THE H_2 CONTROL PROBLEM FOR TDS – AN OPTIMAL OUTPUT FEEDBACK CONTROLLER SYNTHESIS

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Abstract: In the present paper the H_2 control problem objectives are defined in terms of Linear Matrix Inequalities (LMI) for a class of linear systems with multiple delays. An output feedback controller synthesis procedure is then proposed together with an optimization procedure. The classical H_2 output feedback synthesis problem is formalized for TDS in terms of a Lyapunov based approach (for the first time to our knowledge). A new manner of solving the nonlinear bounding condition of the closed loop is proposed without using the Projection Lemma (traditionally employed in similar situations). This is used to compute a feasible initial set of the weight matrices for the Lyapunov functional that will be the starting point of an alternating iterative procedure that solves the H_2 optimization problem, which is proposed in the end. A numerical example illustrating the use of this procedure for a second order system with two delays is then presented with some conclusions. *We declare that the paper is ORIGINAL, that WE ARE THE AUTHORS of the work, that the above work HAS NOT BEEN PREVIOUSLY PUBLISHED ELSEWHERE and that we allow the reproduction and distribution of the article within the proceedings of The International Symposium SINTES 13, 18-20 October 2007 Craiova, Romania and in the publications associated wit this scientific event.*

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1. INTRODUCTION

In the last years the problems related to the Time Delay Systems (TDS) have received an increasing interest from the scientific community due to the big number of potential applications and the development of some modern computational tools. This attention has resulted in significant advances in this field of study. The Lyapunov based approaches have become the backbone of a considerable number of results regarding classical control problems like the stability (see for example (Kolmanovskii and Richard, 1999), (Fridman and Shaked, 2003), (Michiels et al., 2005) or (Dugard and Verriest, 1998)), the quadratic guaranteed cost (Moheimani end Petersen, 1995) or the H_∞ control problem (see (Serbanescu and Popeea, 2004) or (Jeung et al., 1998)). Due to the way in which the final conditions are derived, we can consider that this result is also related to the family of Lyapunov based approaches. The present paper is approaching the H_2 synthesis control by formalizing the control objectives in terms of two Linear matrix Inequalities or LMI's. The approach is original to the extent of our knowledge and it is presenting an optimization procedure for the pure H_2 problem. The computation, in the incipient faze of the optimization procedure, of a feasible set for the Lyapunov weight matrices is done in an original way without using the Projection Lemma, solution traditionally employed in similar situations like for example in the case of the output feedback H_∞ synthesis. Once formalized, the problem is then solved numerically with the help of the Semidefinite Programming (SDP).

2. THE H_2 CONTROL PROBLEM

In the present paper we will try to minimize a norm of the performance output signal in the case of the Time Delay Systems (TDS). In order to do that we will illustrate first how we can formalize the H_2 control problem in the case of TDS by creating an analogy with the classical case of the linear delay free systems.

Let us consider first a simple delay free system, described by the following set of equations:

$$\begin{aligned} \frac{d}{dt}x(t) &= Ax(t) + Bw(t) \\ z(t) &= Cx(t) + Dw(t) \end{aligned} \tag{2.1}$$

where $x \in \mathbf{R}^n$ is the state vector, $w \in \mathbf{R}^m$ is the exogenous input and $z \in \mathbf{R}^p$ is the controlled output.

We will define the set of reachable states with unit-energy inputs as being:

$$R_{ue} = \left\{ x \left| \begin{array}{l} x(t), w(t) \text{ satisfying (2.1), } x(0)=0 \\ \int_0^\infty w^T(t)w(t)dt \le 1, \end{array} \right. \right\} \tag{2.2}$$

and we will try to bound R_{ue} by ellipsoids of the form

$$E_x = \{x \mid x^T P x \le 1, P > 0\}. \tag{2.3}$$

If we define a Lyapunov function $V_{di}(x,t) = x^T(t)Px(t)$, then if the following condition $\frac{d}{dt}V_{di}(x,t) \le w^T(t)w(t)$ holds for all x satisfying (2.1), then the following inequality holds $x^T Px \le 1$, or in other words $R_{ue} \subset E_x$.

