VARIABLE CAUSALITY DYNAMICAL SYSTEMS

Constantin Marin

Faculty of Automation and Computers,
University of Craiova, Romania
E-mail: cmarin@automation.ucv.ro

Abstract: This paper deals with dynamical systems which models physical objects whose causal input-output ordering is changing during their evolution. Such a system is named Variable Causality Dynamical System (VCDS). VCDSs are controlled from outside by a new input called causal ordering signal sharing the same set of state variables. In VCDS all the variables, except the causal ordering signal, are gathered in two forms of so called global variables as current global variable and desired global variable. In this paper different approaches of the causality concept are analysed and there are proposed formal definitions for covariance and causality properties of variables and relations irrespective of the time domain. An example of nonlinear VCDS is presented here but many applications in walking robots of the VCDS approach are developed including simulations in Matlab environment.
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Keywords: dynamical systems, covariance, causality, inference.

1. INTRODUCTION

The causality concept is a very general one and it still is not precisely defined. Causality is a subject of debates in very different scientific communities from philosophy, biology, till technical and social sciences.
The structure of the so-called causal theories and some fundamental forms of scientific inference are developed in /4/. According to this approach, a causal law is a statement that a change in the value of one variable is sufficient to produce a change in the value of another, without the operation of intermediate causes.
There are several different conceptions of cause as Positivist and Essentialists theories of causation, /11/. The first conception stresses the observations of regularities and from this point of view, high correlations demonstrate or are synonymous with causation. Actually, the causation is denied or it is considered a useless concept, saying, " Do not waste your time with unobserved entities.
On the other side in the Essentialists theories of causation, it is considered that, "Cause should only be used to refer to variables that explain phenomena in the sense that these variables, when taken together, are both necessary and sufficient for the effect to occur ", /11/, pp.5.
In the conception Stuart-Mill, /11/, three main factors determine an inference to be causal: a) Cause has to precede the effect; b) The cause and effect have to be related; c) Other explanations of the cause-effect relationship have to be eliminated.
Among the methods of causal inference analysis, in /11/ are mentioned:
1. Method of Agreement states an effect will be present when the cause is present, that means the cause is sufficient for the effect to occur.
2. Method of Differences states the effect will be absent when the cause is absent, that means the cause is necessary for the effect to occur.
2. Method of Concomitant Variations which implies that when both of the above relationships are observed, causal inference will be all the stronger since certain other interpretations of the covariation between the cause and effect can be ruled out.
There are many ways of knowing and different cultures uses different expectations and norms about causality, so much of the research process centers around what are the true causal or independent

variables /8/. Here some rules for cause and effect in nonexperimental studies are established.

One important direction of modeling that rise the problem of causality is that of the bond graphs, /2/, /3/, /10/. The so called hybrid bond graph augment traditional bond graph by a variable causality switching element to facilitate models with mixed continuous/discrete, hybrid behavior.

Bond graphs rely on a small set of basic elements that interact throughout power bonds. These bonds are acausal and connect to ports, /10/.

As it is stipulated in /3/, pp.3, an important consequence of an interconnection of submodels according to the physical structure of the overall system model is that submodels must be acausal. This corresponds to interconnect acausal bond graph submodels and to assign causalities to the overall bond graph after the hierarchy has been resolved.

The causal form of element equations is governed by the interconnections of submodels. If a model contains ideal switches, their state dependent causality is expressed in a textual language /3/.

An investigation of causal state theory and graphical causal models with applications in computational mechanics and the so called ε-machines is developed in /6/.

The problem of variable causality applied to power electronic converters is analyzed in /5/.

In /12/ it is argued that a regularity notion of causality can only be meaningfully defined for systems with linear interactions among their variables, with particular reference to the problem of causal inference in complex genetic systems.

A new approach of dynamical systems conceived by Willems /13/, /14/, ignores the input-output causal ordering. It defines the so called the behavioral approach of dynamical systems formed by the triptych with the behavior of the system in the center, the behavioral equations and latent variables as side notions.

