ON PARAMETERS ESTIMATION OF THE INPUT-OUTPUT CHLORINE RESIDUALS MODELS OF DRINKING WATER DISTRIBUTION NETWORKS

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Abstract: This paper presents some aspects concerning the estimation of the parameters in the input-output chlorine residuals models of drinking water distribution networks (DWDNs) used for transportation of treated drinkable water from the water treating plants to consumers. To kill the microorganisms that can cause the waterborne ills, disinfection is usually the final treatment stage in the drinking water plants. Usually, the disinfectant used in DWDNs is chlorine because it is inexpensive and effectively annihilates a variety of disease-causing organisms. To maintain high-quality drinkable water in such distribution networks it is necessary to regulate the chlorine residual concentration within a prescribed set of bounds. Since the DWDNs are large scale systems with high uncertainties and time varying delays, in order to obtain useful models for control, in this paper, some aspects regarding the estimation of parameters of the input-output chlorine residuals models are analysed. The model parameters are time varying and heavily dependent on the hydraulics which is a main source of uncertainties in chlorine concentration models. Using the input/output data and a priori knowledge of the system, in this paper will be presented a method for the estimation of bounding values corresponding to model parameters so that the obtained model could be used in a robust predictive control method.

Keywords: Nonlinear systems, Time delay dystems, Modelling, Parameter estimation.

1. INTRODUCTION

Drinking water distribution networks (DWDNs) are complex, large-scale systems composed of storage tanks, pumps, valves and pipes that are connected together to supply clean water to industrial and domestic users.

Usually, drinking water is take from ground sources (rivers, lakes), or underground sources (wells, springs). To reduce the risk of human exposure to pathogens, drinking water is treated in the water treating plants to filter out unwanted substances using physical and chemical methods thereby making it safe, clean and healthy for consume. To kill the microorganisms that can cause some waterborne ills, disinfection is usually the final treatment stage in the drinking water plants. Ussualy, the disinfectant used in DWDNs is chlorine because it is inexpensive and effectively annihilates a variety of disease-causing organisms (Brdys *et al*., 2001; Brdys and Chang, 2002a; LeChevallier and Kwok-Keung, 2004). But disinfectant concentration in the water may decay during the transportation, and bacteria growth cannot be controlled if disinfectant concentration is lower than a certain level. As a result, the bacteria and waterborne pathogens may grow. Microorganisms can also grow up on the pipe and tank walls as not all microorganisms are killed in the water treatment plants. The surviving bacteria can grow to harmful levels without further disinfection during water transportation in the DWDNs. Although the magnitude of the waterborne diseases has reduced significantly when the chlorine to treat drinking water does not decrease under a minimum level, at the same time during the water treatment process, chlorine (especially at large level) can react with organic compounds present in the bulk water or pipe walls to form Disinfection By-Products, some of which are suspected carcinogens (LeChevallier and Kwok-Keung, 2004; Polycarpou *et al*., 2002). This leads to a limitation on the chlorine dosage in DWDNs and stricter monitoring of the chlorine residual concentration in the water network. For instance, the U.S. EPA (Environmental Protection Agency of United States) established that the minimum chlorine residual that must be present at points of water consumption is 0.2 mg/l (LeChevallier and Kwok-Keung, 2004; Polycarpou *et al*., 2002), while the maximum residual chlorine in the DWDN is 4 mg/l. As a consequence, distribution network water quality control is an important problem, and the regulation of chlorine residual concentration within a prescribed set of bounds is part of the solution. At the same time, the accurate and reliable control of chlorine residuals within a DWDN is a new and complex problem (Chang and Brdys, 2003; Chang, 2003, Polycarpou *et al*., 2002).

In fact, in the control problem of DWDNs, besides the quality control one major aspect is the quantity control. The quantity control deals with water flows and pressures producing pump and valve schedules. The purpose of quantity control is to satisfy the time-varying water demands, maintain a stable prescribed water pressure throughout the network and minimize the operating costs (Chang, 2003). Since in the DWDNs the chlorine is carried by water, the characteristics of the chlorine transportation and mixing are determined directly by the water flows, velocities and detention times in the reservoirs. Hence, the hydraulics has significant effect on the chlorine concentration, while the chlorine injection's effect on the hydraulics can be neglected because the chlorine mass added is negligible compared to the mass of the water (Brdys and Chang, 2002a). At the same time, the hydraulics is a main source of uncertainties in chlorine concentration model. Such uncertainties are determined by many factors in the system, such as water demand prediction, hydraulic time step, pump characteristic curve, physical coefficients of the pipes and storage facilities, etc. Another source of uncertainty is associated with the chlorine decay caused by chlorine reactions.