Theorem 2.1 If the following LMI in P holds

$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & -I_m \end{bmatrix} \le 0 \tag{2.4}$$

then $\forall x \in R_{ue}$ we have

$$xx^T \le P^{-1}. \tag{2.5}$$

If we denote by T the transfer function from w to z we can define the following induced norm:

$$\|T\| := \sup_{w \in L_2, \|w\|_2 \le 1} \|Tw\| = \sup_{w \in L_2, \|w\|_2 \le 1} \|z\|, \tag{2.6}$$

where $\|z\|$ is a measure of the amplitude of the vector z and it can be for example the ∞ norm or the second norm. If we consider the second norm for the amplitude of z, we get

$$\|T\|^2 = \sup_{w \in L_2, \|w\|_2 \le 1, t \ge 0} (z^T(t)z(t)), \tag{2.7}$$

and after applying the Schur complement we have

$$\|T\|^2 \le \beta \Leftrightarrow \sup_{w \in L_2, \|w\|_2 \le 1, t \ge 0} (z(t)z^T(t)) \le \beta I_p. \tag{2.8}$$

If we consider $D = 0$ for the system described in (2.1) (a common assumption in the H_2 control) then we get $z(t)z^T(t) = Cx(t)x^T(t)C^T$ and from the *Theorem* 2.1 it results that if there is a symmetric and positive definite matrix P such that the LMI (2.4) is satisfied the right inequality in (2.8) holds if and only if

$$CP^{-1}C^T \le \beta I_p. \tag{2.9}$$

Theorem 2.2 If A is exponentially stable and $D = 0$, the inequality $\|T\|^2 \le \beta$ holds if and only if there exists a matrix symmetric and positive definite P such that

$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & -I_m \end{bmatrix} \le 0 \tag{2.10}$$

and

$$CP^{-1}C^T \le \beta I_p. \tag{2.11}$$

We can consider $\|T\|_2^2 := \sup(z^T(t)z(t)) = \text{trace}(CP^{-1}C^T)$ as being the classical H_2 norm of the system.

The *Theorem* 2.2 is still valid for the H_2 norm if we replace the inequality (2.11) with $\text{trace}(CP^{-1}C^T) \le \beta$. The two conditions are actually equivalent since we already proved that $z^T z \le \beta \Leftrightarrow z z^T \le \beta I_p$ and $z^T z = \text{trace}(z z^T)$.

This is the reason why we will consider the condition (2.11) for further investigations in the case of TDS. The *Theorem* 2.2 gives us an upper bound on the H_2 norm in the case of the delay free systems. Let us consider now the case of a TDS described by the following set of equations:

$$\begin{cases} \frac{d}{dt}x(t) = Ax(t) + \sum_{i=1}^{L} A_i x(t-\tau_i) + Bw(t) \\ z(t) = Cx(t) + \sum_{i=1}^{L} C_i x(t-\tau_i) + Dw(t), \end{cases} \quad (2.12)$$

where, $x \in \mathbf{R}^n$ is the state vector, $w \in \mathbf{R}^m$ is the input and $z \in \mathbf{R}^p$ is the output.

We will consider instead of the Lyapunov function $V_{di}(x) = x^T Px$ the following Lyapunov functional

$$V(x,t) = x(t)^T Px(t) + \sum_{i=1}^{L} \int_{-\tau_i}^{0} x(t+s)^T P_i x(t+s) ds, \quad (2.13)$$

where $P, P_i, i = 1:L$ are symmetric and positive definite weight matrices. This is a classical choice for the functional in the delay independent case see for example (Park and Won, 2000) or (Boyd et al., 1994).