2. CAUSAL VARIABLES AND CAUSALITY RELATIONS

The fundamental notion in mathematical modeling of objects (physical or abstract) is that of variable. A variable V is the triple

$$V = \{\mathrm{v}, \mathrm{V}, \mathbb{V}\} \quad , \quad \mathrm{v} \in \mathrm{V} \subseteq \mathbb{V} \tag{1}$$

where: $\mathbb{V}$ named variable universe, is a set endowed with a well defined mathematical structure; V named variable domain, is a subset of $\mathbb{V}$; v named the variable instant, is the generic element of V.

The variable V is finite dimensional of the order p if it exists an one-to-one application $\mathrm{V} \to \mathbb{R}^p$. The instant v of an n-dimensional variable can be presented as a p-tuple of scalar (one dimensional) instants, $\mathrm{v} = \{\mathrm{v}^i\}_{i=1:p}$.

Let $\boldsymbol{R}(V_i, V_j)$ be a binary relation on the Cartesian product $\mathrm{V}_i \times \mathrm{V}_j$, called a relation between the variable V_i and V_j,

$$\boldsymbol{R}(V_i, V_j) \subseteq \mathrm{V}_i \times \mathrm{V}_j \subseteq \mathbb{V}_i \times \mathbb{V}_j . \tag{2}$$

Such a relation, as a crisp relation, can be expressed by its membership function (characteristic function),

$$\mu_R(\mathrm{v}_i, \mathrm{v}_j) = \begin{cases} 1 & \text{if } (\mathrm{v}_i, \mathrm{v}_j) \in \boldsymbol{R}(V_i, V_j) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

A relation can be described by an equation defined as an equilibrium condition /15/, on an equating space $\mathbb{E}$

$$\mathrm{f}_1(\mathrm{v}_i, \mathrm{v}_j) = \mathrm{f}_2(\mathrm{v}_i, \mathrm{v}_j), \quad \mathrm{f}_1, \mathrm{f}_2 : \mathrm{V}_i \times \mathrm{V}_j \to \mathbb{E} . \tag{4}$$

If the equation space $\mathbb{E}$ is a finite m-dimensional linear space, then the equilibrium condition (4) can be expressed as an equation

$$\mathrm{R}(\mathrm{v}_i, \mathrm{v}_j) = 0 \ , \tag{5}$$

where

$$\mathrm{R}(\mathrm{v}_i, \mathrm{v}_j) = \mathrm{f}_1(\mathrm{v}_i, \mathrm{v}_j) - \mathrm{f}_2(\mathrm{v}_i, \mathrm{v}_j) = 0$$
$$\mathrm{R} : \mathrm{V}_i \times \mathrm{V}_j \to \mathbb{E} .$$

The equilibrium equation (5) is called the existence equation for the relation (2), if each pair $(\mathrm{v}_i, \mathrm{v}_j) \in \boldsymbol{R}(V_i, V_j)$ verifies equation (2) and any solution $(\mathrm{v}_i, \mathrm{v}_j)$ of $\mathrm{R}(\mathrm{v}_i, \mathrm{v}_j) = 0$, belongs to $\boldsymbol{R}(V_i, V_j)$. In such a case, there is the equivalence

$$\boldsymbol{R}(V_i, V_j) \Leftrightarrow \mathrm{R}(\mathrm{v}_i, \mathrm{v}_j) = 0 \tag{6}$$

Such a relation is an m-order relation, which involves m restrictions on its variables.

A relation $\boldsymbol{R}(V_i, V_j)$ is well defined if its projection on each variable universe equals to the corresponding variable domain,

$$\Pr\{\boldsymbol{R}(V_i, V_j) / \mathbb{V}_i\} = \mathrm{V}_i \tag{7}$$

$$\Pr\{\boldsymbol{R}(V_i, V_j) / \mathbb{V}_j\} = \mathrm{V}_j \tag{8}$$

where

$$\Pr\{\boldsymbol{R}(V_i, V_j) / \mathbb{V}_i\} = \{\mathrm{v}_i \in \mathrm{V}_i, \exists \mathrm{v}_j \in \mathrm{V}_j, \mu_{\boldsymbol{R}}(\mathrm{v}_i, \mathrm{v}_j) = 1\} \tag{9}$$

$$\Pr\{\boldsymbol{R}(V_i, V_j) / \mathbb{V}_j\} = \{\mathrm{v}_j \in \mathrm{V}_j, \exists \mathrm{v}_i \in \mathrm{V}_i, \mu_{\boldsymbol{R}}(\mathrm{v}_i, \mathrm{v}_j) = 1\} \tag{10}$$

are the projection of $\boldsymbol{R}$ on the universes $\mathbb{V}_i, \mathbb{V}_j$ respectively.

