NONLINEAR MODELING OF A QUANSER FLEXIBLE LINK

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Abstract: This paper presents an approach on the bond graph modeling applied to mechanical oscillating systems, Quanser flexible link, followed by the simulation of the resulted mathematical model. The system was decomposed into smaller parts – subsystems – that were modeled separately. The obtained subsystems generated submodels and the overall model was then built up by combining these separate structures. The linear model was obtained by modifying the flexible link displacement factor. We compared the results of linear and nonlinear models simulation to identify the nonlinear factor influence upon the output parameters evolution.

Keywords: Bond Graphs, Flexible Arm, Modeling, Simulation.

1. INTRODUCTION

The study of system dynamics resides in modeling its behavior. Systems models are simplified, abstracted structures used to predict the behavior of the studied systems. Our interest is pointing towards the mathematical model used to predict certain aspects of the system response to the inputs. In mathematical notations a system model is described by a set of ordinary differential equations in terms of state variables and a set of algebraic equations that relate the state variable to other system variables.

In order to model a system it is usually necessary to decompose the system into smaller parts – subsystems - that can be modeled separately. The subsystem is a part of the system that can be modeled as a system itself obtaining submodels. The overall model can then be built up by combining the separate submodels.

System models will be constructed using a uniform notation for all types of physical systems which is bond graph method based on energy and information flow (Karnopp D., 1975). The method uses the effort-flow analogy to describe physical processes. A bond graph consists of subsystems linked together by lines representing power bonds. Each process is described by a pair of variables, effort (*e*) and flow (*f*), and their product is the power. The direction of power is depicted by a half arrow. In a dynamic system the effort and the flow variables, and hence the power fluctuate in time. Using the bond graph approach it is possible to develop models of electrical, mechanical, magnetic, hydraulic, pneumatic, thermal, and other systems using a small set of variables.

It is remarkable how models of various systems belonging to different engineering domains can be express using a set of only nine elements, called elementary components. These elements are sufficient to describe any physical system regardless of the energy types processed by it.

A classification of bond graph elements can be made up by the number of ports. The ports are places where interactions with other processes take place. There are one port elements represented by inertial elements (I), capacitive elements (C), resistive elements (R), effort sources (SE) and flow sources (SF), two ports elements represented by transformer elements (TF) and gyrator elements (GY), and multi ports elements - effort junctions (J0) and flow junctions (J1).

Two other types of variables are very important in describing dynamic systems and these variables, sometimes called energy variables, are the generalized momentum (*p*) as time integral of effort and the generalized displacement (*q*) as time integral of flow.

Depending on the complexity of systems it is a good idea to use a systematic way of building the model in small steps (Thoma J., 1990).
A first step is to write a word bond graph which contains words instead of standard symbols for the main components and bonds for power and signal exchange. The component name is useful, but it is more important the connection of the components to other components through ports.
There are two types of ports, power ports characterized by power flow into or out of the component, graphically represented by a half arrow, and control ports characterized by negligible power flow and high information content, depicted by a full arrow.
The next step is to replace words by standards elements which contain precise mathematical or functional relations. Each element represents a definite effect or action in the system. When the bond graph model is done it is possible to formulate the state space equations starting from the constitutive relations of elements.

2. QUANSER FLEXIBLE LINK SYSTEM

The SRV02 rotary plant module serves as the base component for the rotary family of experiments. Its modularity facilitates the change from one experimental setup to another. The SRV02 plant consists of a DC motor in a solid aluminum frame equipped with a gearbox whose output drives external gears. The basic unit is equipped with a potentiometer to measure the output/load angular position.

Fig. 1. SRV02 plant – DC motor and gear box

The external gear can be reconfigured in two configurations:
- Low Gear Ratio - this is the recommended configuration to perform the position and speed control experiments with no other module attached to the output.
The only loads that are recommended for this configuration are the bar and circular loads supplied with the system;
- High Gear Ratio - this is the recommended configuration for all other experiments that require an additional module such as the flexible beam, ball and beam, gyro, rotary inverted pendulum etc.

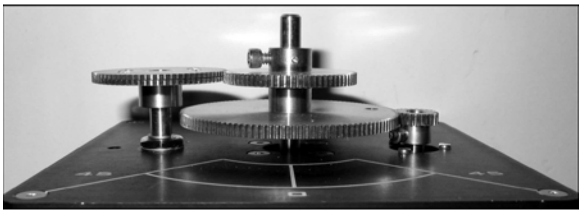

Fig. 2. High gear configuration

The Rotary Flexible Link module is designed as an attachment to the SRV02 plant. The module consists of a thin stainless steel link instrumented with a strain gage. The arm deflection is measured via the strain gage output. The model is designed to accentuate the effects of flexible links in robot control systems.