We will also define the set of reachable states with unit energy inputs as

$$R_{ue\_TDS} = \left\{ x \,\middle|\, \begin{matrix} x(t), w(t) \text{ satisfy } (2.12), x(t) = 0, \forall t \le 0 \\ \int_0^{\infty} w^T(t) w(t) dt \le 1, \end{matrix} \right\} \quad (2.14)$$

Due to the integral terms in the (2.13) the ellipsoid defined by $E = \{x \mid x^T Px \le 1\}$ will be a less tight approximation of the reachable set R_{ue_TDS} compared with the delay free case. The bounding condition will be in this case:

$$\sup_{w \in L_2, \|w\|_2 \le 1, t>0} (z(t)z^T(t)) \le \beta I_p \Leftrightarrow$$

$$\Leftrightarrow \sup_{w \in L_2, \|w\|_2 \le 1, t>0} \left[Cx(t) + \sum_{i=1}^{L} C_i x(t-\tau_i) \right] \cdot$$

$$\cdot \left[Cx(t) + \sum_{i=1}^{L} C_i x(t-\tau_i) \right]^T \le \beta I_p \quad (2.15)$$

Lemma 1 Given a set of vectors $u_1, u_2, \ldots, u_L \in R^n$ where $L > 1$, the following inequality holds:

$$\|u_1 + \ldots + u_L\|^2 \le L\|u_1\|^2 + \ldots + L\|u_L\|^2. \quad (2.16)$$

If we apply the *Lemma* 1 in (2.15) we get:

$$\left[Cx(t) + \sum_{i=1}^{L} C_i x(t-\tau_i) \right] \left[Cx(t) + \sum_{i=1}^{L} C_i x(t-\tau_i) \right]^T \le$$

$$\le (L+1)(x(t)x^T(t)C^T + \sum_{i=1}^{L} Cx(t-\tau_i)x^T(t-\tau_i)C^T. \quad (2.17)$$

If $\frac{d}{dt}V(x,t) \le w^T(t)w(t)$, $\forall x(t), w(t)$ satisfying (2.12) it results that $x^T(t)Px(t) \le 1, \forall t > 0$.

By applying the Schur comlemnet this inequality is equivalent with $x(t)x^T(t) \le P^{-1}$, $\forall t > 0$. As a result we have:

$$\left[Cx(t) + \sum_{i=1}^{L} C_i x(t-\tau_i) \right] \left[x^T C^T + \sum_{i=1}^{L} x^T(t-\tau_i) C_i^T \right] \le$$

$$\le (L+1)\left(CP^{-1}C^T + \sum_{i=1}^{L} C_i P^{-1} C_i^T \right). \quad (2.18)$$

From (2.15) and (2.18) we get the following implication:

$$(L+1)\left(CP^{-1}C^T + \sum_{i=1}^{L} C_i P^{-1} C_i^T \right) \le \beta I_p \Rightarrow$$

$$\Rightarrow \sup_{w \in L_2, \|w\|_2 \le 1, t>0} (z(t)z^T(t)) \le \beta I_p. \quad (2.19)$$

Theorem 2.3 If the for system described by (2.12) we have $D = 0$ and there is a set of matrices symmetric and positive definite P and $P_i, i = 1:L$ such that the following LMI's are satisfied:

$$\begin{bmatrix} A^T P + PA + \sum_{i=1}^{L} P_i & PA & \cdots & PA_L & PB \\ A_1^T P & -P_1 & & & \\ \vdots & & \ddots & & \\ A_L^T P & & & -P_L & \\ B^T P & & & & -I_m \end{bmatrix} < 0 \quad (2.20)$$

and

$$\begin{pmatrix} \frac{-\beta}{L+1} I_p & C & C_1 & \cdots & C_L \\ C^T & -P & & & \\ C_1^T & & \ddots & & \\ \vdots & & & \ddots & \\ C_L^T & & & & -P \end{pmatrix} \le 0, \quad (2.21)$$

then the system is exponentially stable and $\|T\|_2^2 \le \beta$.

3. THE H_2 SYNTHESIS PROBLEM

Let us consider the following control configuration:

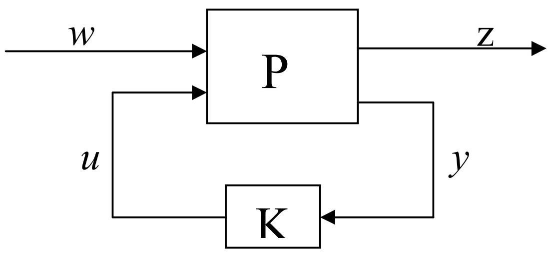

Fig.1 (the closed loop system)

where $x \in \mathbf{R}^n$ is the state vector, $w \in \mathbf{R}^{m1}$ is the exogenous input, $u \in \mathbf{R}^{m2}$ is the control input, $z \in \mathbf{R}^{p1}$ is the performance output and $y \in \mathbf{R}^{p2}$ is the measured output.