Two variables, V_i, V_j are covariant variables if there is a nonempty set of index $\mathrm{A} \ni \alpha$ and a nonempty set variables

$$\boldsymbol{G}_\mathrm{A} = \{G_\alpha\}_{\alpha \in \mathrm{A}}, \ G_\alpha = \{\mathrm{g}_\alpha, \mathrm{G}_\alpha, \mathbb{G}_\alpha\} \tag{11}$$

called the set of intermediate variables, such a way for each intermediate variable G_α there are two families of functions $\mathrm{F}^\alpha_{\mathrm{X}^\alpha_i}$, $\mathrm{F}^\alpha_{\mathrm{X}^\alpha_j}$ called covering functions of the relation

$$\mathbf{F}^\alpha_{\mathrm{X}^\alpha_i} = \{\mathrm{f}^\alpha_{\mathrm{x}^\alpha_i}\}_{\mathrm{x}^\alpha_i \in \mathrm{X}^\alpha_i \subseteq \mathbb{X}^\alpha_i}, \ \mathrm{f}^\alpha_{\mathrm{x}^\alpha_i} : \mathrm{G}_\alpha \to \mathrm{V}_i , \tag{12}$$

$$\mathbf{F}^{\alpha}_{X^{\alpha}_j} = \{f^{\alpha}_{x^{\alpha}_j}\}_{x^{\alpha}_j \in X^{\alpha}_j \subseteq \mathbb{X}^{\alpha}_j} \ , \ f^{\alpha}_{x^{\alpha}_j} : G_{\alpha} \to V_j \qquad (13)$$

so that each family cover the relation

$$v_i = f^{\alpha}_{x^{\alpha}_i}(g_{\alpha}) \, , \ \bigcup_{x^{\alpha}_i \in X^{\alpha}_i} f^{\alpha}_{x^{\alpha}_i}(G_{\alpha}) = V_i \qquad (14)$$

$$v_j = f^{\alpha}_{x^{\alpha}_j}(g_{\alpha}) \, , \ \bigcup_{x^{\alpha}_j \in X^{\alpha}_j} f^{\alpha}_{x^{\alpha}_j}(G_{\alpha}) = V_j \, . \qquad (15)$$

The two families $F^{\alpha}_{X^{\alpha}_i}$, $F^{\alpha}_{X^{\alpha}_j}$ are called parameter families of functions.

The triple $\{\alpha, f^{\alpha}_{x^{\alpha}_i}, f^{\alpha}_{x^{\alpha}_j}\}$ is called an instant of the covariant variables V_i, V_j and relations (14), (15) express one parametric representation of the two covariant variables.

The parametric representation (14), (15) may be expressed also as

$$v_i = f^{\alpha}_i(g_{\alpha}, x^{\alpha}_i) \qquad (16)$$

$$v_j = f^{\alpha}_j(g_{\alpha}, x^{\alpha}_j) \, , \qquad (17)$$

where x^{α}_i, x^{α}_j are instants of the new variables,

$$X^{\alpha}_i = \{x^{\alpha}_i, X^{\alpha}_i, \mathbb{X}^{\alpha}_i\} \, , \qquad (18)$$

$$X^{\alpha}_j = \{x^{\alpha}_j, X^{\alpha}_j, \mathbb{X}^{\alpha}_j\} \qquad (19)$$

called state variables.

