DESIGN OF CONTROLLERS FOR STABLE AND UNSTABLE SYSTEMS WITH TIME DELAY

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Abstract: The paper presents one approach to design of controllers for time delay systems. The proposed method is based on the time delay Padé approximation. The controllers are derived using the polynomial approach and LQ control technique. Resulting proper and stable controllers obtained via polynomial Diophantine equations and spectral factorization techniques ensure setpoint tracking as well as load disturbance attenuation. The procedure is developed for stable and unstable first order time delay systems and the results are verifyed by simulations in MATLAB – Simulink.

Keyword**s**: Time delay system, time delay approximation, polynomial method, LQ control.

1 INTRODUCTION

The existence of a time delay in input-output relations is a common property of many technological processes. Plants with the time delay can often not be controlled using usual controllers designed without a consideration for a presence of the dead-time. The control responses using such controllers then tend to destabilize the closed-loop system.

It is well known that as an effective time-delay compensator especially for stable systems with long time delays, the Smith predictor can be used. However, a part of technological processes containing a time delay can be unstable, such as chemical reactors or bioreactors. Many different approaches have been developed to control such processes. While some methods issue from several modifications of the Smith predictor (De Paor and Egan, 1989), (Majhi and Atherton, 1999), (Liu *et al.*, 2005), other methods employ PI, PD and PID control strategies (Rotstein and Lewin, 1991), (Park *et al.*, 1998) or IMC-based methods (Huang and Chen, 1997), (Tan *et al.*, 2003).

The paper presents one approach to control both stable and unstable time delay systems based on the time delay approximation and the polynomial approach, see, eg. (Kučera, 1993), (Grimble, 1994). The principles of the method have been developed for a stable first order time delay system (FOTDS) and published in (Dostál *et al.*, 2001a). For an unstable FOTDS, the results obtained using first order numerator and Padé time delay approximations in the 1DOF and 2DOF control system configurations were analyzed and compared in (Dostál *et al.*, 2001b). The results demonstrate the priority of the Padé approximation to the numerator approximation together with a utilization of the 2DOF configuration. The method presented here is based on the above combination. For tuning of the controller parameters, the LQ control technique is employed, see, e. g. (Dostál and Bobál, 1999). Resulting stable and proper controllers obtained via polynomial Diophantine equations and spectral factorization technique ensure asymptotic tracking of step references as well as step load disturbance attenuation. Even though any method based on a time delay approximation cannot guarantee the control system stability in general, the simulation results document a usability of the proposed method providing stable control responses of a good quality also for higher values of the time delay.

2 APPROXIMATE TRANSFER FUNCTIONS

Consider the transfer functions of stable and unstable first order time delay system having the form

$$G(s)=\frac{K}{\tau s+1}\,e^{-\tau_d s} \quad (1)$$

$$G(s)=\frac{K}{\tau s-1}\,e^{-\tau_d s} \quad (2)$$

where $K > 0$ is the gain, $\tau > 0$ is the time constant and $\tau_d > 0$ is the time delay. The time delay transform is approximated by the Padé approximation

$$e^{-\tau_d s} \approx \frac{2-\tau_d s}{2+\tau_d s} \quad (3)$$

Using approximation (3), approximate transfer functions take forms

$$G_{AS}(s)=\frac{K(2-\tau_d s)}{(\tau s+1)(2+\tau_d s)}=\frac{b_0-b_1 s}{s^2+a_1 s+a_0} \quad (4)$$

$$G_{AU}(s)=\frac{K(2-\tau_d s)}{(\tau s-1)(2+\tau_d s)}=\frac{b_0-b_1 s}{s^2+a_1 s-a_0} \quad (5)$$

where $b_0=\frac{2K}{\tau\tau_d}$, $b_1=\frac{K}{\tau}$, $a_0=\frac{2}{\tau\tau_d}$,

$a_1=\frac{2\tau+\tau_d}{\tau\tau_d}$ in (4), $a_1=\frac{2\tau-\tau_d}{\tau\tau_d}$ in (5)

and $\tau_d \neq 2\tau$.

Both approximate transfer functions (4) and (5) are strictly proper transfer functions

$$G_A(s)=\frac{b(s)}{a(s)} \quad (6)$$

where b and a are coprime polynomials that fulfill the inequality $\deg b < \deg a$.