We will see the channel from w to z as being the performance channel. The closed loop system will be described by the following system of equations:

$$\begin{cases} \dfrac{d}{dt}x(t) = Ax(t) + \sum_{i=1}^{L} A_i x(t-\tau_i) + B_1 w(t) + B_2 u(t) \\ z(t) = C_z x(t) + \sum_{i=1}^{L} C_{z\_i} x(t-\tau_i) + D_{12} u(t) \\ y(t) = C_y x(t) + \sum_{i=1}^{L} C_{y\_i} x(t-\tau_i) + D_{21} w(t) \\ x(t) = \phi(t), \quad \forall t \in [-\max(\tau_1,\ldots,\tau_L), 0] \end{cases} \tag{3.1}$$

and

$$\begin{cases} \dfrac{d}{dt}x_k(t) = A_k x_k(t) + B_k y(t) \\ u(t) = C_k x_k(t) + D_k y(t) \end{cases} \tag{3.2}$$

The closed loop system will be described by the following system of equations:

$$\begin{cases} \dfrac{d\xi}{dt} = A_{cl}\xi(t) + A_{cl\_1}E\xi(t-\tau_1) + \ldots + \\ \qquad + A_{cl\_L}E\xi(t-\tau_L) + B_{cl}w(t) \\ z(t) = C_{cl}\xi(t) + C_{cl\_1}E\xi(t-\tau_1) + \ldots + \\ \qquad + C_{cl\_L}E\xi(t-\tau_L) + D_{cl}w(t) \end{cases} \tag{3.3}$$

where

$\xi(t) = \begin{bmatrix} x(t) \\ x_k(t) \end{bmatrix}$ is the joint state, $E = \begin{bmatrix} I_n & 0_{n\times k} \end{bmatrix}$ and

$$A_{cl} = \begin{bmatrix} A + B_2 D_k C_y & B_2 C_k \\ B_k C_y & A_k \end{bmatrix}, A_{cl\_i} = \begin{bmatrix} A_i + B_2 D_k C_{y\_i} \\ B_k C_{y\_i} \end{bmatrix},$$

$$B_{cl} = \begin{bmatrix} B_1 + B_2 D_k D_{21} \\ B_k D_{21} \end{bmatrix},$$

$$C_{cl} = \begin{bmatrix} C_z + D_{12} D_k C_y & D_{12} C_k \end{bmatrix}, C_{cl\_i} = C_{z\_i} + D_{12} D_k C_{y\_i},$$

$$D_{cl} = \begin{bmatrix} D_{11} + D_{12} D_k D_{21} \end{bmatrix}.$$

where $i = 1 : L$. (3.4)

The quality output has to be seen as a conventional cost. As we can see from the formulation of the H_2 problem, we are not interested in the direct transfer from w, the exogenous input, to the performance output (see the condition $D = 0$ in the *Theorem* 2.2). The quality output will be defined only in terms depending on current or delayed states of the system, and as a result it will have the following structure

$$z(t) = C_{cl}\xi(t) + C_{cl\_1}E\xi(t-\tau_1) + \ldots + C_{cl\_L}E\xi(t-\tau_L), \tag{3.5}$$

which simply ignores the D_{cl} term.

Theorem 3.1 If there is a set of symmetric positive definite matrices $P \in R^{(n+k)\times(n+k)}$ and $P_1,\ldots,P_L \in \mathbf{R}^{n\times n}$ and a controller *K* such that the following inequalities are satisfied

$$\begin{bmatrix} A_{cl}^T P + P A_{cl} + \sum_{i=1}^{L} E^T P_i E & P A_{cl\_1} & \cdots & P A_{cl\_L} & P B_{cl} \\ A_{cl\_1}^T P & -P_1 & & & \\ \vdots & & \ddots & & \\ A_{cl\_L}^T P & & & -P_L & \\ B_{cl}^T P & & & & I_{m1} \end{bmatrix} < 0 \tag{3.7}$$