The covering conditions on the families of parametric functions $f^{\alpha}_{x^{\alpha}_i}, f^{\alpha}_{x^{\alpha}_j}$ will assure each variable V_i and V_j to be completely involved in the covariance, namely

$$\forall v_i \in V_i \, , \exists (\alpha \in A, g_{\alpha} \in G_{\alpha}, f^{\alpha}_{x^{\alpha}_i} \in \mathbf{F}^{\alpha}_{X^{\alpha}_i}) \qquad (20)$$

so that

$$v_i = f^{\alpha}_{x^{\alpha}_i}(g_{\alpha}) = f^{\alpha}_i(g_{\alpha}, x^{\alpha}_i) \qquad (21)$$

or,

$$\forall v_j \in V_j \, , \exists (\alpha \in A, g_{\alpha} \in G_{\alpha}, f^{\alpha}_{x^{\alpha}_j} \in \mathbf{F}^{\alpha}_{X^{\alpha}_j}) \qquad (22)$$

so that

$$v_j = f^{\alpha}_{x^{\alpha}_j}(g_{\alpha}) = f^{\alpha}_j(g_{\alpha}, x^{\alpha}_j) \, . \qquad (23)$$

So for each pair $(v_i, v_j) \in \boldsymbol{R}(V_i, V_j)$, and each family $F^{\alpha}_{X^{\alpha}_i}$ or $F^{\alpha}_{X^{\alpha}_j}$, it exists a function whose graphic to contain this pair (v_i, v_j). These families are labeled by the variables $X^{\alpha}_i, X^{\alpha}_j$, which become state variables.

If relations (20), (21) are true it is not necessary (22), (23) to be true too and vice-versa.

One instant $\{\alpha, f^{\alpha}_{x^{\alpha}_i}, f^{\alpha}_{x^{\alpha}_j}\}$ of two covariant variables V_i, V_j defines a relation, called covariance relation,

$$\boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}(V_i, V_j) \subseteq V_i \times V_j \subseteq \mathbb{V}_i \times \mathbb{V}_j \qquad (24)$$

$$\boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}(V_i, V_j) =$$

$$= \{(v_i, v_j), v_i = f^{\alpha}_{x^{\alpha}_i}(g_{\alpha}), v_j = f^{\alpha}_{x^{\alpha}_j}(g_{\alpha}), \forall g_{\alpha} \in G_{\alpha}\}$$

whose membership function is,

$$\mu_{\boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}}(v_i, v_j) = \begin{cases} 1 & \text{if } (v_i, v_j) \in \boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}(V_i, V_j) \\ 0 & \text{otherwise} \end{cases} \qquad (25)$$

Because the relation $\boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}(V_i, V_j)$ could be different of $V_i \times V_j$, reflects the covariant character of the two variables.

Two variables V_i, V_j are well covariant if for any instant $\{\alpha, f^{\alpha}_{x^{\alpha}_i}, f^{\alpha}_{x^{\alpha}_j}\}$ the set of correlated pairs $\boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}(V_i, V_j)$ is the same, which denoted $\boldsymbol{R}_{ij}(V_i, V_j)$

$$\boldsymbol{R}^{\alpha}_{X^{\alpha}_i X^{\alpha}_j}(V_i, V_j) = \boldsymbol{R}_{ij}(V_i, V_j) \, , \ \forall \{\alpha, f^{\alpha}_{x^{\alpha}_i}, f^{\alpha}_{x^{\alpha}_j}\} \qquad (26)$$

Between two well covariant variables, V_i, V_j it exists a nonempty binary relation

$$\boldsymbol{R}_{ij}(V_i, V_j) \subseteq V_i \times V_j \subseteq \mathbb{V}_i \times \mathbb{V}_j \qquad (27)$$

but does not exist a function type dependency between them.

The covariance relation (26) can be expressed by an equilibrium equation

$$R_{ij}(v_i, v_j) = 0, (v_i, v_j) \in V_i \times V_j \subseteq \mathbb{V}_i \times \mathbb{V}_j \qquad (28)$$

but has no pysical meaning to withdraw from it the function type dependencies $v_i = f_i(v_j)$ or $v_j = f_j(v_i)$.

Two variables V_i, V_j are called independent variables if it is not posible to establish a covariance relation between them.