For water quality control, an important problem that must to be solved is the modelling of chlorine residual dynamics in DWDNs. It is necessary that the obtained models to be convenient for controller design and at the same time suitable for handling the transport delay, which is inherently associated with the time-varying delivery of water. Note that in (Petre and Selisteanu, 2007b), the input-output relationship between chlorine concentrations at an injection node (input) and at a monitored node (output) was modeled as a linear discrete-time system with time-varying parameters. The model was developed beginning with the case of one single pipe following with a water network with any number of pipes connected in series or in parallel and finally with a complex water network with storage facilities. This model was formulated in discrete-time as an ARMA (auto-regressive moving average) model with time-varying coefficients (heavily depending on the hydraulics). The discrete-time formulation is suitable both for handling the transport delay, and control.

The model parameters are associated with the time delays of chlorine transportation in the water flow paths. Since in a DWDN the calculating the time delays is complicated not only due to this topology, but also because consumer water use rates are varying and unknown, result in inaccurate parameters of the obtained model. At the same time uncertainties in the hydraulic information finally appear also in the chlorine residual model through the detention time and mixing ratio. Then, in order to complete the model design presented in (Petre and Selisteanu, 2007b), its parameters must be estimated. Since operational control of a DWDN is based on the prediction methods, in this paper, some aspects concerning the estimation of the parameters of this model are presented and analysed. Using the input/output data and *a priori* knowledge of the system, in this paper will be presented a method for the estimation of the bounding values corresponding to model parameters so that the obtained model is the least conservative uncertainty model and could be used in a robust predictive control method.

The rest of this paper is organized as follows. An input-output explicit model of chlorine residuals in DWDNs are presented in Section 2. Section 3 describes a method for the estimation of the bounding values corresponding to model parameters so that the obtained model can be used in a robust predictive control method. Concluding remarks in Section 4 complete this paper.

2. THE INPUT-OUTPUT MODEL OF CHLORINE RESIDUALS IN DWDNs

Using the results presented in (Petre and Selisteanu, 2007b), firstly we present the input-output model of a complex DWDN with n_M monitored nodes, n_I chlorine injection nodes and n_T tanks, schematized in Fig. 1. Generally, in this network there could be n_I chlorine transportation path sets from injection nodes to each tank, n_I chlorine transportation path sets from injection nodes to each monitored node, and n_T chlorine transportation path sets from storage tanks to each monitored node. Together, there are maximum $n_M \times n_I + n_M \times n_T + n_T \times n_I$ delays to be identified. These delays can be calculated over a

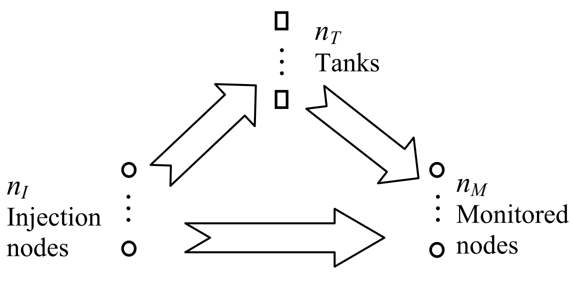

Fig. 1. Illustration of chlorine transportation delays in a DWDN

considered modelling horizon T_m by using the path analysis algorithms described in (Petre and Selisteanu, 2007b). Finally, the delays are discretized and the discrete delay number set F (over the filling cycle), I (over a time slot) and D (over the draining cycle) can be obtained.

