ABOUT A NEW THEORY IN POWER MEASUREMENT
(Part 1 – Concepts)

Traian-Titi Şerban

University of Craiova, Faculty of Automatics, Computers and Electronics

Abstract: Since the world's electronic industry launches more and more ASIC and digital instruments for power measurement on a continuous spreading market, the problem of the theoretical fundamentals of this growth appear as natural. This paper presents some limits of the traditional power theory concepts that are implemented in actual power measurement ASIC and digital instruments. Actual concepts about power components' significances and some new power definitions are exhibited, new approaches of the power measurement are discussed in order to settle the right methods for further power and energy measurement.

Keywords: power measurement, power components, power theory, current components, nonactive power, reactive power, aparent power

1. INTRODUCTION

During last years, a significant progress was made in power metering technology, so the actual instruments are much precise than the traditional ones, even if the waveforms of voltages and currents are more distorted than ever. More digital power meters are now available on the market, more ASIC are designed in order to provide enhanced capabilities of the digital instruments. But what are we measuring? Are we applying the best definitons for powers that we measure?

Previous powers' definitons were applied so many decades since they were first presented and accepted, but were they the best? Further measurements always exhibit differences in power balance? Anyway, most of the actual ASIC and digital power meters are based on the traditional definitions of the power components. Aparent power S, active (real) power P and reactive power Q are most frequently measured. But are the powers that we actually measure the best descriptors of the nature and behaviour of the loads? As all aparent power and active power definitions were the same, the discussions were and will be always about unuseful non-active power components, mainly the reactive power, the distorsion power, the scattered power, and others.

When electrical power flow increased and new kind of loads were introduced in the power networks, the power balance exhibited some inadvertences in the traditional power theory: there is more unuseful power than it was believed, and that limits the capacity of the power lines to transfer the energy from the source to the load.

2. BRIEF TIMELINE

Here is a brief flash-back of the power measurement concepts developed in the last century. In 1892, engineer Charles Proteus Steinmetz revealed a difference between the aparent power and the active power even for resistive nonlinear load (electric arc) due to the waveform distortion. One can say he was the first that officially rised the distortion power problem right in the title of his article: „Is a phase shift in the current of an electric arc?".

In 1927, Constantin Budeanu published his remarcable work „Puissances relatives et fictives" - a reference book that led the power theory over a long period of time, based on the concepts of reactive power Q and distortion power D in terms of the rectangular currents decomposition of the main current and harmonic power decomposition. Both Budeanu's concepts (Q and D) are still supported in the actual IEEE Standard Dictionary of Electrical and Electronical Terms (IEEE, 1997).

In 1931, professor Stanislaw Fryze exhibited a time-domain description of Q and D. The Fryze's definitions for P and Q are still used in the actual ASIC signal processors performing digital power and energy measurements. Based on the Fryze's definitions of non-active powers but also on the Max Buchholz definitons of the collective values of the currents and voltages, professor Manfred Depenbrock presented in 1962 the FBD method (Fryze Buchholz Depenbrock) – a power theory presuming two current components: the active current and the nonactive current.

After some decades, in 1979, Norbert Kusters and William Moore wrote two expresions for the reactive power, one for the inductive reactive power and the other for the capacitive reactive power, respectively.
In 1984, the Instantaneous Reactive Power (IRP) concept was presented by H Akagi, Y Kanazawa and A Nabae. Their theory had as initial purpose not to measure but to control the switching compensators known as „active power filters". In 1993, the IRP theory was developed for three-phase systems by the same authors (Akagi and Nabae, 1993). Known also as „Instantaneous Reactive Power (IRP) p-q Theory", it assumes, by it's mathematical fundamental, the decompositon of the currents into two-dimensions ortogonal system. The first component of the current is the active-like current (i_p), the second – the reactive-like current (i_q).

Based on the vector representation of periodic signals, LaWhite Niels and Ilic Marija presented in 1997 another concept (LaWhite and Ilic, 1997) about the reactive power using the vector space decomposition of the periodic nonsinusoidal signals, aligned to the current decomposition concept.

In 1987, professor Leszek Czarnecki called in question the Budeanu theory, concluding „it should be abandoned" containing something „wrong with the concept of reactive and distorsion power". Later, in 2004 and 2006 respectively, professor Czarnecki objected about the IRP p-q theory (Czarnecki, 2004) and about the description of the power properties of the three-phase systems in terms of the Poynting Vector (Czarnecki, 2006). In 2005 and 2006 respectively, he published two main articles (Czarnecki, 2005a) and (Czarnecki, 2005b) concerning his original concept about currents' physical components (CPC) theory.