The variables V_i, V_j are independent if one of the three conditions takes place:

$$A = \varnothing \ \text{ or } \ \boldsymbol{G}_A = \varnothing \qquad (29)$$

$$\mathbf{F}^{\alpha}_{X^{\alpha}_i} = \varnothing, \forall \alpha \in A \qquad (30)$$

$$\mathbf{F}^{\alpha}_{X^{\alpha}_j} = \varnothing, \forall \alpha \in A \qquad (31)$$

Two any variables V_i, V_j which could be covariant but for which a covariance relation is not established yet are called uncharacterized variables. Uncharacterized variables are considered to be independent variables.

Two covariant variables V_i, V_j characterised by $A, \boldsymbol{G}_A, F^{\alpha}_{X^{\alpha}_i}, F^{\alpha}_{X^{\alpha}_j}$ are called causal variables if

$$A = \{\alpha_1, \alpha_2\} \, , \ \alpha_1 = ij, \ \alpha_2 = ji \qquad (32)$$

$$\boldsymbol{G}_A = \{G_{\alpha_1}, G_{\alpha_2}\} = \{G_{ij}, G_{ji}\} = \{V_i, V_j\} \qquad (33)$$

$$F^{\alpha}_{X^{\alpha}_i} \neq \varnothing, \ F^{\alpha}_{X^{\alpha}_j} \neq \varnothing \, . \qquad (34)$$

The parametric representation (16), (17) takes the following two explicite forms (35), (36) and (39), (40).

For $\alpha = \alpha_1 = ij$

$$v_i = f^{ij}_i(v_i, x^{ij}_i) \, , \ x^{ij}_i \in X^{\alpha_1}_i = X^{ij}_i \qquad (35)$$

$$v_j = f^{ij}_j(v_i, x^{ij}_j) \, , \ x^{ij}_j \in X^{\alpha_1}_j = X^{ij}_j \, , \qquad (36)$$

which illustrates the so called causality $V_i \rightarrow V_j$ where x_i^{ij}, x_j^{ij} are the state instants of the respectively variables V_i, V_j in this causality ordering. The two state variables are,

$$X_i^{\alpha_1} = \{x_i^{\alpha_1}, X_i^{\alpha_1}, \mathbb{X}_i^{\alpha_1}\} = \{x_i^{ij}, X_i^{ij}, \mathbb{X}_i^{ij}\} = X_i^{ij}, \qquad (37)$$

$$X_j^{\alpha_1} = \{x_j^{\alpha_1}, X_j^{\alpha_1}, \mathbb{X}_j^{\alpha_1}\} = \{x_j^{ij}, X_j^{ij}, \mathbb{X}_j^{ij}\} = X_j^{ij} \qquad (38)$$

For $\alpha = \alpha_2 = ji$

$$v_i = f_i^{ji}(v_j, x_i^{ji}),\ x_i^{ji} \in X_i^{\alpha_2} = X_i^{ji} \qquad (39)$$

$$v_j = f_j^{ji}(v_j, x_j^{ji}),\ x_j^{ji} \in X_j^{\alpha_2} = X_j^{ji}, \qquad (40)$$

which illustrates the so called causality $V_j \rightarrow V_i$ where x_i^{ji}, x_j^{ji} are the state instants of the respectively variables V_i, V_j in this causality ordering. The two state variables are,

$$X_i^{\alpha_2} = \{x_i^{\alpha_2}, X_i^{\alpha_2}, \mathbb{X}_i^{\alpha_2}\} = \{x_i^{ji}, X_i^{ji}, \mathbb{X}_i^{ji}\} = X_i^{ji}, \qquad (41)$$

$$X_j^{\alpha_2} = \{x_j^{\alpha_2}, X_j^{\alpha_2}, \mathbb{X}_j^{\alpha_2}\} = \{x_j^{ji}, X_j^{ji}, \mathbb{X}_j^{ji}\} = X_j^{ji} \qquad (42)$$

Each state instant specify a function from the sets $F_{X_i^\alpha}^{\alpha}$, $F_{X_j^\alpha}^{\alpha}$.