Fig. 3. Quanser Flexible Link system

3. BOND GRAPH MODEL OF THE SYSTEM

We proceed to the development of the model by identifying the system components and connecting them as they are in the real system (Damic V., 2002).

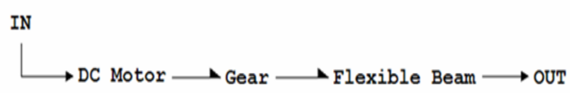

Fig. 4. Block diagram representation of the system

The bond graph model of **DC Motor** component is presented in the figure below:

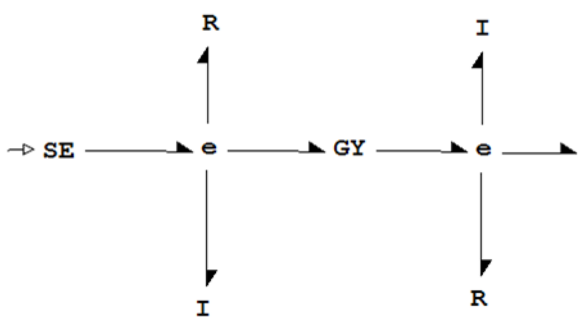

Fig. 5. Bond graph model of DC motor component

The gyrator GY describes the electromechanical conversion in the motor relating the back emf from the electrical part to the angular velocity of the rotor from the mechanical part, respectively the armature current from the electrical part to the torque acting on the rotor. For this reason, the gyrators are called overcrossed transformers.

$$\begin{cases} V_b = k_m \cdot \omega \\ T_m = k_t \cdot i_a \end{cases} \quad (1)$$

where: k_t is the motor torque constant;

k_m is the back emf constant.

The electrical process in the armature is described in bond graph terms by the armature resistance R_m represented using a resistive element (R), and the armature inductance L_m represented using an inertial element (I). These two elements are joined through an effort junction (1 junction). The mechanical process is also described using an inertial element that models the rotation of the rotor mass moment of inertia J_m, and a resistive element that models the linear friction coefficient B_m. These two elements are joined through an effort junction (1 junction).

The gearbox named **Gear** is represented by a transformer (TF) having its parameter equal to the reduction ratio of the gearbox, an inertial element, representing the equivalent high gear inertia J_{hg} and a resistive element to model the viscous friction forces.

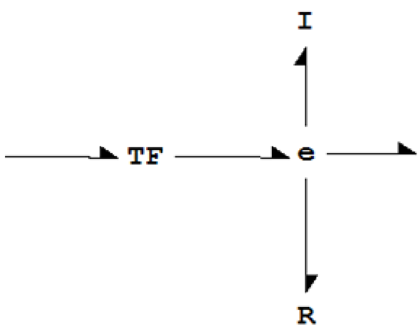

Fig. 6. The bond graph model of Gear component

In order to model the component Flexible it is necessary to write the equations of motion of the rotary flexible link which involves the modeling of gear and flexible link as rigid bodies. The link is modeled as a rod rotating about its endpoint and it has the moment of inertia:

$$J_l = \frac{m_l L^2}{3} \quad (2)$$

where m_l is the mass of flexible link and L is the length of flexible link.

The relation between the natural frequency of the system, moment of inertia and stiffness coefficient is given by

$$\omega_n = \sqrt{\frac{K_{stiff}}{J_l}} \quad (3)$$

with $\omega_n = 2\pi f_n$, where the natural frequency f_n is experimentally computed and k_{stiff} is the equivalent torsion spring constant.

For this system we have neglected the friction effects between the rotational gear and flexible link.

Figure 7 depicts the flexible link in motion

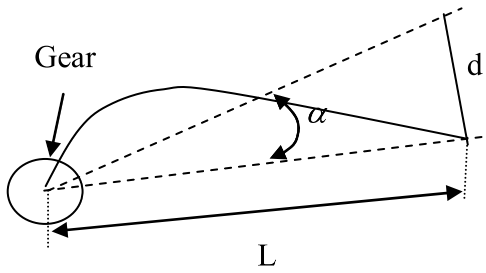

Fig. 7. Schematic of flexible link module

This figure shows the relation between the angular position α and the displacement of the flexible link d, respectively the relation between the angular velocity and the velocity of flexible link.

$$d = L\sin(\alpha) \quad (4)$$

$$\dot{d} = L\cos(\alpha)\dot{\alpha} \quad (5)$$

Using the relations presented above, the bond graph model of flexible link has the following form:

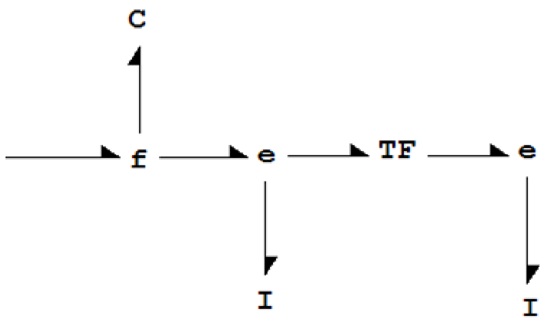

Fig. 8. The bond graph model of Flexible link component

The angular velocity with respect to rotational gear and the velocity of flexible link were introduced by two effort junctions related by a MTF element with the transformer modulus equal to $L\cos(\alpha)$. The capacitive element is used to model the torsion stiffness (Fossard A. J., 1993).

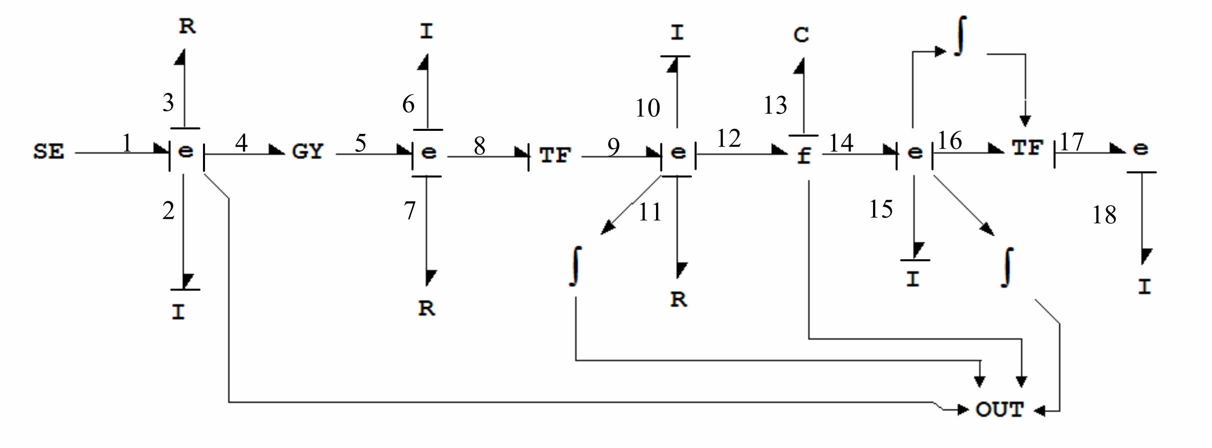

Fig. 9. The complete Bond Graph model of the system

By joining together these three models, we obtained the complete bond graph model of the Quanser flexible link system.

Gawthrop and Bevan (2007) presented a model of this system resuming to a simplified bond graph representation.

Before writing the constitutive equations it is desirable to name all the bonds in the graph, to assign to each bond a reference power direction and to assign causality to each bond. The study of causality is an important feature of bond graph method and it is specified by means of the causal stroke. The causal stroke is a short, perpendicular line made at one of the bond line ends indicating the direction in which the effort signal is directed.

It can be easily seen that we have four elements in integral causality which means that we have four state variables in terms of generalized momentums and generalized displacements $(p_2, p_{10}, q_{13}, p_{15})$.

The constitutive equations of I and C elements in integral causality are given by the following relations:

$$f_2 = \frac{1}{L_m} p_2, f_{10} = \frac{1}{J_{hg}} p_{10}, f_{15} = \frac{1}{J_l} p_{15} \qquad (6)$$

$$e_{13} = \frac{1}{k_{stiff}} q_{13} \qquad (7)$$

For the inertial elements in derivative causality and R elements, the constitutive equations are as follows:

$$e_3 = R_m f_3,\ e_7 = B_m f_7,\ e_{11} = B_{hg} f_{11} \qquad (8)$$

$$p_6 = J_m f_6,\ p_{15} = m_l f_{15} \qquad (9)$$

The effort junctions (J1) and the flow junctions (J0) are characterized by, the flows on all bonds equal to zero and the algebraic sum of the efforts equal to zero, respectively the efforts on all bonds equal to zero and the algebraic sum of the flows equal to zero. For the TF and GY elements we have the following relations:

$$\text{(TF)} \begin{array}{l} f_8 = \frac{1}{n} f_9 \\ e_9 = \frac{1}{n} e_8 \end{array},\ \text{(GY)} \begin{array}{l} e_4 = k_t f_5 \\ e_5 = k_t f_4 \end{array} \qquad (10)$$

