spacing error between the *i*-th and *(i-1)*-th vehicles. This quantity is given by

$$\varepsilon_i(t) = x_{i-1}(t) - x_i(t) - H_i \qquad (1)$$

Many authors for a variety of purposes have developed vehicles models. Assuming that the transmission is locked in gear and ignoring tyre slip, the state equation for the engine speed is:

$$J_e \dot{w}_e(t) = T_t(t-\tau) - T_L(t)$$

Since we have not considered tyre slip, the speed of the wheel can be evaluated by:

$$w_e = \frac{v}{R_g h}$$

Thus, a simple vehicle model is given by:

$$\dot{x}(t) = v(t) \quad (2)$$

$$\dot{v}(t) = \frac{R_g h}{J_e}[T_t(t-\tau) - T_L(t)] \quad (3)$$

where:

- x(t) is the position of vehicle;
- v(t) is the speed of vehicle;
- J_e is the effective rotational inertia of the engine;
- h is the effective tyre radius;
- $w_e(t)$ is the angular velocity of the wheel;
- $T_t(t)$ is the throttle input;
- $T_L(t)$ is the load torque on the engine speed, gear ratio, grade change etc.
- τ - is the total throttle/torque delay

A vehicle following controller should simultaneously guarantee:
- a desired spacing between the vehicles, called also individual vehicle stability, and
- no slinky-effects, that is no amplification of the spacing error between subsequent vehicles, when vehicle index increases.

For the vehicle dynamics (2) - (3) using the following control law:

$$T_{ti} = k_s'\varepsilon_i(t) + k_v'\dot{\varepsilon}_i(t) + k_{sl}(x_L(t) - x_i(t) - \\ - \sum H_i) + k_{vl}'(v_l(t) - v_i(t)) + T_L \quad (4)$$

one gets

$$\ddot{\varepsilon}_i(t) = -(k_s + k_{sl})\varepsilon_i(t-\tau) - (k_v + k_{vl})\dot{\varepsilon}_i(t-\tau) + \\ + k_s\varepsilon_{i-1}(t-\tau) + k_v\dot{\varepsilon}_{i-1}(t-\tau) \quad (5)$$

i=2, 3, …, n

where:

$$k_s = k_s' \cdot k, k_v = k_v' \cdot k, k_{sl} = k_{sl}' \cdot k, k_{vl} = k_{vl}' \cdot k, \quad (6)$$

$$k = \frac{R_g h}{J_e}$$

$k_s^{'}$, $k_v^{'}$ design constants

T_L – the load torque

3. INDIVIDUAL VEHICLE STABILITY

There are basically two directions in which one can develop stability criteria to test the stability of system involving time delays. One is to obtain stability criteria which are independent of the size of the delay. Another way is to get stability conditions which are dependent on the size of the delay. It is well known that the abandonment of information on the delay causes delay-independent criteria to be conservative, especially when delays are small.

To study the individual vehicle stability one uses the following time dependent equation which represent the i*th* closed loop dynamics described as:

$$\ddot{\varepsilon}_i(t) = -(k_s + k_{sl})\varepsilon_i(t-\tau) - (k_v + k_{vl})\dot{\varepsilon}_i(t-\tau) \quad (7)$$

i=1, 2, 3, …, n

which does not include the interconnected terms of other vehicles.

Using the Laplace transform, one gets the following relations:

$$s^2E(s) = -\beta_1 sE(s)e^{-\tau s} - \beta_0 E(s)e^{-\tau s}$$

where E(s) is the Laplace transform of $\varepsilon(t)$.

The transfer function in open loop is

$$H_o(s) = \frac{\beta_1 s + \beta_0}{s^2} e^{-\tau s}$$

and the transfer function in closed loop is:

$$H_c(s) = \frac{(\beta_1 s + \beta_0)e^{-\tau s}}{s^2 + (\beta_1 s + \beta_0)e^{-\tau s}}$$

The characteristic equation has the following form:

$$s^2 + (\beta_1 s + \beta_0)e^{-s\tau} = 0 \quad (8)$$

where

$$\beta_1 = k_v + k_{vl}$$
$$\beta_0 = k_s + k_{sl}$$

Denoting

$$Q(s) = s^2$$
$$P(s) = \beta_1 s + \beta_0$$

then the characteristic equation has the following form:

$$Q(s) + P(s)e^{-s\tau} = 0 \quad (9)$$

This is a transcendental equation having an infinite number of solutions. The analysis of such a system is done in the parameter space (β_0, β_1, τ).