3 CONTROL SYSTEM DESCRIPTION

The 2DOF control system configuration is depicted in Fig. 1. Here, a controller contains next to the feedback part Q also the feedforward part R. In the scheme, w is the reference signal, v is the load disturbance, y is the controlled output and u is the control input. Both w and v are considered to be step functions. The transfer function of the controlled system G represents one from approximate transfer functions G_A .

For the stepwise reference and the step load disturbance, both controller parts contain an integrator and their transfer functions can be found in the form of polynomial fractions

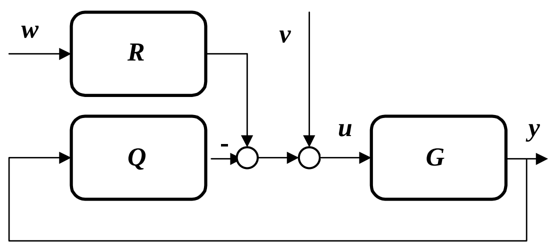

Fig. 1. 2DOF control system configuration.

$$Q(s)=\frac{q(s)}{s\,p(s)}\;,\;R(s)=\frac{r(s)}{s\,p(s)} \quad (7)$$

where q, r, p are polynomials in s.

4 APPLICATION OF POLYNOMIAL METHOD

The controller design described in this section follows from the polynomial approach. The general conditions required to govern the control system properties are formulated as follows:

- Strong stability of the control system (in addition to the control system stability, also the controller stability is required).
- Internal properness of the control system.
- Asymptotic tracking of the reference.
- Load disturbance attenuation.

The procedure to derive admissible controllers can be carried out as follows:

A feedback controller given by a solution of the polynomial Diophantine equation

$$a(s)s\,p(s)+b(s)q(s)=d(s) \quad (8)$$

with a stable polynomial d on the right side ensures the control system stability and the load disturbance attenuation.

A stable polynomial $p(s)$ in denominators of (7) ensures the controller stability.

Asymptotic tracking of the step reference is provided by the controller feedforward part given by a solution of the polynomial Diophantine equation

$$t(s)s+b(s)r(s)=d(s) \quad (9)$$

where $t(s)$ is an auxiliary polynomial which does not enter into controller design but which is necessary for calculation of equation (9).

The control system satisfies the condition of internal properness when the transfer functions of all components are proper. The degrees of the controller polynomials then must fulfill inequalities

$$\deg q \le \deg p+1\,,\;\deg r \le \deg p+1\,. \quad (10)$$

Taking into account (10), the condition $\deg b \le \deg a$ and a solvability of (8) and (9), the degrees of polynomials q, p and r can be derived as

$$\deg q=\deg a\,, \deg p \ge \deg a-1\,, \deg r=0\,. \quad (11)$$

The controller parameters then follow from solutions of polynomial equations (8) and (9) and depend upon coefficients of the polynomial d. Now, the next problem means to find a stable polynomial d that enables to obtain the acceptable stabilizing and stable controllers.

5 LQ CONTROL TECHNIQUE

In this section, the polynomial d is considered as a product of stable polynomials g and n in the form

$$d(s) = g(s)n(s). \quad (12)$$

The first polynomial g is obtained by spectral factorization

$$(sa(s))^* \varphi(sa(s)) + b^*(s)b(s) = g^*(s)g(s) \quad (13)$$

where the asterisk denotes a conjugate polynomial.

Remark: It is well known from the LQ control theory that the polynomial g is used to minimization of the quadratic cost function

$$J = \int_0^\infty \left\{ e^2(t) + \varphi \dot{u}^2(t) \right\} dt \quad (14)$$

where $e(t)$ is the tracking error, $\dot{u}(t)$ is the control input derivative and $\varphi > 0$ is the weighting coefficient.

The second polynomial n, ensuring properness of the controller, is given for a stable FOTDS as

$$n(s) = a(s). \quad (15)$$

For an unstable FOTDS, this polynomial is determined in two different ways.

In the first case, this polynomial is given as a stable part of spectral factorization

$$n^*(s)n(s) = a^*(s)a(s). \quad (16)$$

Then, the procedure leads to a strictly proper controller. The degree of d on the right sides of (8) and (9) is given as

$\deg d(s) = \deg[g(s)n(s)] = 2\deg a(s) + 1$.

