A DIRECT- AND QUARATURE-AXIS APPROACH TO TORQUE ESTIMATION IN AN EDDY CURRENTS BRAKE

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Abstract: The paper presents a torque estimator for an eddy currents brake dynamic model based on the two axis theory and on the excitation current and angular speed direct measurements. The main idea is to proceed as in the induction machine sensorless speed estimation. The basic application of torque estimator would be the on-vehicle brakes optimization. In previous works, the author has validated the dynamic model by comparing experimental and predicted results in off-line operation. However, as shown in this paper, in on-line operation, in presence corrupting measurement noise, the estimator is inconsistent**.**

Keywords: systems identification, estimators, eddy current brakes.

1. INTRODUCTION

The eddy currents brakes are synchronous machines that transform by electrical means the mechanical energy received to the shaft into thermal energy transferred to a cooling environment.

Eddy currents brakes are used to determine, on test-bands, the characteristics of various active loads such as electrical motors, thermal engines, transmission components: gear boxes, power assemblies and so. They are also used as braking element for heavy vehicles such as rails and lorries to enhance braking into high-speed domain.

Today eddy currents brakes are less used to testing active loads because of their poor energetic efficiency; variable-speed induction motors supplied from four-quadrant three-phase inverters are used within such applications. Instead, eddy currents brakes have interesting features to braking pedestrian vehicles. It is proven that eddy current brakes are twice efficient than the classical mechanical brakes; additionally these brakes are pollution free, they don't transmit mechanical vibrations and they are simple and reliable.

When used as torque measuring element on test bands the eddy currents brakes' operation is optimized through a control system to maintain either constant braking torque, M_{el}, or constant angular speed, n, or constant ΔM/Δn ratio.

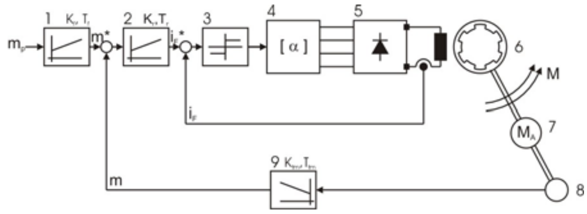

Fig. 1. The eddy currents brake's control system. (1) – input filter; (2) – torque compensator; (3) – excitation current compensator; (4), (5) phase-controlled voltage supply; (6) – eddy currents brake; (7) – active load ; (8) – torque transducer; (9) - transducer's signal conditioner.

The control system's structure is depicted in Fig. 1. In on-vehicle applications the eddy currents brake's operation is simply controlled through the excitation current modification using a controller. The basic problem in such applications is the on-line braking torque measurement. However torque measurement requires an adequate torque transducer and a conditioner that are not suitable for on-vehicle applications. An alternative to the direct torque measurement would be torque estimation using a dynamic brake model and the angular speed and excitation current direct measurements such as in the case of induction machines, see (Campbell *et al*., 2007a). The electromechanical variables estimation is closely related to the electrical parameters estimation. In the literature, the experimental identification of the electric machines parameters is presented in (Vas, 1998) for the induction machines and in (Söderstrom and Stoica, 1989) for the case of the DC machines. The sensorless speed estimation for the induction machine is widely presented in the literature; an algebraic approach in presented in (Campbell *et al*., 2007b).

The paper presents a torque estimator based on the two axis dynamic model of the brake. Subsequently, an analysis of the estimator consistency when taking into account the measurement noise effect onto the estimate is performed.

2. DIRECT AND QUADRATURE AXIS MODEL FOR THE EDDY CURRENTS BRAKES

The general direct and quadrature axis theory is the adequate approach to determine the dynamic models of all classical electric machines, (Henneberger, 2004). The two axis theory approach in the particular case of the eddy currents brakes with transversal excitation field is presented in (Dănilă, 2006), starting from the salient poles synchronous generator. The dynamic set of equations is as follows.