We need to find conditions on the triplet (β_0, β_1, τ) such that the characteristic equation (8) has no solution in the right-half plane C^+. First, one needs the stability guaranteed for $\tau = 0$, that is the Hurwitz stability of the polynomial:

$$s^2 + \beta_1 s + \beta_0 = 0 \tag{10}$$

Evidently, we have stability if and only if

$$\begin{aligned} \beta_0 &> 0 \\ \beta_1 &> 0 \end{aligned} \tag{11}$$

Next, for delay-independent stability, using Tsypkin's criterion, one needs to find pairs (β_0, β_1) such that (10) is satisfied and the characteristic equation (8) has no roots on the imaginary axis for all $\tau > 0$. That means the equation:

$$|Q(j\omega)| = |P(j\omega)| \tag{12}$$

has no roots on the imaginary axis.
One obtains the following equation:

$$\omega^4 - \beta_1^2\omega^2 - \beta_0^2 = 0 \tag{13}$$

that has the real solution:

$$\omega_0 = \sqrt{\frac{\beta_1^2 + \sqrt{\beta_1^4 + 4\beta_0^2}}{2}} \tag{14}$$

In conclusion, we have roots on the imaginary axis and all we can expect is a delay-dependent type result.
To characterise the fact if some root crosses the imaginary axis from left to right (towards instability) or from right to left (towards stability) we have to analyse the sign of the relation

$$\mathrm{Re}\left(\frac{ds}{d\tau}\right)\Bigg|_{s=j\omega_0} \tag{15}$$

- if it is positive, we have one switch from stability to instability, and
- if it is negative, we have one reversal from instability to stability.

In our case, we have:

$$2s\frac{ds}{d\tau} + \beta_1\frac{ds}{d\tau}e^{-\tau s} + e^{-\tau s}(\beta_1 s + \beta_0)\left(-s - \tau\frac{ds}{d\tau}\right) = 0$$

$$\Rightarrow \frac{ds}{d\tau}\left(2s + \beta_1 e^{-\tau s} + s^2\tau\right) = -s^3$$

$$\left(\frac{ds}{d\tau}\right)^{-1} = -\frac{2s + \beta_1 e^{-\tau s} + s^2\tau}{s^3} =$$

$$-\frac{2}{s} - \frac{\beta_1(-s^2)}{s^3(\beta_1 s + \beta_0)} - \frac{\tau}{s}$$

$$\mathrm{Re}\left(\frac{ds}{d\tau}\right)^{-1} = \frac{1}{\omega^2} + \frac{\left(\frac{\beta_0}{\beta_1}\right)^2}{\omega^2\left(\omega^2 + \left(\frac{\beta_0}{\beta_1}\right)^2\right)} > 0 \tag{16}$$

that means that by increasing the delay value, some root will cross the imaginary axis, this root will always crosses from left to right, that is towards instability.

Thus, if the stability is guaranteed for delay $\tau = 0$, it follows that we can find a bound τ_{switch} function of β_0 and β_1 such that the stability is guaranteed for all $\tau \in [0, \tau_{switch})$.

This bound can be calculated from the equation:

$$\frac{P(j\omega)}{Q(j\omega)} = -e^{-j\omega\tau} \tag{17}$$

By substituting

$$e^{-j\omega\tau} = \cos(\omega\tau) - j\sin(\omega\tau),$$

one gets:

$$\tau_{switch} = \frac{1}{\omega_0}\arccos\left(\frac{\beta_0\omega_0^2}{\beta_1^2 + \beta_0^2\omega_0^2}\right) \tag{18}$$

The representation of τ_{switch} in the parameter plane (β_0, β_1) is presented in the figure 2. As it can be seen the bound of the time delay is increasing at smaller values of the parameters (β_0, β_1). But these parameters must accomplish the following goals:

- the individual vehicle system should be asymptotically stable
- the transient errors should not amplify with vehicle index due to any lead vehicle manoeuvre. This is referred to no slinky-effects in the platooning. One way to satisfy this requirement is that the maximum absolute spacing error of *jth* vehicle should be less than or equal to that of the (j-1)*th* vehicle.

