Stable Fuzzy Control of an Electromagnetic Actuated Clutch

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Abstract: This paper presents aspects concerning the stable design of fuzzy control solutions for the position control of an electromagnetic actuated clutch. The mathematical modelling of the plant is first solved offering the plant models expressed as first principle nonlinear models, linearized models, dynamic Takagi-Sugeno fuzzy models and Tensor Product models derived from Linear Parameter-Varying models. The PI controllers are tuned by the Modulus Optimum method. The Takagi-Sugeno PI-fuzzy controllers are tuned on the basis of the modal equivalence principle which maps the parameters of the linear PI controllers onto the parameters of the fuzzy ones. The Takagi-Sugeno state feedback fuzzy controllers are designed by Parallel Distributed Compensation to obtain the state feedback gain matrices in the consequents of the rules. The stability of all fuzzy control systems is guaranteed in terms of deriving stability conditions expressed as Linear Matrix Inequalities (LMIs) and solving the LMIs.

Keywords: Electromagnetic actuated clutch, stable fuzzy control, Takagi-Sugeno fuzzy models, Takagi-Sugeno PI-fuzzy controllers, Tensor Product models.

1. INTRODUCTION

The electromagnetic actuated clutch is an important system in the framework of automotive applications (Kiencke and Nielsen, 2005; Isemann, 2005). Very good control system performance indices should be ensured by the control systems of electromagnetic actuated clutches viewed as actuators in automotive control systems. Some current control solutions for such applications concern internal model control with two-degree-of-freedom (2 DOF) PID controllers (Zhang et al., 2006), model predictive control (Di Cairano et al., 2007a), and one-degree-of-freedom (1 DOF) and 2 DOF fuzzy control (Dragoş et al., 2010).

This paper is developed starting with the previous results on the nonlinear and linearized models of the controlled plant and on the 1 DOF Takagi-Sugeno PI-fuzzy controllers discussed in (Dragoş et al., 2010). The stable design of these fuzzy control systems is suggested. Using the Tensor Product (TP) models derived from Linear Parameter-Varying (LPV) models proposed in (Precup et al., 2010) this paper offers new dynamic Takagi-Sugeno fuzzy models of the controlled plant on the basis of the modal equivalence principle. Original Takagi-Sugeno state feedback fuzzy controllers are next designed. The stable design of all fuzzy control systems is ensured by the combination between the Parallel Distributed Compensation (PDC) and the stability conditions expressed as Linear Matrix Inequalities (LMIs) which are popular in the fuzzy control systems design (Tanaka and Wang, 2001; Lam, 2009; Sala, 2009; Lendek et al., 2010).

This paper is organized as follows. The mathematical modelling of the controlled plant is presented in the next section. Section 3 is dedicated to the stable design of the fuzzy controllers. A sample of digital simulation results for a case study concerning the position control of an electromagnetic actuated clutch is presented in Section 4. The conclusions are pointed out in Section 5.

2. MATHEMATICAL MODELLING OF CONTROLLED PLANT

The state-space mathematical model (MM) of the electromagnetic actuated clutch built around a magnetically mass spring damper system is (Di Cairano et al., 2007b; Dragoş et al., 2009; Dragoş, 2009a; Dragoş, 2009b; Lazăr et al., 2009; Dragoş et al., 2010)

$$\begin{aligned}
&\dot{x}_1(t) = x_2(t),\\
&\dot{x}_2(t) = -\frac{k}{m}x_1(t) - \frac{c}{m}x_2(t) + \frac{k_a}{m\,k_b^2}x_3^2(t),\\
&\dot{x}_3(t) = -\frac{R\,k_b}{2k_a}x_3(t) + \frac{1}{k_b}x_2(t)x_3(t) + \frac{k_b}{2k_a}V(t),\\
&z(t) = x_1(t),
\end{aligned} \tag{1}$$

where: $x_1 = x$ – the mass position, $x_2 = \dot{x}$ – the mass speed, $x_3 = i$ – the current, V – the control signal, z – the controlled output, k – the stiffness of the spring, c – the coefficient of the damper, R – the resistance of the resistive circuit subjected to magnetic flux variations according to Faraday's law, k_a, k_b – the constants which are defined in the relation between the magnetic flux and the current, and t – the independent (continuous) time variable. The parameter values corresponding to the state-space MM presented in (1), which plays the role of controlled plant, are illustrated in (Dragoş, 2009a; Dragoş, 2009b; Lazăr et al., 2009).