1. Voltage equations:

$$\begin{aligned} -u_d &= \frac{d\Psi_d}{dt} - \omega \cdot \Psi_q \\ -u_q &= \frac{d\Psi_q}{dt} + \omega \cdot \Psi_d \end{aligned} , \qquad (1.a\text{-}b)$$

$$u_{F'} = R'_F \cdot i'_F + \frac{d\Psi'_F}{dt} . \qquad (2)$$

2. Electromagnetic torque equation:

$$M_{el} = p \cdot \left(\Psi_q \cdot i_d - \Psi_d \cdot i_q\right) = -M_A + J \cdot \frac{d\Omega}{dt} . \qquad (3)$$

3. Flux linkage equations:

$$\begin{aligned} \Psi_d &= L_d \cdot i_d + L_{hd} \cdot i'_F \\ \Psi_q &= L_q \cdot i_q \\ \Psi'_F &= L'_F \cdot i'_F + L_{hd} \cdot i_d \end{aligned} . \qquad (4.a\text{-}c)$$

4. Voltage equations of the equivalent load:

$$\begin{aligned} u_d &= R_{OL} \cdot i_d + L_{OL} \cdot \frac{di_d}{dt} \\ u_q &= R_{OL} \cdot i_q + L_{OL} \cdot \frac{di_q}{dt} \end{aligned} \qquad (5.a\text{-}b)$$

$$\omega = \frac{\Omega}{p} = const. \qquad (6)$$

where:

Ψ_d, Ψ_q, Ψ'_F : flux linkages components with respect to Od and Oq axis and the excitation flux linkage respectively;

u_d, i_d, u_q, i_q, i'_F : induced voltages and induced currents components with respect to the Od and Oq axis referred to the induced;

Ω, ω, p : the rotor angular speed, the angular frequency of the electric variables within the induced and the pair poles of the brake, respectively;

M_{el}, M_A, J : the electromagnetic torque, the torque of the active load and the axial inertia moment of the load, respectively;

R_{OL}, L_{OL} : equivalent resistance and inductance of the brake's induced;

L_{hd}, L_d, L_q, L'_F : the magnetization inductance, the self-inductance with respect to the Od and Oq axis of the induced and the self-inductance of the excitation winding.

The input variable, or the command, is the excitation current, i'_F, the output variable, or the measuring variable is either the angular speed, Ω or the electromagnetic torque of the brake, M_{el}.

The set of equations (1 - 6) is non-linear and cannot be solved analytically. Due to the electrical variables within the induced that are not accessible to direct measurements, the set of equations cannot be solved numerically as it is. In (Dănilă *et al*., 2007), a field analysis with FEM method is performed for the eddy currents brakes with transversal excitation field.

The main conclusion of that study is that the demagnetizing effect of the induced currents on the Od axis is much less important than the transversal magnetizing effect of these currents. Therefore, in the set of equations (1 - 6) the following approximation is valid: $u_d = 0, i_d = 0$.

With the time constants:

$$\begin{aligned} T_d &= \frac{L_{OL} + L_d}{R_{OL}} \\ T_F &= \frac{L'_F}{R'_F} \end{aligned} \qquad (7.a\text{-}b)$$

= i.e. induced and excitation winding time constants, respectively, the set of equations (1 - 6) is modified as follows:

$$T_d \cdot \frac{di_q}{dt} + i_q = -\omega \cdot \frac{L_{hd}}{R_{OL}} \cdot i'_F$$
$$T_F \cdot \frac{di'_F}{dt} + i'_F = \frac{u'_F}{R_F} \qquad (8.a\text{-}c)$$
$$M_{el} = -p \cdot \Psi_d \cdot i_q = -p \cdot L_{hd} \cdot i'_F \cdot i_q$$

Induced and inductor circuitry are first-order systems. The first equation proves that the excitation current controls the induced current. The third equation gives the link between the excitation current, the induced current and the electromagnetic torque. The command is the excitation current and the controlled variable is the electromagnetic torque.

The set of equations still is non-linear but the electromagnetic torque may be computed with iterations with the hypothesis: the angular speed or the angular frequency of induced variables is constant. This hypothesis is always valid because the electromechanical time constant is much greater than the electric time constant of the induced.

In this approach, in the set of equations (8) the derivatives are approximate with the Euler-Cauchy formula as follows:

$$\frac{dy(t_k)}{dt} = \frac{y(t_k + \Delta t) - y(t_k)}{\Delta t} \qquad (9)$$

where:
$y(t_k + \Delta t), y(t_k)$: are the values of function y at instances $t_k + \Delta t$, t_k respectively, and Δt is the sampling period.

The following algorithm is obtained:

$$\hat{i}_q(k+1) = \frac{\Delta t}{T_d} \cdot \left[-\hat{i}_q(k) - \omega(k) \cdot \frac{L_{hd}}{R_{OL}} \cdot \hat{i}'_F(k) \right] + \hat{i}_q(k)$$
$$\hat{i}'_F(k+1) = \frac{\Delta t}{T_F} \cdot \left[\frac{u'_F(k)}{R'_F} - \hat{i}'_F(k) \right] + \hat{i}'_F(k)$$
$$\hat{M}_{el}(k+1) = -p \cdot L_{hd} \cdot \hat{i}'_F(k+1) \cdot \hat{i}_q(k+1)$$

(10.a-c)

For a consistent estimation of the electromagnetic torque with algorithm (10), both electric parameters of the brake must be known and the effect of noise on the measurements must be evaluate.