The input-output model corresponding of these n_I inputs and n_M outputs takes the form (Chang, 2003):

$$y_n(t) = \sum_{j=1}^{n_I} \sum_{i \in I_{n,j}(t)} a_{n,j,i}(t) u_j(t-i) + \varepsilon_n(t), \quad t \in S_{n,f} \tag{1a}$$

and

$$y_n(t) = \sum_{i=1}^{n_T} b_{n,i}(t) y_n(t) + \sum_{j=1}^{n_I} \sum_{i \in I_{n,j}(t)} a_{n,j,i}(t) u_j(t-i) + \varepsilon_n(y), \quad t \in S_{n,d} \tag{1b}$$

where:

- $y_1(t), \ldots, y_{n_M}(t)$ are chlorine concentrations at the n_M monitored nodes;
- $u_1(t), \ldots, u_{n_I}(t)$ are chlorine concentrations at the n_I injection nodes;
- $S_{n,f}$ and $S_{n,d}$ denote time-slots of filling cycle and draining cycle, respectively;
- $I_{n,j}(t)$ denotes delay number set that corresponds to $y_n(t)$ associating with the j^{th} input, $u_j(t)$;
- $a_{n,j,i}$ are the model parameters that corresponds to the n^{th} output $y_n(t)$ and the j^{th} input $u_j(t)$ associating with delay number I;
- $b_{n,i}$, $i = 1, \ldots, n_T$, are the parameters corresponding to the n^{th} output that describe dynamics caused by n_T tanks;
- $\varepsilon(t)$ denotes the modelling error in $y_n(t)$.

The n^{th} output of the input-output model (1a), (1b) can be written in a compact form as:

$$y_n(t) = \psi_n(t)^T \theta_n(t) + \varepsilon_n(t), \quad n = 1, \ldots, n_M \tag{2}$$

where $\psi_n(t)^T$ and $\theta_n(t)$ are defined as:

$$\phi_n(t)^T = [u_1(t - i_{n,1,l}) \ldots u_1(t - i_{n,1,u}) \ldots u_{n_I}(t - i_{n,n_I,l}) \ldots u_1(t - i_{n,n_I,u})]$$

$$\theta_n(t) = [a_{n,1,l}(t) \ldots a_{n,1,u}(t) \ldots a_{n,n_I,l}(t) \ldots a_{n,n_I,u}(t)],$$

for $t \in S_{n,f}$

and

$$\psi_n(t)^T = [y_n(t-1) \ldots y_n(t-n_T) \;\; u_1(t - i_{n,1,l}) \ldots u_1(t - i_{n,1,u}) \ldots u_{n_I}(t - i_{n,n_I,l}) \ldots u_1(t - i_{n,n_I,u})]$$

$$\theta_n(t) = [b_{n,1}(t) \ldots b_{n,n_T}(t) \;\; a_{n,1,l}(t) \ldots a_{n,1,u}(t) \ldots a_{n,n_I,l}(t) \ldots a_{n,n_I,u}(t)]^T,$$

for $t \in S_{n,d}$, where $\{i_{n,j,l}, \ldots, i_{n,j,u}\} = I_{n,j}$ for $j = 1, \ldots, n_I$.

Note, that (2) is in the same ARMA form like the model presented in (Petre and Selisteanu, 2007b). Since in a DWDN the interactions between the inputs and outputs are inevitable and there are not interactions between outputs, then a DWDN can be simplified as a number of n_M multiple inputs – single output (MISO) systems. So, each MISO model structure can be identified by repeating the procedure individually for each output. Hence, the parameter estimation problem can be formulated for a MISO time-varying dynamical model with delayed inputs under uncertainties, which is given in a general ARMA form as:

$$y(t) = \psi(t)^T \theta(t) + \varepsilon(t) \tag{3}$$

where the model parameters $\theta(t)$ are time varying, and the model error $\varepsilon(t)$ is unknown but bounded. It must be noted that the structure error $\varepsilon(t)$ is operating point dependent and could be very large under certain inputs.