The linearization of the MM in (1) around the ten operating points

$$\begin{aligned}
&1:\{x_{10}=0.0033, && x_{30}=1, && V_0=1.2\},\\
&2:\{x_{10}=0.0027, && x_{30}=2, && V_0=2.4\},\\
&3:\{x_{10}=0.0023, && x_{30}=3, && V_0=3.6\},\\
&4:\{x_{10}=0.0021, && x_{30}=4, && V_0=4.8\},\\
&5:\{x_{10}=0.002, && x_{30}=5, && V_0=6\},\\
&6:\{x_{10}=0.0021, && x_{30}=6, && V_0=7.2\},\\
&7:\{x_{10}=0.0023, && x_{30}=7, && V_0=8.4\},\\
&8:\{x_{10}=0.0027, && x_{30}=8, && V_0=9.6\},\\
&9:\{x_{10}=0.0033, && x_{30}=9, && V_0=10.8\},\\
&10:\{x_{10}=0.0038, && x_{30}=9.8, && V_0=11.76\},
\end{aligned} \tag{2}$$

leads to ten linearized state-space MMs of the controlled plant with the general expression

$$\begin{aligned}
&\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,\Delta V(t),\\
&\Delta z(t) = \mathbf{c}^T\mathbf{x}(t),\\
&\mathbf{x} = [x_1 = x \quad x_2 = \dot{x} \quad x_3 = \lambda]^T,\\
&\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -k/m & -c/m & 2k_a x_{30}/(mk_b^2) \\ 0 & x_{30}/k_b & -x_{20}/k_b - Rk_a/(2k_b) \end{bmatrix},\\
&\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ k_b/(2k_a) \end{bmatrix}, \mathbf{c}^T = [1 \quad 0 \quad 0].
\end{aligned} \tag{3}$$

However considering separately the coordinates of the operating points in (2) this leads to a maximum of 1000 linearized state-space MMS of the controlled plant of type (3).

Using the following notations for the variations of the variables with respect to the coordinates of the operating points

$$u = \Delta V, \; y = \Delta z, \tag{4}$$

the state-space MM in (3) will obtain the well accepted form

$$\begin{aligned}
&\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t),\\
&y(t) = \mathbf{c}^T\mathbf{x}(t),
\end{aligned} \tag{5}$$

where u is the control signal and y is the controlled output, i.e. the mass position.

The transfer functions (t.f.s) of the linearized MMs of the controlled plant in (3) or (5) are

$$P(s) = \frac{y(s)}{u(s)} = \frac{k_P}{(1+sT_1)(1+sT_2)(1+sT_3)}, \tag{6}$$

where the zero initial conditions are considered, and the three time constants fulfil the condition

$$T_1 > T_2 > T_3. \tag{7}$$

The dynamic Takagi-Sugeno fuzzy models of the controlled plant are based on the three input variables which set the coordinates of the operating points in (2), x_1, x_3 and V (which can be replaced by u according to (4)). All membership functions of the input linguistic terms are defined such that their modal values are the coordinates of a subset of the operating points in (2). Selecting o operating points in (2) the complete rule base of this fuzzy model contains $n_R = o^3$ rules, R^i, $i = \overline{1, n_R}$. The consequents of the rules correspond to the linearized state-space MMs defined in (5). The modal equivalence principle (Galichet and Foulloy, 1995) guarantees the equivalence between the fuzzy models and the nonlinear state-space ones.

The linguistic terms $T_{x_1,l}$, $l = \overline{1, o}$, $T_{x_3,m}$, $m = \overline{1, o}$, and $T_{V,n}$, $n = \overline{1, o}$, are defined for x_1, x_3 and V, respectively such that to respect the modal equivalence principle. Fig. 1 exemplifies the membership functions of the linguistic terms afferent to the input variable x_1 for $o = 3$ operating points, where $x_{10,1}$, $x_{10,2}$ and $x_{10,3}$ are the modal values of the input variable x_1.