There is one more transformer element, a modulated or controlled transformer, denoted MTF. This component satisfies the power conservation requirement. This is satisfied not only by the transformer modulus, but also by the ratios dependent on a control variable. In our case the modulated transformer modulus depends on the relative angular velocity of the flexible link. Thus, the modulated transformer is described by the following constitutive relations:

$$\begin{array}{l} e_{16} = k_1 e_{17} \\ f_{17} = k_1 f_{16} \end{array} \qquad (11)$$

Combining all these equations and using the following notations

$$J_e = n^2 J_{hg} + J_m,\ R_e = n^2 R_{hg} + R_m \qquad (12)$$

$$J_{eb} = k_1^2 m_l + J_l \qquad (13)$$

$$k_1 = L\cos(\alpha) \qquad (14)$$

we arrive at the state space equations in terms of energy variables p and q.

$$\begin{cases} \dot{p}_2 = -\frac{R_m}{L_m} p_2 - \frac{k_t}{nJ_{hg}} p_{10} + e_1 \\ \dot{p}_{10} = \frac{nJ_{hg} k_t}{J_e L_m} p_2 - \frac{r_e}{J_e} p_{10} - \frac{k^2 J_{hg}}{k_{stiff} J_e} q_{13} \\ \dot{q}_{13} = \frac{1}{J_{hg}} p_{10} - \frac{1}{k_1 m_l} p_{18} \\ \dot{p}_{18} = \frac{k_1 m_l}{J_{eb} k_{stiff}} q_{13} \end{cases} \qquad (15)$$

Deriving the constitutive equations of I and C elements we will obtain the state space equations in terms of power variables. Taking into account the physical significance of the effort *e* and flow *f*:

$$f_2 = i_a\,,\; f_{10} = \omega\,,\; f_{18} = \omega_l \tag{16}$$

$$e_{13} = \tau \tag{17}$$

we arrive at the final form of state space equations:

$$\begin{cases} \dfrac{di_a}{dt} = -\dfrac{R_m}{L_m} i_a - \dfrac{k_t}{nL_m}\omega + \dfrac{1}{L_m} u_a \\ \dfrac{d\omega}{dt} = \dfrac{nk_t}{J_e} i_a - \dfrac{r_e}{J_e}\omega - \dfrac{n^2}{J_e}\tau \\ \dfrac{d\tau}{dt} = \dfrac{1}{k_{stiff}}\omega - \dfrac{1}{k_{stiff}}\omega_l \\ \dfrac{d\omega_l}{dt} = \dfrac{1}{J_{eb}}\tau \end{cases} \tag{18}$$

Where: i_a - the armature current, u_a - the armature voltage, ω - angular velocity of gear shaft, τ - the torque action on the rotor, ω_l - angular velocity of the flexible link.

4. SIMULATION RESULTS

For the simulation the following values were used: u_a=2V motor input voltage, R_m=2.6 Ω armature resistance, $L_m = 0.18e-3H$ armature inductance, $k_t = 0.00767\,N \cdot m$ motor torque constant, $J_m = 3.87e-7\,Kg \cdot m^2$ motor inertia, $J_{hg} = 2e-3\,Kg \cdot m^2$ equivalent high gear inertia, $B_{hg} = 4e-3\,Nm/(rd/s)$ viscous damping coefficient, $L = 0.45m$ the flexible link length, $m_l = 0.08\,Kg$ flexible link mass; $f_c = 3.2\,Hz$ natural frequency;

Figures 10 – 14 represent the variations of main model parameters.