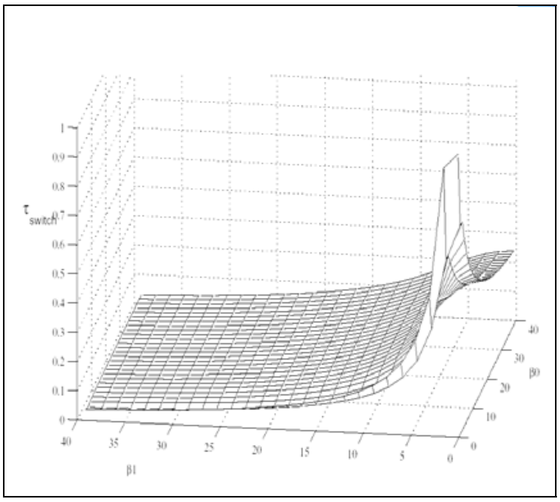

Fig. 2. The representation of τ_{switch} in the parameter plane (β_0, β_1)

Sensor measurements and communications between vehicles are required as follows:

(1) *Sensor measurements*: The speed and distance of the controlled vehicle relative to its preceding vehicle is sensed by onboard sensors like radar or sonar.

(2) *Communications between vehicles:* The lead vehicle velocity may be transmitted to all the following vehicles in the platoon, and the control state of the lead vehicle, such as throttle control or brake control, may also be broadcast to all the following vehicles. Thus, the position of the j*th* vehicle relative to the lead vehicle can be obtained by summing the position of j*th* vehicle relative to the (j-1)*th* vehicle and the position of the (j-1)*th* vehicle relative to the lead vehicle.

In August 1997, the National Automated Highway Systems Consortium (NAHSC) proof of technical feasibility demonstrations, called *Demo'97*. The demo was a complete success. This demonstration was performed by an eight-vehicle platoon. The eight vehicles traveled at a fixed separation distance of 6.5 m at all speed up to full highway speed. The sensor measurements and communications between vehicles described above were achieved by combining range information from a forward-looking radar with information from a radio communication system that provided lead vehicle's position and speed updates 50 times per second. The Demo showed that platoon travel should be technically feasible in the near future.

Although early platooning is expensive, it enables vehicles to operate much closer together than is possible under manual driving conditions, each lane carrying at least twice as much traffic as it can today. Also, at close spacing, aerodynamic drag is significant reduced, which can lead to major reductions in fuel consumption and exhaust emissions. The high performance automation control system also increases the safety of highway travel.

4. SIMULATION RESULTS

The global behavior of the DDE model (1) - (2) in the (β_0, β_1)-plane can be investigated with the the Matlab package DDE-BIFTOOL; see (Engelborghs *et al.*, 2001).

DDE-BIFTOOL is a collection of Matlab routines for numerical bifurcation analysis of systems delay differential equation with several constant and state-dependent delays. The package allows to compute, continue and analyse stability of steady state solutions and periodic solutions. It further allows to compute and continue steady state fold and Hopf bifurcations and to switch, from the latter, to an emanating branch of periodic solutions.

Homoclinic and hetero-clinic orbits can also be computed. To analyse the stability of steady state solutions, approximations are computed to the rightmost, stability-determining roots of the characteristic equation which can subsequently be used as starting values in a Newton procedure. For periodic solutions, approximations to the Floquet multipliers are computed.

The constants used in our case for the vehicle are:

$R_g=0,3058$;

$J_e=0,2630\ kgm^2$;

$h=0,33m;$

$T_L=67,7\ Nm.$

The controller parameters values are:

$k_v=3,$

$k_{vl}=0.9,$

$k_s= 4.5$

$k_{sl}=0.5.$

By using the Lyapunov method with $\varepsilon=0.3055$ and

$$P = \begin{pmatrix} 1,1592 & 0,1000 \\ 0,1000 & 0,1538 \end{pmatrix}$$

one found

$$\tau_{switch} = 0.0514.$$

As we can see in the following figure, the system is stable for $\tau = 0,0514$ because all the roots are in the left half plane.