Taking into account (11) and the relation $\deg[s\,p(s)] = \deg d(s) - \deg a(s) = \deg a(s) + 1$, strict properness of (7) is evident.

In the second case, the polynomial n is chosen as a stable part of the polynomial a

$$n(s) = a^+(s) = s + \frac{2}{\tau_d}. \quad (17)$$

Now, the procedure leads to a nonstrictly proper feedback part of the controller. Since equalities $\deg d(s) = \deg[g(s)n(s)] = 2\deg a(s)$ and $\deg[s\,p(s)] = \deg d(s) - \deg a(s) = \deg a(s)$ hold, $Q(s)$ in (7) is nonstrictly proper.

A preference of such determination of polynomial d lies in the fact that all controller parameters can be tuned by only single selectable parameter. Taking into account that all parameters except φ in (13) are given by properties of the controlled system, the coefficients of g depend upon single selectable parameter φ. Since polynomial n does not contain any selectable parameters, also the coefficients of polynomial d, and, the controller parameters given by solutions of (8) and (9) depend next to fast given parameters K, τ and τ_d only upon φ.

6 CONTROLLER DESIGN

For both stable and unstable FOTDS, normed polynomial g has the form

$$g(s) = s^3 + g_2 s^2 + g_1 s + g_0 \quad (18)$$

with coefficients

$$g_0 = \frac{2K}{\tau\tau_d}\sqrt{\frac{1}{\varphi}}$$

$$g_1 = \frac{1}{\tau\tau_d}\sqrt{4\left(K\tau\tau_d\sqrt{\frac{1}{\varphi}}\,g_2 + 1\right) + K^2\tau_d^2\frac{1}{\varphi}} \quad (19)$$

$$g_2 = \frac{1}{\tau\tau_d}\sqrt{2\tau^2\tau_d^2\sqrt{\frac{1}{\varphi}}\,g_1 + 4\tau^2 + \tau_d^2}.$$

In behalf of shortness of the writing, other important equations and derived formulas for considered systems are introduced in the form of tables in the following order:

- Form of used polynomial n.
- Formulas for computation of coefficients n.
- Transfer functions of the resulting controller.
- Formulas for computation of controller parameters.
- Condition of the resulting controller stability.

Tab. 1. Stable FOTDS

$n(s) = s^2 + n_1 s + n_0$
$n_0 = \frac{2}{\tau\tau_d}$, $n_1 = \frac{2}{\tau_d} + \frac{1}{\tau}$
$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(s^2 + p_1 s + p_0)}$, $R(s) = \frac{r_0}{s(s^2 + p_1 s + p_0)}$
$p_0 = g_1 + \frac{\tau_d}{2} g_0$, $p_1 = g_2$; $q_0 = \frac{1}{K} g_0$, $q_1 = \frac{2\tau + \tau_d}{2K} g_0$, $q_2 = \frac{\tau\tau_d}{2K} g_0$
$p_0 > 0$, $p_1 > 0$ for all τ_d

Tab. 2. Unstable FOTDS – Strictly proper controller

$n(s) = s^2 + n_1 s + n_0$
$n_0 = \frac{2}{\tau\tau_d}$, $n_1 = \frac{2}{\tau_d} + \frac{1}{\tau}$
$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(s^2 + p_1 s + p_0)}$, $R(s) = \frac{r_0}{s(s^2 + p_1 s + p_0)}$
$p_0 = \frac{4g_2 + (2\tau + \tau_d)\left(g_1 + \frac{\tau_d}{2} g_0\right) + \frac{4}{\tau}}{2\tau - \tau_d}$; $p_1 = g_2 + \frac{2}{\tau}$, $q_0 = \frac{1}{K} g_0$

$$q_1 = \frac{1}{K}\left[p_0 + g_1 + (\tau + \tau_d) g_0\right]$$

$$q_2 = \frac{1}{K}\left[\tau(p_0 - g_1) - 2\left(g_2 + \frac{1}{\tau}\right)\right]$$