In (3) the regressor vector and parameter vector are defined as:

$$\psi(t)^T = [y(t-1) \ldots y(t-n) \;\; u_1(t - i_{1,l}) \ldots u_1(t - i_{1,u}) \ldots u_m(t - i_{m,l}) \ldots u_m(t - i_{m,u})]$$

$$\theta(t) = [b_1(t) \ldots b_n(t) \;\; a_{1,i_{1,l}}(t) \ldots a_{1,i_{1,u}}(t) \ldots a_{m,i_{m,l}}(t) \ldots a_{m,i_{m,u}}(t)]^T \tag{4}$$

It must be noted that the operational control of a DWDN is repeated over certain period (typically, the control horizon used in practice is 24 hours) and is based on a nominal water demand prediction over this period. The values of $y(t-i), i = 1, \ldots, n$ and $u_k(t - i_{k,l}), \ldots, u_k(t - i_{k,u})$, $k = 1, \ldots, m$ in the regressor $\psi(t)$ can be obtained by using a DWDN simulator (named the implicit model) which is used to predict the real DWDN outputs, see Fig. 2, where $y_s(t)$ is the simulator output, $\varepsilon_s(t)$ represents the simulator error and $y_{DWDN}(t)$ is the output that equals to real DWDN output. It is clear that DWDN simulator is considered the “plant” in parameter estimation process, and $y_s(t)$ is the plant output during estimation process. For clarity, in the following $y_p(t)$ will represent the simulator output in the presenting of parameter estimation algorithm.

A parametric model is the mapping from the input to the output space with proper parameters associated with the model structures such as transfer function or ARMA model. Considering a time-invariant system modeled by an ARMA expression:

$$y(t) = \psi(t)^T \theta + \varepsilon(t), \tag{5}$$

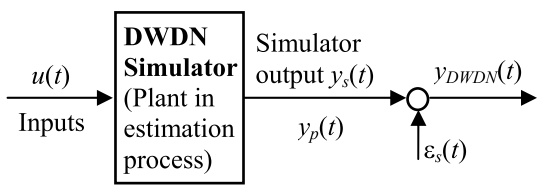

Fig. 2. DWDN simulator in model estimation

where θ is the model parameter vector and $\varepsilon(t)$ is the modelling error, with proper values of these parameters the mapping can make model output $y(t)$ equal to the plant output $y_p(t)$. Parameter estimation problem is the process of obtaining the value of θ based on a series of observations of $y(t)$ on a time horizon $[1,\ldots,N]$. Since the operational control of a DWDN is usually based on the prediction methods, the requirements of a robust predictive control do not impose exact parameter values of the model parameters. A robust predictive controller could be designed based on a bounded parameter model.

If in model (3) we can find a bound of the error $\varepsilon(t)$, that is $\|\varepsilon(t)\| \le \varepsilon_{\max}$ with $\varepsilon_{\max} > 0$, then the observations $\{y(1), y(2),\ldots, y(N)\}$ over modelling horizon N confines the parameter vector θ to a feasible parameter set given by:

$$y(i) - \varepsilon_{\max} \le \psi(t)^T \theta \le y(i) + \varepsilon_{\max}\ ,\ i = 1,\ldots,N \quad (6)$$

Since in (6) both the observations $y(i)$ and the regressor vector $\psi(t)$ are deterministic and bounded, we obtain that the value of each parameter θ_i in parameter vector θ is bounded.

This method named *bounded concept* (Chang, 2003) uses the a priori knowledge on modelling error or even parameter variation that usually is derived by mathematically analysis of physical system. Unfortunately, for the chlorine residual model (1) is not straightforward to obtain in advance the prior bounds on $\varepsilon(t)$. Then, in order to make the model more feasible and efficient for control purposes, in the following section we present another approach to find the parameters' bounds. Since in model (5) the uncertainties are located in two parts of model (parameters θ and modelling error $\varepsilon(t)$), then instead of trying to construct a model with unique parameter vector, multiple parameter vectors are assumed that are associated with the inputs and modelling error, which leads to the introduction of point-parametric model.

3. POINT-PARAMETRIC MODEL FOR PARAMETER ESTIMATION

As stated before instead of trying to obtain in advance the prior bounds on $\varepsilon(t)$, during estimation process it will be handled together with model parameters. The uncertainties in the parameter and structure error can be better explained by the *point-parametric model* (Chang and Brdys, 2003).