This short story is far from depleting all tries and authors of most interesting concepts about power definitons and power measurement. Only few names may be mentioned: M Milic and his generalised powers concept in 1970, A Ferrero and L Cristaldi, and their mathematical foundations of the instantaneous power concept in 1996, F Peng and J Lai and their generalized instantaneous reactive power theory for three phase power systems in 1996, and many, many others noticeable contributors.

3. MAIN CONCEPTS' ILLUSTRATIONS

The clasical power theory presumes three main power definitions used to describe a certain load:
- the aparent power S – the product of the load voltage multiplied by the load current:

$$S = U\,I \qquad (1)$$

Here „I" stays for the biggest effective load current supplied at the rated voltage;
- the active (real) power P – associated with the energy conversion in a certain load;
- the reactive power Q – associated with the energy exchange between the source and the load;
- the distorsion power D – associated mainly with the interaction of the harmonic components of the load current and voltage. It is called also harmonic power.

The clasical concept about these powers assumes that:

$$S^2 = P^2 + Q^2 + D^2 \qquad (2)$$

For three-phase systems, various presumtions about the main power components were developed, based on different theories and concepts, resulting in other power cathegories: the fictious power F, the nonactive power N, the total vectorial power and other. Nevertheless, it is unanimously accepted that only aparent power S and real power P have a physical meaning. The actual IEEE concept about the power definitions may be presented by mean of a geometrical illustration in figure 1. The significances of the vectors in the diagram is:
P – the real power;
Q – the reactive power;
Ss – the aparent power in sinusoidal state;
D – the distorted power;
F – the fictious power;
N – the nonactive power;
Sns – the aparent power in nonsinusoidal state.

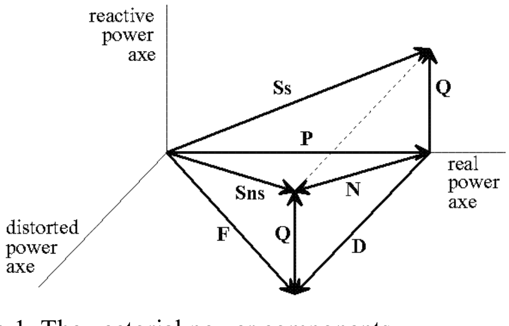

Fig.1. The vectorial power components

This concept was the source of different interpretations, the simplest one presuming that the reactive power corresponds only to the electromagnetic field energy exchanges between the source and the load, and the distorsion power signifies the sum of all non-active powers generated by all interactions between the harmonic components of the voltage and current of the load, other than the ones that generate the real power.

Most of the producers offer, as the main design specification for the generator and/or the electrical power network, only the total (vectorial) aparent power Ss in the sinusoidal state, letting the user to decide how this aparent power will be used, dealing with the balance of the power components. Nevertheless, the generator's and/or the network capacity may be severely dimished if the non-active power is high. The presence of the harmonic components of the nonsinusoidal currents and voltages associated to a certain load produce also a distortion power D; but that distortion power is demosntrated to be associated not only to the nonsinusiodal state. In the three-phase sistems the situation is even more complex, since the load unbalance produces more unuseful nonactive powers even with resistive unbalanced loads and/or asymetrical supply voltages.

The clasical concept about power definitons may be supported by various mathematical models, as presented in paragraph 2. One of them is presented in (Czarnecki, 2006). Considering only the active power, the reactive power and the distorsion power described in fig.1, a mathematical model may be developed based on three ortogonal components of the current „i" flowing through supplying conductors from source to the load:

- the vectorial active current i_a;
- the vectorial reactive current i_r;
- the vectorial harmonic current i_h;

$$\underline{i_a} + \underline{i_r} + \underline{i_h} = \underline{i} \qquad (4)$$

with effective (rms) values:

$$I^2_a + I^2_r + I^2_h = I^2 \qquad (5)$$

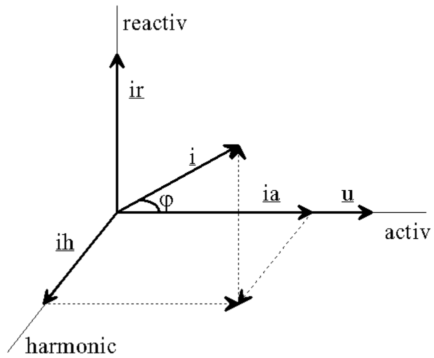

Fig.2. The current decomposition

This concept presumes that active, reactive and harmonic power are generated by the same name currents, respectively. Nonetheless, not all these current components may be associated to physical currents. They are only mathematical entities, so the nonactive powers Q and D are not strictly related to some physical phenomena (mainly energy oscillations between the source and the load) but to a certain increase of the aparent power due to the current fundamental harmonic phase-shift (Czarnecki, 2006).