As the state variaable internally characterise a variable, the state variables attached to a variable must be the same irrespective of the causality ordering, so

$$X_i^{ij} = X_i^{ji} = X_i = \{x_i, X_i, \mathbb{X}_i\}$$

$$X_j^{ij} = X_j^{ji} = X_j = \{x_j, X_j, \mathbb{X}_j\}$$

and the parametic equations

Any binary relation between two covariant variables is called covariance relation.

Two variables V_i, V_j are called causal variables if they are covariant variables with the intermediate variables set having one of the three form

$$\boldsymbol{\emptyset} = \{V_i\},\ \boldsymbol{\emptyset} = \{V_j\},\ \boldsymbol{\emptyset} = \{V_i, V_j\}. \qquad (11)$$

Any binary relation between two causal variables is called a causal relation.

All the above notions can be extended to n-ary relations,

$$\boldsymbol{R}(V_1, V_2 ..., V_n) = \boldsymbol{R}(V),\ V = \{V_1, V_2 ..., V_n\} \quad (12)$$

In addition, n variables $V_1, V_2 ..., V_n$ are covariant, independent or causal if any two variables of them have the above-defined properties.

3. CAUSAL ORDERING IN DYNAMICAL SYSTEMS

Several definitions of the system notion there are well known from any System Theory textbook /1/, /7/, /16/. According to the Webster's definition, "a system is a set of physical objects or abstract entities connected or related by different forms of interactions or interdpendencies as to form an entirety or a whole" /9/. According to the definition issued from thermodynamics, " a system is a part (a fragment) of the universe for which one inside and one outside can be delimited from behavioral point of view".

The mathematical model of a physical system is a pair

$$\boldsymbol{S} = \{V, \boldsymbol{R}\} \qquad (13)$$

where $V = \{V_i\}_{i=1:n}$ is a set of variables and $\boldsymbol{R} = \boldsymbol{R}(V)$ is a causal relation between them.

If p_i is the order of the variable V_i, then there are $p = \sum_{i=1}^{n} p_i$ scalar components of the variables involved in system. Suppose that the relation $\boldsymbol{R}$ is of the order m.

The system acts as a restriction among its variables and delimits its inside. All the other variables and relations different of V and $\boldsymbol{R}$, denoted $\bar{V}$ and $\bar{\boldsymbol{R}}$ belong to the outside of the system denoted by $\bar{\boldsymbol{S}}$

$$\bar{\boldsymbol{S}} = \{\bar{V}, \bar{\boldsymbol{R}}\} \qquad (14)$$

as depicted in Fig. 1.

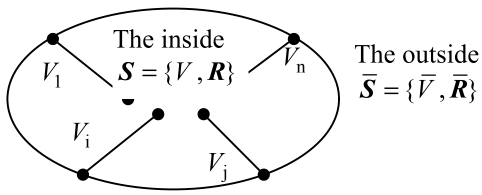


Fig 1. The structure of an un-oriented system.

This is the so-called an un-oriented system and it is similar to the model of Willems in his behavioral approach of systems /13/, /14/, /15/.

As the model represents a physical system, the relation $\boldsymbol{R}$, called also the existence relation, is true as far as the physical system exists.

At this stage, looking at the un-oriented system $\boldsymbol{S}$, we can observe only the instants of the variables $v = \{v_i\}_{i=1:n}$. One instant v is a realization of the physical system and it verifies the existence relation $\boldsymbol{R}$.

The un-oriented system looks like an isolated one from the universe it belongs to, so any variable $W \in \bar{\boldsymbol{S}}$ is independent with respect to any variable $\{V_i\}_{i=1:n}$ of $\boldsymbol{S}$.

But, any physical system whose model is $\boldsymbol{S}$, as a part of the universe, is not isolated one, it has changes of energy, material and information with the outside.