From the models presented in (Petre and Selisteanu, 2007b), one can see that there are internal links between the uncertainty in the parameters and the uncertainty in structure error of the model which can expressed as:

$$y(t) = M(u, \theta(u,t), t) + \varepsilon(u,t) \quad (7)$$

where M denotes the model function, and u is the input (as function of time). In the *point-parametric model* (7), the parameter θ and the error ε are input dependent. Uncertainty in the parameter can now be linked to the structure error uncertainty directly. Now it is possible to trade off the uncertainty distribution between the parameter and structure error and estimate them jointly. However, in order to get sufficiently rich information in the system outputs it is necessary to excite the plant with some specially inputs (Chang, 2003, Rossiter, 2003). It means that with this information, it will be possible to find a parameter set in the parameter space so that for any output there exists a parameter in this set such that the plant output can be produced by the model with this parameter.

Since the uncertainties in the system may locate in the parameters and/or structure error part of the model, this results in two types of possible model structure differing in uncertainty allocation. The first type of model allocates all of the uncertainties in the process into model parameters, resulting in:

$$y(t) = M(u, \theta(u,t), t) \quad (8)$$

where the parameters is time-varying. . The second type of model explains the uncertainties in the process by constant parameters and a time-varying modelling error, as:

$$y(t) = M(u, \theta(u), t) + \varepsilon(u,t) \quad (9)$$

In both of the cases, the parameters and modelling error are input dependent. In the subsequence, based on the observations corresponding to some applied inputs, for the models (8) and (9) will be presented the algorithms for finding the bounds on parameters θ and modelling error ε.

3.1. Time-varying parameter

First, we consider the model (8) with all uncertainties in the process locating only in the time-varying parameters.

Problem formulation 1. For any plant input $u(t)$, there exists a pair $\{\theta(t), \psi(t)\}$ so that equation $y = \psi(t)^T \theta(t)$ generates $y(t)$ that equals to the plant output $y_p(t)$, for any t over the considered time horizon.

Let $y^j(\cdot)$ denote the model response to input $u^j(\cdot)$ over the time interval $[t_0, t_0 + T_m]$. According to the above problem formulation, there exist trajectories of $\theta^j(\cdot)$ so that the model response equals to the plant response $y(t) = y_p(t)$. Different inputs require different scenarios of $\theta^j(\cdot)$ in order to match the plant responses, giving:

$$y^j = \psi^j(t)^T \theta^j(t) \quad (10)$$

It is assumed that the control input is valued on a compact set so that the trajectories of $\theta^j(\cdot)$ can be bounded above and below over the time interval $[t_0, t_0 + T_m]$. For robust control purposes, obtaining the exact envelopes over time is not essential. A constant least conservative envelope that can bound the exact envelope trajectory over time is estimated, which is sufficient for robust control design in the water quality control (Brdys and Chang, 2002b).

To estimate the envelope of the parameters it is necessary to perform a series of experiments under input $u^j(\cdot)$, $j = 1,\ldots,E$. The corresponding outputs are collected as $y^j(\cdot)$ from $t = 1$ to $t = N$. A feasible parameter set corresponding to E inputs can be defined as:

$$\theta^j(t) \in \tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u) \tag{11}$$

$$\tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u) \stackrel{\Delta}{=} \{\theta^j(t) \in R^M : y^j(t) = \psi^j(t)^T \theta^j(t)\}, \tag{12}$$

with $\tilde{\theta}^l \le \theta^j(t) \le \tilde{\theta}^u$, $j = 1,\ldots,E$, $t = 1,\ldots,N$,

where $\tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u)$ is the union of the parameter sets that are consistent with the inputs and corresponding measurements, $\tilde{\theta}^l$ and $\tilde{\theta}^u$ are the lower and the upper parameter bounds, M is the dimension of the parameter vector, E is the experiment number, and N is the number of observations under an experiment.

Example 1. A two dimension $\theta = [\theta_1, \theta_2]$ example at time instant $t = k$ over time horizon $[1,\ldots,N]$ is shown in Fig. 3, where $\theta^1,\ldots,\theta^5$ are parameters corresponding to five inputs $u(\cdot)^1,\ldots,u(\cdot)^5$, respectively:

$$\begin{aligned} &y^1(k) = \psi^1(k)^T \theta^1(k) \\ &\vdots \\ &y^5(k) = \psi^5(k)^T \theta^5(k) \end{aligned} \tag{13}$$

According to problem formulation 1, there could exist multiple parameter values corresponding to one input. Values of θ^j are not unique as shown in the figure.