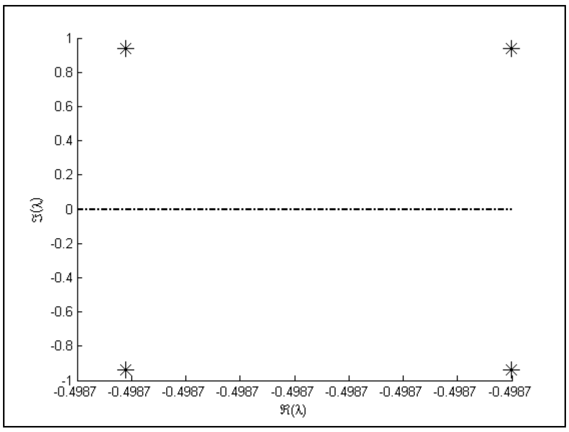

Fig. 3. The roots of the characteristic equation for $\tau = 0,0514$

Using the Tsypkin's cryterion one found τ_{switch} = *0,3099*

As we can see in the following figure the system is steel stable for τ = *0,3,* all the are in the left-half plane.

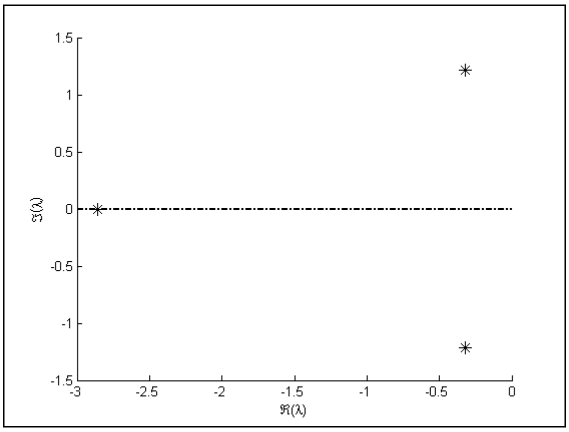

Fig. 4. The roots of the characteristic equation for τ = *0,3*

For τ = *0,32* some roots cross the imaginary axis as it can be seen in the following figure.

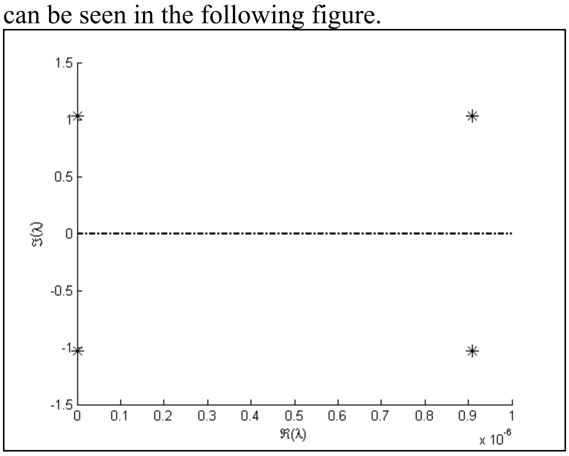

Fig. 5. The roots of the characteristic equation for τ = *0,32*

The spacing error evolution is presented in the following figure:

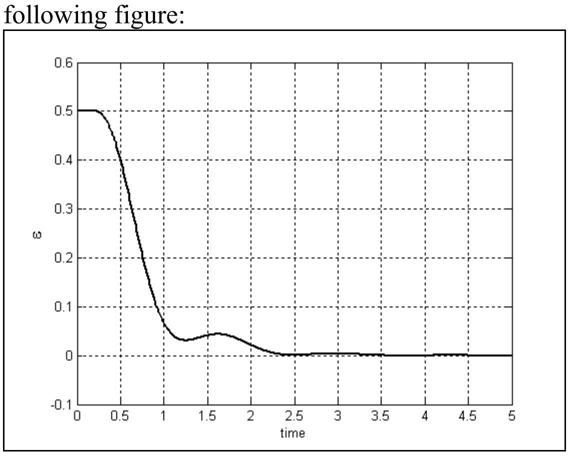

Fig. 6. Time evolution of spacing error

5. CONCLUSIONS

In this paper we had presented a comparative study for the stability of a time delay system obtained by modelling a vehicle following control system. For the proposed model some roots of the characteristic equation will cross the imaginary axis from left to right (from stability to instability). The superior border for τ is greater using Tsypkin's cryterion that using the Lyapunov method. The car-following model presented here is valid for any number of cars.

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