The complete rule base of the continuous-time dynamic T-S fuzzy model is

$$\begin{aligned}
&R^i: \text{IF } x_1(t) \text{ IS } T^i_{x_1,l} \text{ AND } x_3(t) \text{ IS } T^i_{x_3,m} \text{ AND}\\
&V(t) \text{ IS } T^i_{V,n} \text{ THEN} \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{b}_i u(t) \\ y(t) = \mathbf{c}_i^T\mathbf{x}(t) \end{cases}, \; i = \overline{1, n_R},
\end{aligned} \tag{8}$$

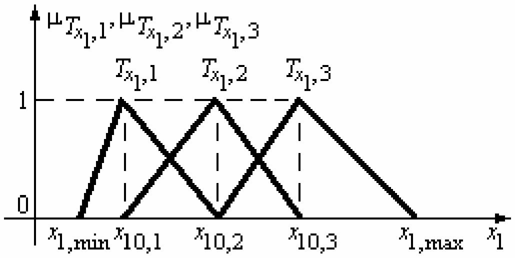

Fig. 1. Membership functions of the linguistic terms afferent to the input variable x_1 for $o = 3$.

where the matrices are expressed in (3).

The discretization of the n_R models in the consequents of the Takagi-Sugeno fuzzy model (8) accepting the zero-order hold and setting the value of the sampling period leads to the following rule base of the discrete-time dynamic Takagi-Sugeno fuzzy model of the controlled plant:

$$\begin{aligned}&R^i : \text{IF } x_{1,k} \text{ IS } T^i_{x_1,l} \text{ AND } x_{3,k} \text{ IS } T^i_{x_3,m} \text{ AND}\\ &V_k \text{ IS } T^i_{V,n} \text{ THEN} \begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_{d,i}\mathbf{x}_k + \mathbf{b}_{d,i}u_k \\ y_k = \mathbf{c}_i^T \mathbf{x}_k \end{cases}, \; i = \overline{1, n_R},\end{aligned} \tag{9}$$

where k is the index of the current sampling interval.

All Takagi-Sugeno fuzzy models of the plant and all Takagi-Sugeno fuzzy models of the PI-fuzzy controllers and of the state-feedback Takagi-Sugeno fuzzy controllers as well use the SUM and PROD operators in the inference engine and the weighted average defuzzification method.

Accepting the bounded parameter vector $\mathbf{p}$ which is the following particular scalar:

$$\mathbf{p} = [p_1] = [x_3] \in R, \tag{10}$$

the state-space MM defined in (1) is expressed as the following LPV state-space MM:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u}(t),\\ y(t) &= \mathbf{C}(\mathbf{p})\mathbf{x} + \mathbf{D}(\mathbf{p})\mathbf{u}(t),\end{aligned} \tag{11}$$

where

$$y = z, \; \mathbf{u} = V \in R, \tag{12}$$

and the matrices are expressed as

$$\begin{aligned}\mathbf{A}(\mathbf{p}) &= \begin{bmatrix} 0 & 1 & 0 \\ -k/m & -c/m & k_a p_1/(m k_b^2) \\ 0 & p_1/k_b & -R k_b/(2k_a) \end{bmatrix},\\ \mathbf{B}(\mathbf{p}) &= \begin{bmatrix} 0 \\ 0 \\ k_b/(2k_a) \end{bmatrix},\\ \mathbf{C}(\mathbf{p}) &= \mathbf{C} = \mathbf{I}_3, \mathbf{D}(\mathbf{p}) = \mathbf{D} = [0],\end{aligned} \tag{13}$$

where $\mathbf{I}_3$ is the third order identity matrix.

Introducing the general parameter-varying system matrix

$$\mathbf{S}(\mathbf{p}) = \begin{pmatrix} \mathbf{A}(\mathbf{p}) & \mathbf{B}(\mathbf{p}) \\ \mathbf{C}(\mathbf{p}) & \mathbf{D}(\mathbf{p}) \end{pmatrix} \in R^{6\times 4} \tag{14}$$

the LPV MM presented in (11) is transformed into the following synthesized form:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ y(t) \end{pmatrix} = \mathbf{S}(\mathbf{p}) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}. \tag{15}$$

Since the matrices $\mathbf{C}$ and $\mathbf{D}$ are constant with respect to $\mathbf{p}$ the following simpler expression of the matrix $\mathbf{S}(\mathbf{p})$ in (14) will be used as follows:

$$\mathbf{S}(\mathbf{p}) = (\mathbf{A}(\mathbf{p}) \quad \mathbf{B}(\mathbf{p}) \in R^{3\times 4}. \tag{16}$$