$p_1 > 0$ for all τ_d, $p_0 > 0$ for $\tau_d < 2\tau$

Tab. 3. Unstable FOTDS – Nonstrictly proper feedback part of the controller

$$n(s) = s + n_0$$

$$n_0 = \frac{2}{\tau_d}$$

$$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(s + p_0)}, \quad R(s) = \frac{r_0}{s(s + p_0)}$$

$$p_0 = \frac{\tau\left[2g_2 + \tau_d\left(g_1 + \frac{\tau_d}{2} g_0\right)\right] + 2}{2\tau - \tau_d}$$

$$q_0 = \frac{\tau}{K} g_0, \; q_1 = \frac{1}{K}\left[p_0 + \tau(g_1 + \tau_d g_0)\right]$$

$$q_2 = \frac{1}{K}\left[\tau(p_0 - g_2) - 1\right]$$

$p_0 > 0$ for $\tau_d < 2\tau$

7 EXAMPLES

All simulation experiments in this section were performed by MATLAB-Simulink tools. In all cases, the reference signal $w(t) = 1(t)$ was used.

7.1 Stable FOTDS

Consider a stable first order time delay system with transfer function (1) where $K = 1$ and $\tau = 4$. In realized simulations, the step disturbance $v(t) = -0.2$ was incorporated into the controlled system at time t_v. The values of t_v may be seen below each figure. The step reference and load disturbance responses for $\tau_d = 4$ ($\tau_d = \tau$), and $\tau_d = 12$ ($\tau_d = 3\tau$) and $\tau_d = 24$ ($\tau_d = 6\tau$) are shown in Figs. 2, 3 and 4. The responses in all figures clearly document an effect of the parameter φ upon the control responses. An increasing φ improves the control stability and by choosing of its higher value aperiodic responses can be obtained. The responses in Fig. 5 demonstrate the robustness of the proposed method against changes of τ_d. The controller parameters were computed for a nominal model with $\tau_d = 24$ and subsequently used for perturbed models with the ± 20% estimation error in τ_d value ($\tau_d = 28.8$ and $\tau_d = 19.2$).

7.2 Unstable FOTDS

Let in transfer function (2) $K = 1$ and $\tau = 4$. The responses for $\tau_d = 2$ ($\tau_d / \tau = 0.5$) are shown in Fig. 6. These document a control of good quality by both strictly and nonstrictly controllers for a small value τ_d. The responses to the step load disturbance using both controllers are compared in Fig. 7. These show that a nonstrictly controller gives approximately half-length overshoot. Such as in the above case, an effect of parameter φ upon the control responses is shown in Figs. 8 and 9. Again, it can be seen that an increasing value of φ results in aperiodic responses without overshoots during tracking as well as during disturbance attenuation. Responses for $\tau_d = 5$ ($\tau_d / \tau = 1.25$) are shown in Fig. 10. Especially, the setpoint

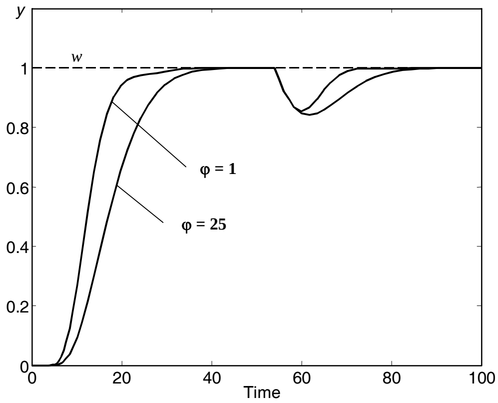

Fig. 2. Step reference and load disturbance responses ($\tau_d = 4$, $t_v = 50$).

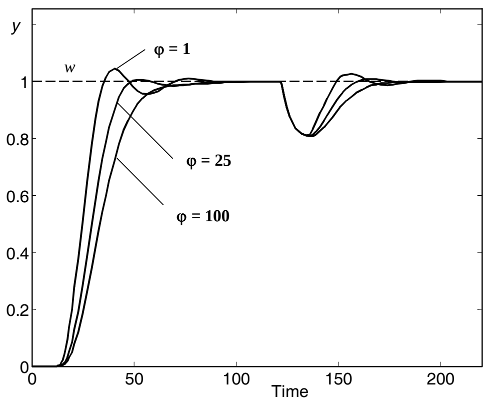

Fig. 3. Step reference and load disturbance responses ($\tau_d = 12$, $t_v = 110$).