$\tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u)$ defines a union of the parameters bounded by an axis-parallel box, which is composed of at least one value of $\theta^1(k),\ldots,\theta^5(k)$ respectively.

When the box bounded space is the "minimum", it is called the least conservative bounding of the union, that is $\tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u)^*$. The least conservative estimation for set $\tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u)$ can be calculated by optimizing an index function :

$$[\tilde{\theta}^l, \tilde{\theta}^u]^* = \underset{[\tilde{\theta}^l, \tilde{\theta}^u, \theta^j(t)]}{\arg\min}\ J(\tilde{\theta}^l, \tilde{\theta}^u] \tag{14}$$

with $\theta^j \in \tilde{\Theta}(\tilde{\theta}^l, \tilde{\theta}^u)$, where

$$J(\tilde{\theta}^l, \tilde{\theta}^u) = (\tilde{\theta}^u - \tilde{\theta}^l)^T P(\tilde{\theta}^u - \tilde{\theta}^l) \tag{15}$$

A possible choice of the matrix P could be the $(M \times M)$-dimensional identity matrix, M being the dimension of the parameter vector. The formulation of the index is related to finding the minimum diagonal distance of the axis-parallel box that contains parameters that are consistent with the experiment data, in other word, we are looking for the "uncertainty-minimum" parameter sets that can explain the experiment data. When $P = I$, where I is an identity matrix, the performance index exactly represents the diagonal. Apparently, the result of (14) depends on the experiments. As the inputs are evaluated on a compact set it is possible to find a set of experiments so that the result of (14) is consistent with all the inputs. Finally a time-varying point-parametric model with constant bounded parameter is obtained as:

$$y = \psi(t)^T \theta(t) \tag{16}$$

$$\theta(t) \in \tilde{\Theta}, \quad \tilde{\Theta} \stackrel{\Delta}{=} \{\theta \in R^M : \tilde{\theta}^{l^*} \le \theta(t) \le \tilde{\theta}^{u^*}\} \tag{17}$$

This is a time-varying model in which parameters are bounded by constant envelopes.

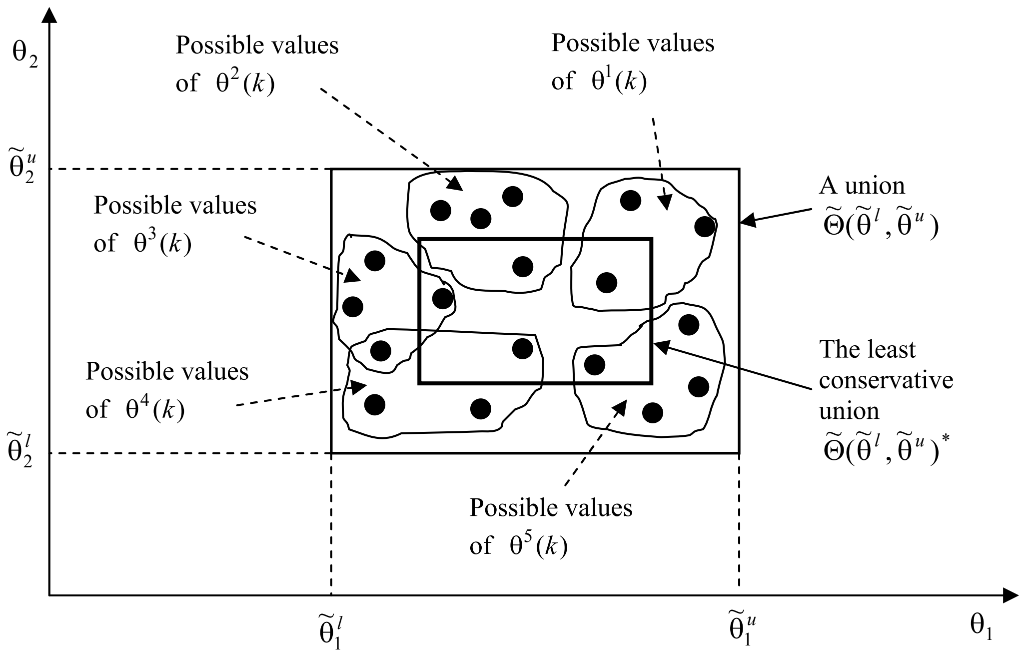

Fig. 3. A two dimension parameter example

3.2 Time invariant parameter formulation

The uncertainties in the parameter and structure error can also be handled by the point-parametric model with time-invariant parameters and time-varying structure error. Let $y^j(\cdot)$ denote the model response to input $u^j(\cdot)$:

$$y^j(t) = \psi^j(t)^T \theta^j + \varepsilon^j(t) \qquad (18)$$

Notice that $\varepsilon^j(t)$ must be time-varying as the parameters are constant.