Therefore the following LPV MM is derived:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{S}(\mathbf{p}) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix},\\ y(t) &= \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t).\end{aligned} \tag{17}$$

The goal of the TP-based model transformation is to transform the LPV state-space model of the controlled plant expressed in (17) into the following parameter-varying combination of Linear Time-Invariant (LTI) system matrices $\mathbf{S}_i = [\mathbf{A}_i \quad \mathbf{B}_i]$ called vertex systems (Baranyi, 2004; Baranyi et al., 2006; Matszangosz et al., 2008; Nagy et al., 2008; Baranyi et al., 2009):

$$\begin{aligned}&\dot{\mathbf{x}}(t) = S \overset{N}{\underset{n=1}{\otimes}} \mathbf{w}_n(\mathbf{p}_n) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix} =\\ &\sum_{i=1}^{I} w_{1,i}(\lambda) \mathbf{S}_i \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix},\\ &y(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t),\end{aligned} \tag{18}$$

where the row matrix $\mathbf{w}_n(\mathbf{p}_n)$ contains a bounded variable and its continuous weighting functions, N is the tensor's dimension, and S is the $(N+2)$-dimensional coefficient tensor.

The TP model expressed in (18) is convex because the weighting functions fulfil certain conditions.

The TP Tool (Nagy et al., 2007) is employed transform the TP model defined in (18) into different polytopic forms which depend on the number of singular values and the number of shapes of weighting functions. The following polytopic form results (for $I = 2$ in (18)) when the maximum singular values are kept the normal weighting functions presented in Fig. 2 are used:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{2} w_{1,i}(\lambda)[\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t)],\\ y(t) &= \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t).\end{aligned} \tag{19}$$

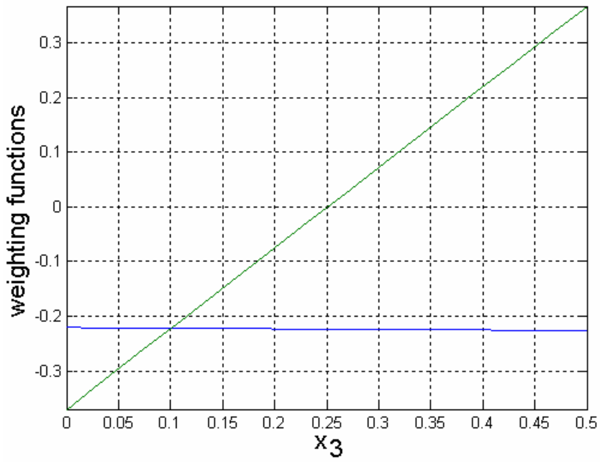

Fig. 2. Weighting functions of the TP model expressed in (18).

Digital simulation results concerning the behaviour of the TP model expressed in (10) are given in (Precup et al., 2010).

It should be pointed out that the models presented in this section can be applied with no major difficulties to other controlled plants and controller structures (Škrjanc et al., 2004; Johanyák and Kovács, 2006; Hermann, 2007; Kovács and Paláncz, 2007; Bellomo et al., 2008; Mok and Chan, 2008; Ferreira and Ruano, 2009; Harmati and Skrzypczyk, 2009; Vaščák, 2009; Haber et al., 2010; Ahn and Anh, 2010; Kurnaz et al., 2010).

3. STABLE DESIGN OF FUZZY CONTROLLERS

The design starts with the Takagi-Sugeno PI-fuzzy controller which is based on the fuzzy control system structure presented, where *r* is the reference input, *e* is the control error, *d* is the disturbance input, *y* is the controlled output, P is the controlled plant and the nonlinear block FB is the fuzzy block.