Let W_k be a variable from the ouside of $\boldsymbol{S}$,

$$W_k = \{w_k, W_k, \mathbb{W}_k\} \in \bar{\boldsymbol{S}}. \qquad (15)$$

A variable $W_k \in \bar{\boldsymbol{S}}$ is assigned to a variable $V_i \in \boldsymbol{S}$

$$V_i = \{v_i, V_i, \mathbb{V}_i\} \in \boldsymbol{S}, \qquad (16)$$

denoted $W_k \rightarrow V_i$, if

$$\mathbb{W}_k = \mathbb{V}_i,\ W_k \subseteq V_i,\ v_i = w_k \qquad (17)$$

that means the instant v_i takes the value of the instant w_k. Through this assignment process, a new variable $U_j \in \boldsymbol{S} \cap \overline{\boldsymbol{S}}$ is created,

$$U_r = \{u_r, U_r, \mathbb{U}_r\} = \{w_k, V_i, \mathbb{V}_i\}. \tag{18}$$

The variable U_j is a cause for the system $\boldsymbol{S}$ or an input variable. The set of variables $\{W_k\}_{k=1,2,..}$ assigned to the system $\boldsymbol{S}$, could be independent variables which is not a necessary condition.

Let $Y_k = \{y_k, Y_k, \mathbb{Y}_k\} \in \overline{\boldsymbol{S}}$ be a variable from outside. If $\mathbb{Y}_k = \mathbb{V}_j$, $Y_k = V_j$, $y_k = v_j$ then the variable $V_j \in \boldsymbol{S}$ is assigned to the outside world so it becomes an output of the system $\boldsymbol{S}$. This process of variables causal ordering is illustrated in Fig. 2.

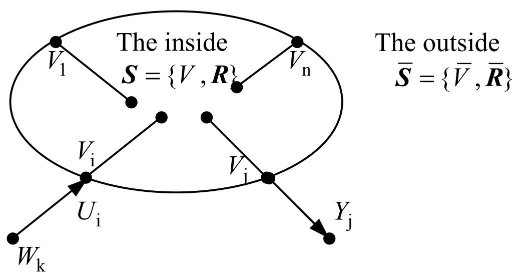


Fig 2. The structure of a system with oriented variables.

4. VARIABLE CAUSALITY DYNAMICAL SYSTEMS

There are many physical systems where the causal ordering is controlled from outside expressed by a new variable

$$q \in Q = \{q_\beta\}_{\beta \in B} \tag{19}$$

and the terminal variables $V = \{V_1, V_2 ..., V_n\}$ are some times inputs, and another time outputs. Let $X = \{X_1, X_2 ..., X_n\}$ be the state variables attached to each terminal variable in such a way to reestablish the univocity of the selected variables as being outputs with respect to the variables selected as to be inputs. They are internal variables.

In the case of VCDS description, there are no explicitly input and output variables. All the variables, terminal variables $V = \{V_1, V_2 ..., V_n\}$ and internal variables, satisfy the existence relation of the system named System Existence Relation (ER) $\boldsymbol{R}(V_1, V_2 ..., V_n) = \boldsymbol{R}(V)$.

As the system exist, the SER is true according to the causality ordering available at that time instant.

Particularly let we consider that any variable without time index (t, k), depending of the context, to be interpreted its value at the continuous time or dicrete time.

Let we denote by

$$\xi = \{V_1, V_2 ..., V_n, X_1, X_2 ..., X_n\} \tag{20}$$

the current value of the so called global variable of the system and denote by

$$\hat{\xi} = \{\hat{V}_1, \hat{V}_2 ..., \hat{V}_n, \hat{X}_1, \hat{X}_2 ..., \hat{X}_n\} \tag{21}$$

the desired value of the global variable.

The VCDS evolution equation is of the form,

$$\xi = f(\hat{\xi}, q) \tag{22}$$

For algebraic dynamical systems, the equation (22) takes the form,

$$\xi_{t+\varepsilon} = f(\xi_t, \hat{\xi}_t, q_t) \tag{23}$$

5. EXAMPLE

Let us consider a sytem about it is known having a causal relation $\boldsymbol{R}_{ij}(V_i, V_j)$ as in Fig. 3.a.