and

$$\begin{pmatrix} \dfrac{-\beta}{L+1} I_{p1} & C_{cl} & C_{cl\_1}E & \cdots & C_{cl\_L}E \\ C_{cl}^T & -P & & & \\ E^T C_{cl\_1}^T & & \ddots & & \\ \vdots & & & \ddots & \\ E^T C_{cl\_L}^T & & & & -P \end{pmatrix} \le 0 \tag{3.8}$$

then $\|T_{wz}\|_2^2 \le \beta$, where norm $\|T_{wz}\|$ is the induced norm of the performance channel defined in (2.7).

Since we have proven in the previous chapter that $\forall z \in R^p$, we have that $z z^T \le \beta I_p \Leftrightarrow \text{trace}(zz^T) \le \beta$ we will try to solve the H_2 problem as presented in the *Theorem* 3.1.

4. CONTROLLER SYNTHESIS AND NUMERICAL RESULTS

Due to the availability of some Semidefinite Programming Toolboxes the main target of this chapter is to formalize the optimization objectives in terms of some SDP and then to create an optimization procedure that would compute the optimal H_2 controller. An SDP problem is in practical terms a set of LMI restrictions together with a linear optimization objective and in it's dual form can be represented as:

$$\max_{\hat{y}} \hat{b}^T \hat{y} \text{ under the restrictions}$$

$$\sum_{k=1}^{m} \hat{y}_k \hat{A}_k - \hat{C} \le 0. \qquad (4.1)$$

where $\hat{y}$ is the vector of the unknown scalar variables $\hat{y}_k$ and $\hat{C}$, $\hat{A}_k$ are symmetric matrices of same size. There are several Toolboxes available with Matlab on the internet, most of them for free. For the use of this research we used the SDPT3.2 developed by Toh, Todd and Tutuncu and for more details see (Toh, 1998).

In order to solve the mixed problem we will bring first the inequalities present in *Theorem* 3.1 in an equivalent form, by separating the controller matrices from the rest.
If we denote

$$K := \begin{bmatrix} D_k & C_k \\ B_k & A_k \end{bmatrix} \qquad (4.2)$$

and we define

$$A_{00} = \begin{bmatrix} A & 0 \\ 0 & 0_k \end{bmatrix}, A_{0\_i} = \begin{bmatrix} A_i \\ 0_{k\times n} \end{bmatrix}, i = 1 : L$$

$$B_{00} = \begin{bmatrix} B_2 & 0 \\ 0 & I_k \end{bmatrix}, B_{10} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad (4.3)$$

$$C_{00} = \begin{bmatrix} C_y & 0 \\ 0 & I_k \end{bmatrix}, C_{0\_i} = \begin{bmatrix} C_{y\_i} \\ 0 \end{bmatrix},$$

$$E = \begin{bmatrix} I_n & 0_{n\times k} \end{bmatrix}, D_{20} = \begin{bmatrix} D_{21} \\ 0 \end{bmatrix},$$

we can express the state matrices (3.4) in terms of (4.2) and (4.3)

$$A_{cl} = A_{00} + B_{00} K C_{00}, A_{cl\_i} = (A_{0\_i} + B_{00} K C_{0\_i}) E,$$
$$B_{cl} = B_{10} + B_{00} K D_{20},$$
$$C_{cl} = C_{z0} + D_{10} K C_{00}, C_{cl\_i} = (C_{z\_i} + D_{10} K C_{0\_i}) E. \qquad (4.4)$$