A low-cost design of the Takagi-Sugeno PI-fuzzy controller is based on the t.f.s $P(s)$ expressed in (6). The continuous-time PI controllers with the t.f. $C(s)$:

$$C(s)=\frac{k_c}{s}(1+T_c s)=k_C(1+\frac{1}{sT_i}),\quad T_c=T_i,\ k_c=\frac{k_C}{T_i}, \tag{20}$$

with the controller gain k_c and the integral time constant T_c, can ensure good control system performance indices when controlling $P(s)$ in (6) if the Modulus Optimum method is used. The tuning equations specific to the Modulus Optimum method applied to (6) are

$$T_c=T_1,\ k_c=\frac{1}{2k_P(T_2+T_3)}. \tag{21}$$

The continuous PI controller with the t.f. defined in (20) is discretized using Tustin's method after setting the value of the sampling period T_s. Five quasi-continuous digital PI controllers with the following recurrent equations are obtained when a low-cost fuzzy controller:

$$\begin{aligned}\Delta u_k^i&=\gamma(k_I^i\Delta e_k+\alpha k_P^i e_k)=\\ &-p^i e_{k-1}-q^i\Delta e_k,\\ p^i&=-\gamma\alpha k_P^i,\ q^i=-\gamma(k_I^i+\alpha k_P^i),\end{aligned} \tag{22}$$

where $i\in\{3,5,6,7,8\}$ (Dragoş et al., 2010) is the rule index which corresponds also to the index of the operating points defined in (2), Δe_k is the increment of control error, and Δu_k^i is the increment of control signal. The expressions of the parameters in the recurrent equations (22) are

$$k_P^i=k_C^i(1-\frac{T_s}{2T_i^i}),\ k_I^i=\frac{k_C^i T_s}{T_i^i},\ \alpha=\frac{k_I^i}{k_P^i}, \tag{23}$$

and the parameter γ, $0<\gamma<1$, is introduced to reduce the overshoot.

The fuzzification of the two input variables of the block FB, e_k and Δe_k, is performed in terms of three linguistic terms with the membership functions presented in Fig. 4 which shows that $o=3$ according to the notation accepted in the previous section.

The tuning equations of this Takagi-Sugeno PI-fuzzy controller resulted from the modal equivalence principle are (Dragoş et al., 2010)

$$B_e=0.01,\ B_{\Delta e}=(k_P^i/k_I^i)B_e=\alpha B_e=0.0002. \tag{24}$$

The rule base of the block FB is

$$\begin{aligned}&R^i:\text{IF } e_k \text{ IS LTE}_l^i \text{ AND } \Delta e_k \text{ IS LTDE}_m^i\\ &\text{THEN } \Delta u_k=\Delta u_k^i,\ i=\overline{1,5},\end{aligned} \tag{25}$$

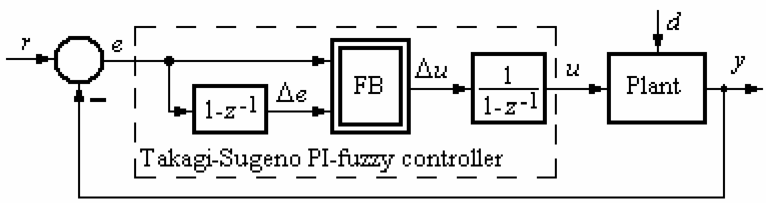

Fig. 3. Structure of fuzzy control system with Takagi-Sugeno PI-fuzzy controller.

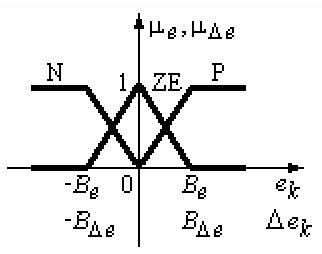

Fig. 4. Membership functions of the linguistic terms afferent to the input variables of the block FB.

where $\mathrm{LTE}_l^i, \mathrm{LTDE}_m^i \in \{\mathrm{N}, \mathrm{ZE}, \mathrm{P}\}$ are the linguistic terms of the two input linguistic variables.

The stable design of the Takagi-Sugeno PI-fuzzy controller requires all $n_R = o^2 = 3^2 = 9$ local quasi-continuous digital PI controllers resulted from (2) and (6). It ensures the calculation of the parameters in the rule consequents (22) such that to guarantee the global stability of the fuzzy control system. Accepting that the discrete-time state-space MMs in the consequents of the rules are those presented in (9):