In the causality ordering $V_i \to V_j$ the causal relation can be covered by a unique function so the label set X_j contains only one element, let we denote it s_j^1 so $x_j \in X_j = \{s_j^1\}$ and the input-output relation, in this causal ordering is covered by a unique function $v_j = f_j^{ij}(v_i, s_j^1)$ and the state transition equation is $x_j = s_j^1, \forall v_j$.

The input-state-output relations in $V_i \to V_j$ causality are:

$$v_j = f_j^{ij}(v_i, x_j)$$

$$x_j = s_j^1, \forall v_j$$

In the causal ordering $V_j \to V_i$, the causal relation $\boldsymbol{R}_{ij}(V_i, V_j)$ is covered by three functions $v_i = f_i^{ji}(v_j, s_i^1)$, $v_i = f_i^{ji}(v_j, s_i^2)$, $v_i = f_i^{ji}(v_j, s_i^3)$ as depicted in Fig 3.b, c, d. Now the label set X_i contains three elements labeled s_i^1, s_i^2, s_i^3 so the state instant is $x_i \in X_i = \{s_i^1, s_i^2, s_i^3\}$.

The input-state-output relation in $V_j \to V_i$ causality are:

$$v_i = f_i^{ji}(v_j, x_i)$$

$$x_i = \begin{cases} s_i^1, & v_i \in [v_i^0, v_i^1) \\ s_i^2, & v_i \in [v_i^1, v_i^2) \\ s_i^3, & v_i \in [v_i^2, v_i^3] \end{cases}$$

The global variable is

$$\xi = \{V_i, V_j X_i, X_j\}$$

where

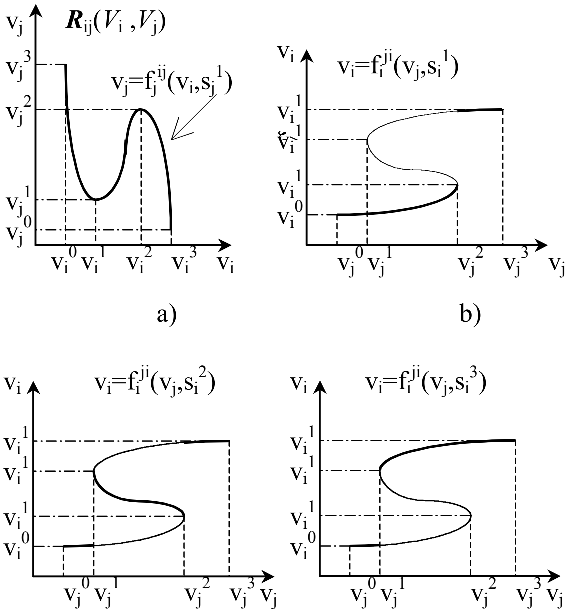
$$V_i = \{v_i, V_i, \mathbb{V}_i\} = \{v_i, [v_i^0, v_i^3], \mathbb{R}\}$$

$$V_j = \{v_j, V_j, \mathbb{V}_j\} = \{v_j, [v_j^0, v_j^3], \mathbb{R}\}$$

$$X_i = \{x_i, X_i, \mathbb{X}_i\} = \{x_i, \{s_i^1, s_i^2, s_i^3\}, \{s_i^1, s_i^2, s_i^3\}\}$$

$$X_j = \{x_j, X_j, \mathbb{X}_j\} = \{x_j, \{s_j^1\}, \{s_j^1\}\}$$

Such a variable causality structure is very easy to implement for simulations as an element interconnected with other dynamical subsystems.



c) d)
d)

Fig 3. Example of a nonlinear VCDS.

6. CONCLUSIONS

This paper deals with dynamical systems which models physical objects whose causal input-output ordering is changing during their evolution. Such a system is named Variable Causality Dynamical System (VCDS). VCDSs are controlled from outside by a new input called causal ordering signal sharing the same set of state variables. In VCDS all the variables, except the causal ordering signal, are gathered in two forms of so called global variables as current global variable and desired global variable. There are proposed formal definitions for covariance and causality properties of variables and relations irrespective of the time domain. Applications in walking robots of the VCDS approach developed including simulations in Matlab environment prove the advantages of this approach.

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