The first inequality in the *Theorem* 3.1 will be equivalent with

$$\Sigma + \Lambda \Pi K \Theta^T + \Theta K^T \Pi^T \Lambda < 0 \qquad (4.5)$$

where Σ, Λ, Π and Θ are defined as:

$$\Sigma = \begin{bmatrix} A_{00}^T P + P A_{00} + \sum_{i=1}^{L} E^T P_i E & P A_{0\_1} & \cdots & P A_{0\_L} & P B_1 \\ A_{0\_1}^T P & -P_1 & & & \\ \vdots & & \ddots & & \\ A_{0\_L}^T P & & & -P_L & \\ B_1^T P & & & & -I_{m1} \end{bmatrix}, \qquad (4.6)$$

$$\Lambda = diag\left\{ P \quad \overbrace{I_n \quad \ldots \quad I_n}^{L} \quad I_{m1} \right\}, \qquad (4.7)$$

$$\Pi = \begin{bmatrix} B_2 & 0_{n\times k} \\ 0_{k\times m2} & I_k \\ 0_{n\times(m2+k)} & \\ \vdots & \\ 0_{n\times(m2+k)} & \\ 0_{m1\times(m2+k)} & \end{bmatrix}, \Theta = \begin{bmatrix} C_y^T & 0_{n\times k} \\ 0_{k\times p2} & I_k \\ C_{y1}^T & 0_{n\times k} \\ \vdots & \vdots \\ C_{yL}^T & 0_{n\times k} \\ D_{21}^T & 0_{m1\times k} \end{bmatrix}. \qquad (4.8)$$

In a similar way the second inequality in the *Theorem* 3.1 can be rewritten as

$$\Psi + \Omega K \Phi^T + \Phi K^T \Omega^T \le 0 \qquad (4.9)$$

where

$$\Phi = \begin{bmatrix} 0_{p1\times p2} & 0_{p1\times k} \\ C_y^T & 0_{n\times k} \\ 0_{k\times p2} & I_k \\ C_{y1}^T & 0_{n\times k} \\ \vdots & \\ C_{yL}^T & 0_{n\times k} \end{bmatrix},$$

$$\Omega = \begin{bmatrix} D_{12} & 0_{p1\times k} \\ 0_{((L+1)n+k)\times(m2+k)} & \end{bmatrix}$$

and

$$\Psi = \\ = \begin{pmatrix} \frac{-\beta}{L+1} I_{p1} & C_z E & C_{z\_1} & \cdots & C_{z\_L} \\ E^T C_z^T & -P & & & \\ C_{z\_1}^T & & -P_{(1:n,1:n)} & & \\ \vdots & & & \ddots & \\ C_{z\_L}^T & & & & -P_{(1:n,1:n)} \end{pmatrix},$$

where we denote by $P_{(1:n,1:n)}$ the upper left $n \times n$ block of the P matrix.

(4.10)

Since in (4.5) the inequality is not linear in both the weight matrix P and the controller K we cannot optimize with respect of both of them in the same time and we need an iterative procedure. Before being able to start this iterative process however we need to compute an initial feasible value for the P matrix. In order to do so we can consider the following inequality

$$\Sigma + \Lambda \Pi K \Theta^T + \Theta K^T \Pi^T \Lambda - \lambda I_{(L+1)n+k+m1} < 0,$$

(4.11)

and then alternately optimize with respect P and K till λ becomes smaller or equal than 0. This step could be seen as a test of checking if we can find a stabilizing controller. The idea is that we can choose a K arbitrarily, then the computed value of P at the first step can be use to the next step when we optimize in terms of K and then use the new computed value of K for the optimization of P. We could repeat then till $\lambda < 0$ or no progress anymore. Since both problems are convex ($\min(\lambda)$ where either P or K are variable in (4.11)), a smaller or equal value of λ will result for each step, which ensures the convergence. The resulting values of P, $P_1, \ldots, P_L$ and K could be seen as a good starting point for the H_2 optimization problem.

5. CONCLUSIONS

The present approach offers, for the first time to our knowledge, the possibility to formalize the H_2 control objectives for TDS in terms of two LMI's. The conditions are constructed from the initial requirements and several alternative objectives are analyzed and related to each other. A methodology for the computation of the feedback controller is also presented together with a two stages alternating optimization procedure. An example is presented in the end. One new feature of the optimization procedure is the solution chosen for the computation of the a feasible set of the weight matrices present in the Lypunov functionals, which doesn't make use of the Projection Lemma us done in similar situations in the past (see for example [7,8,11]). A numerical example is presented in the end. The procedure is related to the family of the Lyapunov based approaches and it offers the mainframe for future extensions. The mixed problem H_2 / H_∞ will be presented in a future paper.

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