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}_{d,i}\mathbf{x}_k + \mathbf{b}_{d,i}u_k, \\ y_k &= \mathbf{c}_i^T \mathbf{x}_k,\ i = \overline{1, n_R}, \end{aligned} \tag{26}$$

with the general form of the state vector $\mathbf{x}_k$

$$\mathbf{x}_k = [x_1 \quad x_2 \quad \dots \quad x_{nP}]^T, \tag{27}$$

where nP is the system order, $nP = 3$, and T indicates the matrix transposition, the following three additional state variables are defined such that to transfer the dynamics of the two linear blocks inside the Takagi-Sugeno PI-fuzzy controller to the (extended) controlled plant:

$$\begin{aligned} x_{nP+1,k} &= u_{k-1}, \\ x_{nP+2,k} &= e_{k-1}, \\ x_{nP+3,k} &= \Delta e_k. \end{aligned} \tag{28}$$

Using the equations (26) and (28), the fuzzy control system structure (Fig. 3) and assuming that $r = \mathrm{const}$ the following extended state-space MM of the controlled plant is derived:

$$\begin{aligned} \mathbf{x}_{E,k+1} &= \mathbf{A}_{dE,i}\mathbf{x}_{E,k} + \mathbf{b}_{dE,i}u_k, \\ y_k &= \mathbf{c}_i^T \mathbf{x}_{E,k},\ i = \overline{1, n_R}, \end{aligned} \tag{29}$$

where

$$\begin{aligned} \mathbf{x}_{E,k} &= [\mathbf{x}_k^T \quad x_{nP+1,k} \quad x_{nP+2,k} \quad x_{nP+3,k}]^T, \\ \mathbf{A}_{dE,i} &= \begin{bmatrix} \mathbf{A}_{d,i} & \mathbf{b}_{d,i} & 0 & 0 \\ \mathbf{0}_{nP,nP} & 1 & 0 & 0 \\ \mathbf{0}_{1,nP} & 0 & 1 & 1 \\ -\mathbf{c}_i^T(\mathbf{A}_{d,i} - \mathbf{I}_{nP}) & -\mathbf{c}_i^T \mathbf{b}_{d,i} & 0 & 0 \end{bmatrix}, \\ \mathbf{b}_{dE,i} &= [\mathbf{b}_{d,i}^T \quad 0 \quad 0 \quad -\mathbf{c}_i^T \mathbf{b}_{d,i}]^T. \end{aligned} \tag{30}$$

The dynamic Takagi-Sugeno fuzzy model of the extended controlled plant is

$$\begin{aligned} & R^i: \mathrm{IF}\ e_k = x_{nP+2,k} + x_{nP+3,k}\ \mathrm{IS}\ \mathrm{LTE}_l^i\ \mathrm{AND} \\ & \Delta e_k = x_{nP+3,k}\ \mathrm{IS}\ \mathrm{LTDE}_m^i \\ & \mathrm{THEN} \begin{cases} \mathbf{x}_{E,k+1} = \mathbf{A}_{dE,i}\mathbf{x}_{E,k} + \mathbf{b}_{dE,i}u_k \\ y_k = \mathbf{c}_i^T \mathbf{x}_{E,k} \end{cases},\ i = \overline{1, n_R}, \end{aligned} \tag{31}$$

where $l = \overline{1, o}$ and $m = \overline{1, o}$. This MM is important as it allows the convenient design of the remaining Takagi-Sugeno fuzzy controller without dynamics i.e. the block FB.

Using the notations defined in (28) and the fuzzy control system structure defined in Fig. 3 the block FB of the Takagi-Sugeno PI-fuzzy controller can be expressed as a Takagi-Sugeno state feedback fuzzy controller with the following rule base:

$$\begin{aligned} & R^i: \mathrm{IF}\ e_k = x_{nP+2,k} + x_{nP+3,k}\ \mathrm{IS}\ \mathrm{LTE}_l^i\ \mathrm{AND} \\ & \Delta e_k = x_{nP+3,k}\ \mathrm{IS}\ \mathrm{LTDE}_m^i \\ & \mathrm{THEN}\ \Delta u_k = r_k - \mathbf{K}_i \mathbf{x}_{E,k},\ i = \overline{1, n_R}. \end{aligned} \tag{32}$$

The PDC framework (Tanaka and Wang, 2001) justifies the separate design of the local state feedback controllers in the rule consequents presented in (32). The following normalized firing strengths (membership functions of fuzzy sets) h_i are defined to enable the formulation of the LMIs:

$$\begin{aligned} h_i(\mathbf{z}) &= w_i(\mathbf{z}) / [\sum_{i=1}^{n_R} w_i(\mathbf{z})], \\ w_i(\mathbf{z}) &= \prod_{\alpha=1}^{nv} T_{v,\alpha}^i(\mathbf{z}),\ i = \overline{1, n_R}, \end{aligned} \tag{33}$$

where

$$\mathbf{z} = [e_k \quad \Delta e_k]^T,\ nv = 2,\ v \in \{e_k, \Delta e_k\}, \tag{34}$$

and the notations for the linguistic terms defined in Section 2 are kept.