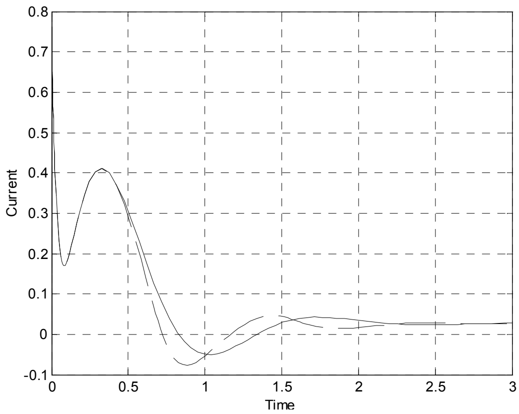

Fig. 10. The current variation in the DC motor

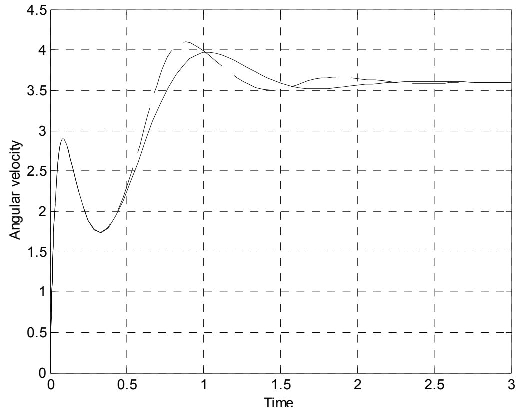

Fig. 11. The gear shaft angular velocity as time function

In the diagrams the continuous line was used to present the results of linear system model. With dashed lines were represented the results of nonlinear system model. The nonlinearity is introduced by the angular factor $\sin(\alpha)$ in the free end of flexible link displacement (eq. 4).

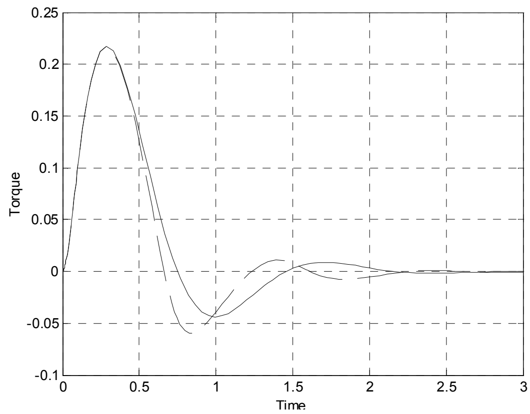

Fig. 12. The shaft torque variation

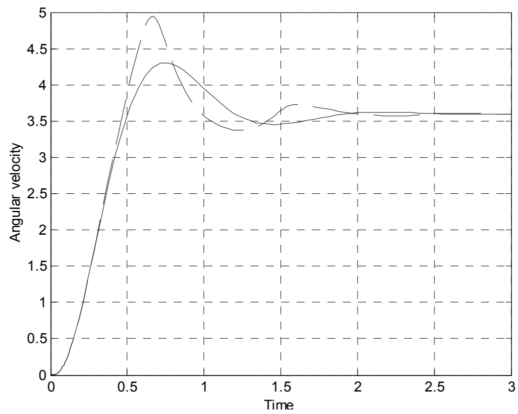

Fig. 13. The angular velocity of the flexible link deflection

Previous works on oscillating mechanical units such as the flexible beam, ball and beam, gyro, rotary inverted pendulum etc. carried out (Ionete C., 2003) showed similar evolutions of the system behavior.

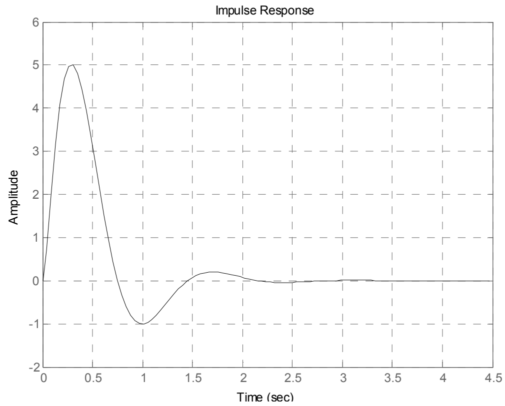

Fig. 14. The impulse response of the flexible link

Based on parameters evolution in the diagrams above we notice a slight difference between linear and nonlinear response. The settling time is almost the same for both systems even if the amplitude for nonlinear model is higher. The influence of system nonlinearity is more visible in the variation of angular velocity of the flexible link deflection due to angular factor existent in its mathematical expression.

5. CONCLUDING REMARKS

In this work our interest was pointed towards the mathematical model used to predict certain aspects of the system response to the inputs.

The system model was constructed using a uniform notation for all types of physical systems, the bond graph method based on energy and information flow. Using this method, models of various systems belonging to different engineering domains can be express using a set of only nine elements.

First we wrote a word bond graph containing words instead of standard symbols for the main components and bonds for power and signal exchange. The next step was to replace words by standards elements that contain precise mathematical or functional relations.

The system was decomposed into three subsystems that were modeled separately. By joining together these three models, we obtained the complete bond graph model of the Quanser flexible link system.

The model was simulated in the MatLab environment the results showing differences between linear and nonlinear systems in terms of amplitude for the output parameters due to angular factor existent in the mathematical expression of the last one. Despite these differences the settling time is almost the same for both systems.

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