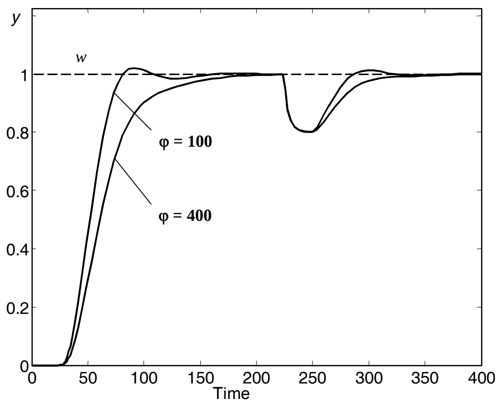

Fig. 4. Step reference and load disturbance responses. ($\tau_d = 24$, $t_v = 200$).

responses without any overshoot document the usability of the proposed method. Note that for instance De Paor and Egan's method (1989) provides stable setpoint responses for $\tau_d/\tau < 0.5163$ and Majhi and Atherton (1999) require this ratio as $\tau_d/\tau < 1$. Simulation results in Fig. 11 show a robustness of the method. Here, a nominal model with the exact value $\tau_d = 3$ ($\tau_d/\tau = 0.75$) has been used for a $\pm$ 10% estimation error in τ_d ($\tau_d = 3.3$ and $\tau_d = 2.7$). The responses demonstrate a sufficient robustness of the controller also for a relatively higher ratio between τ_d and τ, though the sensitivity to an estimation error in τ_d grows up according to an increasing ratio τ_d/τ .

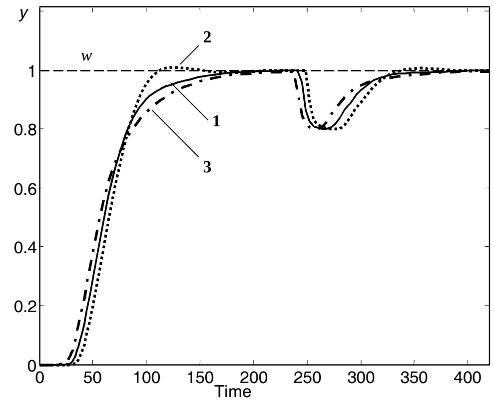

Fig. 5. Nominal and perturbed system responses (τ_d = 24 (1) ,28.8 (2), 19.2 (3), φ = 400, t_v = 220).

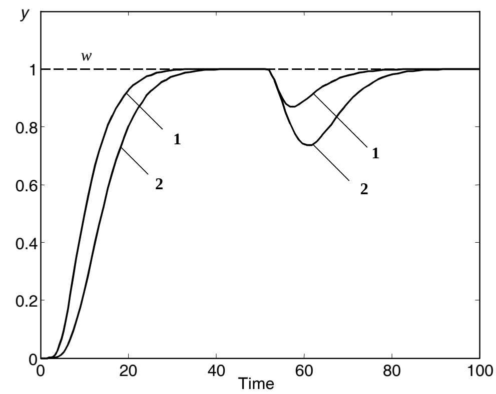

Fig. 6. Step reference and load disturbance responses for nonstrictly (1) and strictly (2) proper controller (τ_d = 2, φ = 25, v = - 0.1, t_v = 50).

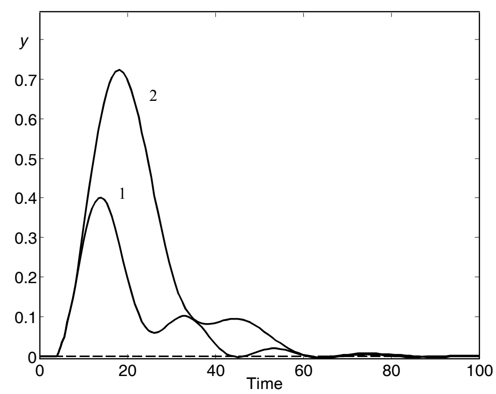

Fig. 7. Step load disturbance responses for nonstrictly (1) and strictly (2) proper controller (τ_d = 4, φ = 100, v = 0.1, t_v = 0).