The equilibrium point of the fuzzy control system is globally asymptotically stable if there exists a common positive definite matrix $\mathbf{X}$ and the matrices $\mathbf{M}_i$, $i = \overline{1, n_R}$, such that the following LMIs are fulfilled:

$$\begin{bmatrix} \mathbf{X} & \mathbf{X}(\mathbf{A}_{dE,i})^T - (\mathbf{M}_i)^T(\mathbf{b}_{dE,i})^T \\ \mathbf{A}_{dE,i}\mathbf{X} - \mathbf{b}_{dE,i}\mathbf{M}_i & \mathbf{X} \end{bmatrix} > 0,\ i = \overline{1, n_R},$$
$$\begin{bmatrix} \mathbf{X} & [(\mathbf{A}_i\mathbf{X} + \mathbf{A}_j\mathbf{X} - \mathbf{b}_{dE,i}\mathbf{M}_j - \mathbf{b}_{dE,j}\mathbf{M}_i)/2]^T \\ (\mathbf{A}_{dE,i}\mathbf{X} + \mathbf{A}_{dE,j}\mathbf{X} - \mathbf{b}_{dE,i}\mathbf{M}_j - \mathbf{b}_{dE,j}\mathbf{M}_i)/2 & \mathbf{X} \end{bmatrix} \ge 0, \tag{35}$$
$$\forall\, 1 \le i < j \le n_R,\ \text{such that}\ h_i \cap h_j \ne \varnothing,$$

and

$$\mathbf{K}_i = \mathbf{M}_i\mathbf{X}^{-1} = [\mathbf{0}_{1,nP} \quad 0 \quad p^i \quad q^i]^T,\ i = \overline{1, n_R}\ . \tag{36}$$

Concluding, the parameters in the consequents of the rules of the Takagi-Sugeno PI-fuzzy controllers are determined in terms of obtaining the state feedback gain matrices $\mathbf{K}_i$, $i = \overline{1, n_R}$, in (36) such that to solve the LMIs (35) and also to fulfil equality-type constraints. These equality-type constraints in (36) make these PI-fuzzy controllers act like Takagi-Sugeno partial state feedback fuzzy controllers. This transformation guarantees the global stability of the fuzzy control systems.

The design of the discrete-time Takagi-Sugeno state feedback fuzzy controllers for the controlled plant modelled in the previous section is based on the same equations (35) and (36), but the consequents of the rules of the fuzzy models of the controlled plant are those presented in (9). Therefore the system order is 3 instead of 6 as for the Takagi-Sugeno PI-fuzzy controllers. In addition, no inequality-type constraints are imposed in (36).

The continuous-time Takagi-Sugeno state feedback fuzzy controller is characterized by the complete rule base

$$\begin{aligned} &R^i: \text{IF } x_1(t) \text{ IS } T^i_{x_1,l} \text{ AND } x_3(t) \text{ IS } T^i_{x_3,m} \text{ AND} \\ &V(t) \text{ IS } T^i_{V,n} \text{ THEN } u_k = r_k - \mathbf{K}_i\mathbf{x}_k,\ i = \overline{1, n_R}. \end{aligned} \tag{37}$$