Problem formulation 2. For any plant input $u(t)$, there exists $\{\theta, \psi(t), \varepsilon(t)\}$ so that equation (18) generates $y(t)$ that equals to the plant output $y_p(t)$, for any *t* over the considered time horizon.

Similar to time-varying formulation, the feasible parameter set Θ can be defined as (Chang, 2003):

$$[\theta^j, \varepsilon^j(t)] \in \Theta(\theta^l, \theta^u, \varepsilon^l, \varepsilon^u) \qquad (19)$$

$$\Theta(\theta^l, \theta^u, \varepsilon^l, \varepsilon^u) \overset{\Delta}{=} \{[\theta^j, \varepsilon^j(t)] \in R^{M+1} : y^j(t) = \psi^j(t)^T \theta^j + \varepsilon^j(t)\} \qquad (20)$$

where $\theta^l \le \theta^j \le \theta^u$ and $\varepsilon^l \le \varepsilon^j \le \varepsilon^u$ for $j = 1, \dots, E,\ t = 1, \dots, N$.

The bound estimation can be calculated as follows:

$$[\theta^l, \theta^u, \varepsilon^l, \varepsilon^u]^* = \underset{[\theta^l, \theta^u, \theta^j, \varepsilon^l, \varepsilon^u, \varepsilon^j]}{\arg\min} J(\theta^l, \theta^u, \varepsilon^l, \varepsilon^u) \qquad (21)$$

with $[\theta^j, \varepsilon^j(t)] \in \Theta(\theta^l, \theta^u, \varepsilon^l, \varepsilon^u)$, where

$$J(\theta^l, \theta^u, \varepsilon^l, \varepsilon^u) = (\theta^u - \theta^l)^T P(\theta^u - \theta^l) + (\varepsilon^u - \varepsilon^l) Q(\varepsilon^u - \varepsilon^l) \qquad (22)$$

The choice of P and Q is the compromise of distributing uncertainty between the parameters and structure error. The selection is "optimal" that is with such P and Q the obtained model uncertainty "radius" is minimum, where the uncertainty radius can be defined using the worst-case output prediction (for details see Chang and Brdys, 2003; Chang, 2003). A time-invariant model with constant bounded parameter was obtained as:

$$y(t) = \psi(t)^T \theta + \varepsilon(t) \qquad (23)$$

$$\Theta \overset{\Delta}{=} \{[\theta, \varepsilon(t)] \in R^{M+1} : \theta^{i^*} \le \theta \le \theta^{u^*}, \varepsilon^{l^*} \le \varepsilon(t) \le \varepsilon^{u^*}\} \qquad (24)$$

4. CONCLUSIONS

In order to obtain useful mathematical models for control, in this paper some aspects regarding the estimation of the parameters in the input-output chlorine residuals models of drinking water distribution networks were presented. Since the model parameters are time varying and heavily dependent on the hydraulics, which is a main source of uncertainties in chlorine concentration model, the model uncertainty parameters must to be estimated. Uncertainty of the model locates both in the parameters and modelling error part of the process model. Through inputs these two parts are linked and a point-parametric model is defined.

Since the operational control of a DWDN is ussualy based on the prediction methods, then using the input/output data and a priori knowledge of the process, in this paper a method for the estimation of the bounding values corresponding to model parameters was presented. The obtained model is the least conservative uncertainty model and could be used in a robust predictive control method.

ACKNOWLEDGEMENT

This work was created in the frame of the CNCSIS research grant No. 29C/2007, Code 621 and the research project CEEX-MENER No. 717/P1/2006, financed by the Education and Research Ministry, Romania.

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