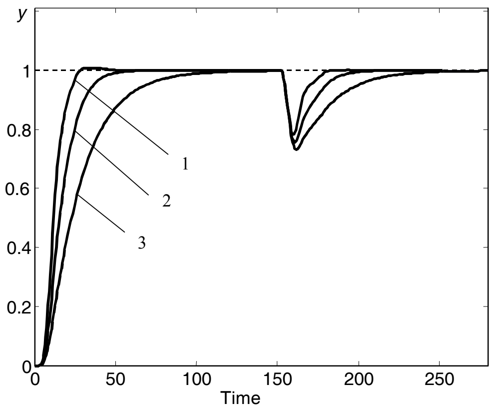

Fig. 8. Nonstrictly proper controller: Step reference and load disturbance responses (τ_d = 3, φ = 25 (1), 100 (2), 400 (3), v = - 0.1, t_v = 150).

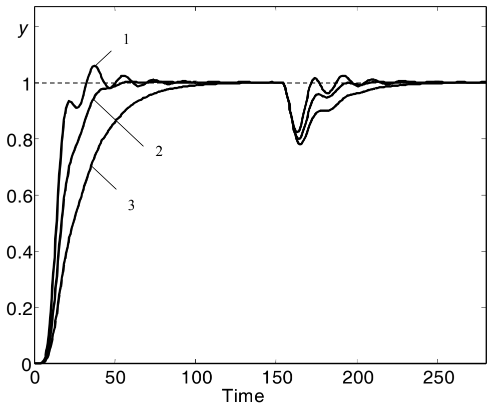

Fig. 9. Nonstrictly proper controller: Step setpoint and load disturbanceresponses (τ_d = 4, φ = 25 (1), 100 (2), 400 (3), v = - 0.1, t_v = 150).

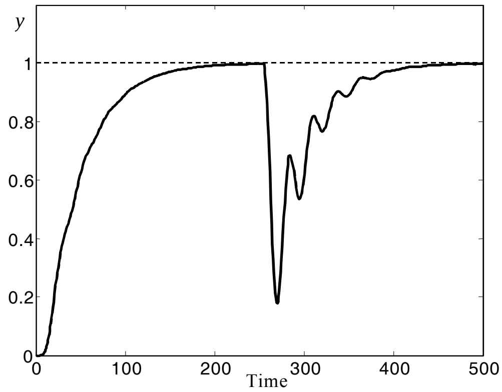

Fig. 10. Nonstrictly proper controller: Step reference and load disturbance response (τ_d = 5, φ = 1600, v = - 0.1, t_v = 250).

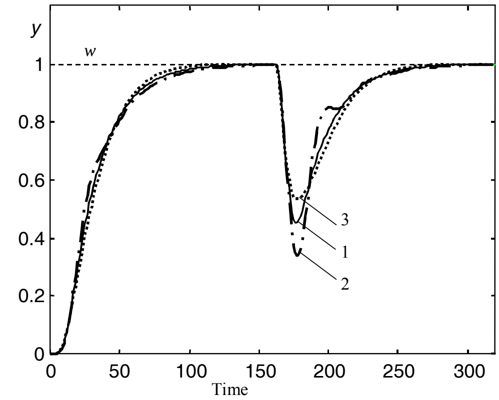

Fig. 11. Nominal and perturbed system responses (τ_d = 3 (1), 3.3 (2), 2.7 (3), φ = 400, v = - 0.1, t_v = 150).

8 CONSLUSION

The problem of control design for stable and unstable time delay systems has been solved and analyzed. The proposed method is based on the first order Padé time delay approximation. The controller design uses the polynomial method and a controller setting employs results of the LQ control theory. The presented procedure provides satisfactory control responses in the tracking of the step reference as well as in step load disturbance attenuation. The presented results have demonstrate the usability of the method and the control of a good quality for relatively high ratio between the time delay and the time constant ($\tau_d /\tau \approx 6$ for a stable and $\tau_d /\tau \approx 1.25$ for an unstable FOTDS). The procedure makes possible a tuning of the controller parameters by a single selectable parameter. Using derived formulas, the controller parameters can be automatically computed. From this reason, the method could also be used for an adaptive control.

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