The equilibrium point of this fuzzy control system is globally asymptotically stable if there exists a common positive definite matrix $\mathbf{X}$ and the matrices $\mathbf{M}_i$, $i = \overline{1, n_R}$, such that the following LMIs are fulfilled (Tanaka and Wang, 2001):

$$\begin{aligned} &-\mathbf{X}(\mathbf{A}_i)^T - \mathbf{A}_i\mathbf{X} + (\mathbf{M}_i)^T(\mathbf{b}_i)^T + \\ &\mathbf{b}_i\mathbf{M}_i > 0, i = \overline{1, n_R}, \\ &-\mathbf{X}(\mathbf{A}_i)^T - \mathbf{A}_i\mathbf{X} - \mathbf{X}(\mathbf{A}_j)^T - \mathbf{A}_j\mathbf{X} + \\ &(\mathbf{M}_j)^T(\mathbf{b}_i)^T + \mathbf{b}_i\mathbf{M}_j + (\mathbf{M}_i)^T(\mathbf{b}_j)^T + \\ &\mathbf{b}_j\mathbf{M}_j \ge 0,\ \forall\, 1 \le i < j \le n_R, \\ &\text{such that } h_i \cap h_j \ne \varnothing. \end{aligned} \tag{38}$$

The state feedback gain matrices $\mathbf{K}_i$, $i = \overline{1, n_R}$, are next calculated as follows:

$$\mathbf{K}_i = \mathbf{M}_i\mathbf{X}^{-1},\ i = \overline{1, n_R}\ . \tag{39}$$

The results presented in this section can be applied to the design of Takagi-Sugeno fuzzy controllers on the basis of the TP models. The advantage of such design concerns the reduced numbers of local models in the conclusions if the TP models are viewed as Takagi-Sugeno fuzzy models. The polytopic models derived with this regard can be used in the controller design. The design controller design is convenient because it can be supported by the manipulation of the convex hulls beside the manipulation of the LMIs.

4. A SAMPLE OF DIGITAL SIMULATION RESULTS

The fuzzy control system with the Takagi-Sugeno PI-fuzzy controller characterized by (24) and (25) was tested with respect to several step modifications of *r*. A sample of digital simulation results is presented in Fig. 5.

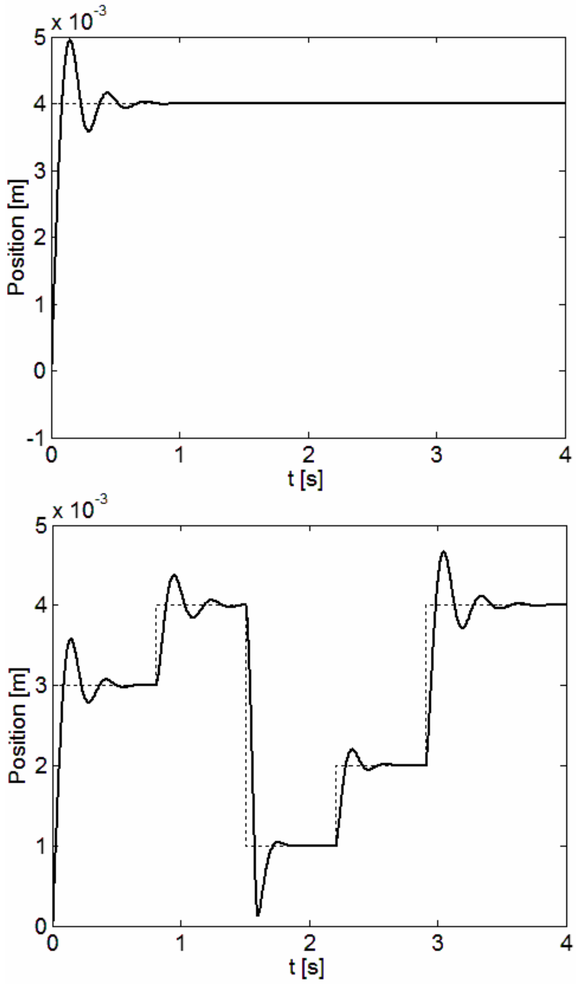

Fig. 5. Digital simulation results of fuzzy control system with Takagi-Sugeno PI-fuzzy controller.

5. CONCLUSIONS

The paper has suggested the stable design of several new Takagi-Sugeno fuzzy controllers for or the position control of an electromagnetic actuated clutch. They are based on different mathematical models of the controlled plant.

The limitation of our new approach is related to the numerical problems associated to solving the stability conditions i.e. with the determination of the common positive matrix to fulfil certain inequalities (LMIs) and eventually equalities as well. It is compensated by the strength of the plant models and of the fuzzy controllers when coping with large classes of nonlinear plants.

The future research will be focused on real-time experimental tests in several conditions which involve various reference input and disturbance input modifications. The low-cost design and implementation of the controllers will be aimed.

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