In the sinusoidal state, the current and the voltage waveforms associated with a reactive load (inductive or capacitive) define a time lag known as the phase difference φ. Current vector $\underline{i}$ may be decomposed into two components: one with the same direction as the voltage, said the *active current* i_a , another orthogonal to the voltage, said the *reactive current* i_r :

$$\angle(\underline{i_a}, \underline{u}) = 0 \ ; \qquad \angle(\underline{i_r}, \underline{u}) = 90^o \qquad (6)$$

$i_a(t)$ has the same zero crossings as voltage u(t) and $i_r(t)$ is in quadrature with u(t), so one can write the relations between the effective (RMS) values:

$$I_a = I\cos\varphi \ ; \qquad I_r = I\sin\varphi \qquad (7)$$

The active current produces the active power and the reactive current produces the reactive power:

$$P = UI_a = UI\cos\varphi \qquad (8)$$

and reactive current produce the reactive power:

$$Q = UI_r = UI\sin\varphi \qquad (9)$$

For a resistive load, current and voltage reach the maximum values at the same time (φ=0), so there is no reactive power and the active power equals the aparent power. This explains the definition of the active power as maximum power that a load can take from a certain source.

4. POWER DEFINITIONS

4.1. The active (real) power

The most accepted definition of the active power was developed by Fryze, based on the mean of the instant power:

$$P = \frac{1}{T}\int_t^{t+T} u(t)i(t)dt \qquad (11a)$$

or, in the numerical format:

$$P = \frac{1}{N}\sum_{k=1}^{N} u(t_k)\, i(t_k) \qquad (11b)$$

where T represents the period of the alternate current and N represents the number of samples picked during a voltage period time.

In nonsinusoidal environment, the voltage and current waveforms include a DC component, a fundamental frequency component and a sum of harmonic components:

$$u(t) = U_0 + \sum_{n=1}^{\infty} u_n(t) \;; \qquad i(t) = I_0 + \sum_{n=1}^{\infty} i_n(t) \qquad (12)$$

where U_0, I_0 represent the DC components and u_n, i_n represent the fundamental and higher order harmonic components of the voltage and current.

The Budeanu definition of the real power involves all harmonic components:

$$P = \sum_{n=0}^{\infty} P_n \qquad (13)$$

computed as a mean of the instantaneous values of the harmonic components of the load voltage and current:

$$P_n = \frac{1}{nT} \int_{\tau}^{\tau + \frac{T}{n}} u_n(\tau)\, i_n(\tau) d\tau \qquad (14)$$

where „n” represents the multiple of the fundamental frequency.

The active (real) power becomes:

$$P = \frac{1}{T} \int_{t}^{t+T} \left[U_0 + \sum_{n=1}^{\infty} u_n(t) \right] \left[I_0 + \sum_{n=1}^{\infty} i_n(t) \right] dt \qquad (15)$$

with the final result:

$$P = U_0 I_0 + \sum_{n=1}^{\infty} U_n I_n \cos\varphi_n \qquad (16)$$

The contribution of the cross-products of the harmonics with different orders is always zero.
In the simbolic representation:

$$P = P_0 + \sum_{n=1}^{\infty} P_n \qquad (17)$$

Where P_0 represents the DC (constant) component of the real power and P_n represent the n-th order harmonic component of the real power defined as product of the same order harmonics of the voltage and active current.

4.2. The aparent power

If U_Σ the effective (rms) value of the nonsinusoidal voltage and I_Σ - the effective (rms) value of the load current, excluding the DC components:

$$U_\Sigma^2 = \sum_{1}^{\infty} U_n^2 \;\; ; \;\; I_\Sigma^2 = \sum_{1}^{\infty} I_n^2 \qquad (18)$$

The aparent power defined by (1) may be evaluated as product of the harmonic components of the voltages and currents defined in relation (12):

$$S_\Sigma^2 = U_\Sigma^2\, I_\Sigma^2 = \sum_{n=1}^{\infty} U_n^2 \sum_{n=1}^{\infty} I_n^2 \qquad (19)$$

4.3. The reactive power

In nonsinusoidal state, both voltage and curent may be decomposed in series of sinusoidal components. Each harmonic component of the supply voltage $u_n(t)$ $= \sqrt{2} U_n \cos n\omega_1 t$ produces a load current:

$i_n(t) = \sqrt{2} I_n \cos\left(n\omega_1 t - \varphi_n\right)$ and an instantaneous power component $p_n(t) = u_n(t)\, i_n(t)$ as follows:

$$p_n(t) = 2\, U_n\, I_n \cos(n\omega_1 t) \cos(n\omega_1 t + \varphi_n) \qquad (20)$$

or:

$$p_n(t) = P_n\,(1 + \cos 2n\omega_1 t) + Q_n \sin 2n\omega_1 t \qquad (21)$$

where: $P_n = U_n\, I_n \cos\varphi_n$ and $Q_n = U_n I_n \sin\varphi_n$ (22)

Q_n represents the reactive power of the n-th order harmonic.

Budeanu theory defines the total reactive power Q_B as the sum of the contributions of each harmonic component:

$$Q_B = \sum_{n=1}^{\infty} Q_n = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n \qquad (23)$$

Obviously,

$$P^2 + Q_B^2 = \left(\sum_{k=1}^{\infty} U_k I_k \cos\varphi_k \right)^2 + \left(\sum_{k=1}^{\infty} U_k I_k \sin\varphi_k \right)^2 \le S^2$$

so Budeanu had to define the power difference, named the distorsion power:

$$\mathrm{D} = \sqrt{S^2 - (P^2 + Q_B^2)} \qquad (24)$$

Fryze's definition stands for the reactive power as the quadratic difference of the aparent power and the active (real) power:

$$Q_F = \sqrt{S^2 - P^2} \qquad (25)$$

From the physical point of vue, the reactive power appears as produced by the generator but not properly used by the consumer to be converted into an active power. This concept identifies the reactive power as the total nonactive power, that includes the distorsion power defined by Budeanu:

$$Q_F^2 = Q_B^2 + D^2 \qquad (26)$$

Professor Czarnecki shown some faults in the previous theories regarding the physical meaning of the reactive power.

One of the main faults of the Budeanu theory is that it cannot support the distorsion power as defined by its name; distorsion power D appears to be not related to the waveform distorsion of the load voltage and/or current (Czarnecki, 2005a). The lack of distorsion is not always associated with zero distorsion power. That is the reason for the absence of a method of compensation of the distorsion power as defined by Budeanu.

Neither the reactive power Q_B is associated directly to the energy oscillation between the source and the load, so the power factor cannot be improved using the Budeanu definition of Q.

The Fryze's definition of the reactive power, even if based on the current decomposition into an active and an inactive (said reactive) currents:

$$i(t) = i_a(t) + i_{rF}(t) \tag{27}$$

provides no solution to reduce the reactive power by mean of the load compensation. Fryze's definition of the reactive power Q_F includes all the power that is not related to the direct energy conversion. The compensation of the reactive power Q_F is not supported by the Fryze's definition, because a separation of the main causes of energy oscillation and current distorsion is not made. The compensation of the reactive current component i_{rF} by mean of active compensators (fig.3) may produce more distorsion in the source curent, affecting directly the quality of energy supplied the other consumers.

Another power theory that is shown to be not reliable for measurement and load power properties is the p-q theory. The concepts of instantaneous active (i_a) and reactive (i_r) currents used to describe the load behaviour are very useful only for the control of the active power filters, but they have nothing in common with the same name currents used to describe the power properties of a certain load. Moreover, the calculated values for ia and ir show nonsinusoidal waveforms of these currents even for sinusoidal supply voltages and linear loads, and some active current for purely reactive circuits.

The Current's Physical Components (CPC) concept developed by Professor Czarnecki use the same names for the active and reactive currents, but it gives them some different significances (Czarnecki, 2005). The CPC concept starts from Fryze's separation of the active and the reactive current components, considering also its harmonic structure (12) but in a slightly modified form:

$$\begin{aligned} u(t) &= U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} U_n e^{jn\omega t} \\ i(t) &= Y_0 U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} Y_n U_n e^{jn\omega t} \end{aligned} \tag{28}$$

The *active current* $i_a(t)$ is associated to a resistive equivalent load having the same active power as the real load:

$$i_a(t) = G\,u(t) = \sqrt{2}\,\mathrm{Re}\sum_{n=0}^{\infty} G U_n e^{jn\omega t} \tag{29}$$

The remaining current does not contribute to the energy transmission, and can be decomposed in other two components: the first is the *reactive current* $i_r(t)$:

$$i_r(t) = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n U_n e^{jn\omega t} \tag{30}$$

that flows through the load when susceptance B is different from zero for at least one harmonic frequency, and the other is the *scattered current*, $i_s(t)$, flowing through the load with a frequency-dependent conductance G_n:

$$i_s(t) = (G_0 - G)U_0 + \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} (G_n - G) U_n e^{jn\omega t} \tag{31}$$

Index n represents the order of the harmonic frequency. The value of G_n is scattered around G. Obviously, one can write the equation for the instantaneous values:

$$i(t) = i_a(t) + i_r(t) + i_s(t) \tag{32}$$

The physical meanings of these three components are: $i_a(t)$ – the current associated to the useful energy conversion; $i_r(t)$ – the current associated to the phase shift between the same order harmonic voltage and current; $i_s(t)$ – the current associated to the change of the load conductance with the harmonic frequency.

The effective values (RMS) of these currents are calculated by professor Czarnecki as follows:

$$I_a = \frac{P}{U};\quad Is = \sqrt{\sum_{n=1}^{\infty} (G_n - G)^2 U_n^2}\;;\quad I_r = \sqrt{\sum_{n=1}^{\infty} B_n^2 U_n^2} \tag{33}$$

The currents' components described above are reciprocally orthogonal, so the effective (RMS) values satisfy the relation:

$$I^2 = I_a^2 + I_r^2 + I_s^2 \tag{34}$$

that leads to the powers' relation:

$$S^2 = P^2 + Q_C^2 + D_s^2 \tag{35}$$

Where Q_C represents the Czarnecki reactive power and D_s represents the scattered power.

One can remark the similarity between the relations (4) and (32) or (5) and (34), but keeping in mind the great difference between the significances of terms. In (4) there is no physical meaning of the i_h while in (32) the same place term (the scattered current) has a much more realistic and clear illustration.

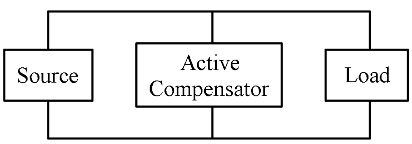

Fig.3. The active compensation

5. CONCLUSION

Some different points of vue about power definitions are presented in this paper. All of these conceptions are important in order to know what kind of power one can measure using different types of power meters.

Currently, the concept of Currents' Physical Components (CPC) developed by professor Czarnecki supports the most advanced power theory.

The CPC concept provides physical interpretation for all power components in all kind of circuits: single-phase and three-phase circuits, balanced and unbalanced, sinusoidal and nonsinusoidal circuits with linear and nonlinear loads.

The CPC-based power theory also provides a better theoretical background for the reactive power compensation in the systems with unbalanced loads than the IRP concept and the p-q theory.

The CPC concept and the power definitions developed by professor Czarnecki in his theory have to be considered in the future designs involving power and energy metering.

6. REFERENCES

Akagi H., A. Nabae (1993), The p-q Theory in Three-Phase Systems under Nonsinusoidal Conditions, *ETEP*, **vol. 3**, pp. 27-31

Czarnecki L. (2006) Could Power Propetries of Three-Phase Systems Be Described in Terms of the Poynting Vector ?, *IEEE Transactions on Power Delivery*, **Vol. 21**, **No.1**, pp. 339-344

Czarnecki L (2004), On Some Misinterpretations of the Instantaneous Reactive Power p-q Theory, IEEE Transactions on Power Electronics, **Vol. 19**, **No. 3**, pp. 828-836;

Czarnecki L. (2005a), Currents' Physical Components (CPC) in Circuits with Nonsinusoidal Voltages and Currents – Part 1: Single Phase Linear Circuits, *Electric Power Quality and Utilization Journal*, **Vol. XI**, **No.2**

Czarnecki L. (2005b), Currents' Physical Components (CPC) in Circuits with Nonsinusoidal Voltages and Currents – Part 2: Three-Phase Three Wire Linear Circuits, *Electrical Power Quality and Utilization Journal* **Vol. XII**, **No.1**, pp. 3-13

Czarnecki L. (2006), Physical Interpretation of the Reactive Power in terms of the CPC Power Theory, *Seventh International Workshop on power Definitions and Measurements under Non-Sinusoidal Conditions*, Cagliari, Italy

LaWhite N. and M. Ilic Marija (1997), Vector Space Decomposition of Reactive power for Periodic Nonsinusoidal Signals, *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, **Vol. 44**, **No. 4**, pp. 338-346

IEEE (1997) *The new IEEE Standard Dictionary of Electrical and Electronic Terms, Institute of Electrical and Electronic Engineers*, Inc., New York.