3. TORQUE OBSERVER

In (Campbell *et al.*, 2007b) is presented an algebraic method for the estimation of the rotor time constant in the induction motor. The proposed method is based on the deduction of a polynomial equation in T_R whose coefficients depend on the stator currents, stator voltages and their derivatives. For the eddy currents brake time constant T_d determination, the induced voltage and current components u_q and i_q should be directly measured. Because this action is practically impossible another method is used to determine the electrical parameters of the brake by analyzing field distribution within the air gap and the induced with FEM method. Validation of the results is performed by comparing the computational data with a steady-state set of measurements.

The analysis of measurements noise on the torque estimate may be founded on the ideas presented in (Söderstrom and Stoica, 1989). The set of differential equations (8) is split into two subsystems as shown below.

3.1. The dynamic model of the induced

The continuous-time process described through the constant-coefficients differential equation (8.a) is converted into a discrete-time process with the Euler-Cauchy method - T_e is the sampling period – and the following difference equation is obtained:

$$i_q[t] - \frac{T_d}{T_d + T_e} \cdot i_q[t-1] = -\omega \cdot \frac{L_{hd}}{R_{OL}} \cdot \frac{T_e}{T_d + T_e} \cdot i'_F[t] \qquad (11)$$

With the one-step-delay operator the equation (11) is converted into the operational equation:

$$\left(1 - a \cdot q^{-1}\right) \cdot i_q[t] = -b \cdot i'_F[t] \qquad (12)$$

where: $a = \frac{T_d}{T_d + T_e}$ and $b = \omega \cdot \frac{L_{hd}}{R_{OL}} \cdot \frac{T_e}{T_d + T_e}$

3.2. The dynamic model of the electromagnetic torque

To obtain the dynamic model of the electromagnetic torque, the excitation current from equation (8.a) is substituted into equation (8.c):

$$M_{el} = -p \cdot L_{hd} \cdot i'_F \cdot i_q = = \frac{p \cdot R_{OL}}{\omega} \cdot \left(T_d \cdot i_q \cdot \frac{di_q}{dt} + i_q^2 \right) = = \frac{p \cdot R_{OL}}{\omega} \cdot \left(T_d \cdot \frac{1}{2} \cdot \frac{di_q^2}{dt} + i_q^2 \right) \qquad (13)$$

The continuous-time linear process give by equation (13) is converted into a discrete-time linear process and results the equation:

$$y[t] = \frac{p \cdot R_{OL}}{\omega} \cdot \left(\frac{T_d}{2 \cdot T_e} + 1 \right) \cdot u[t] - \frac{p \cdot R_{OL} \cdot T_d}{\omega \cdot T_e} \cdot u[t-1] \qquad (14)$$

where $M_{el}[t] = y[t]$ and $u[t] = i_q^2[t]$.

In relation (14) the one-step-delay operator is introduced and results:

$$y[t] = \left(c - d \cdot q^{-1}\right) \cdot u[t] \qquad (15)$$

where: $c = \dfrac{p \cdot R_{OL}}{\omega} \cdot \left(\dfrac{T_d}{2 \cdot T_e} + 1\right)$ and $d = \dfrac{p \cdot R_{OL} \cdot T_d}{\omega \cdot T_e}$.

3.3. Evaluation of measurements noise impact on the estimate

In purpose to evaluate the effect of noise due to given measurement conditions we assume that to the inputs of systems (12) and (15) a normal distributed white noise signal $e[t]$ with zero mean and λ^2 variance is applied instead of true signal.

From equation (12) results:

$$i_q[t] = \frac{-b}{1 - a \cdot q^{-1}} \cdot e[t] \qquad (16)$$

that describe an AR(1) process. Equation (15) describes a MA(1) process instead.

Noise effect evaluation is performed through output signal variance of both systems computation. In (Söderstrom and Stoica, 1989) the following methods are described to compute functions covariance: (1) division method; (2) Yule-Walker method (3) integration around the unit circle in complex plane and (4) state-space representation method.

Because processes (15), (16) are first order processes, division method may be used to compute the variance of signals. From (16) results:

$$y[t] = \frac{C(q)}{A(q)} \cdot e[t] = = -b \cdot \sum_{j=0}^{\infty} a^j \cdot e[t-j] \qquad (17)$$

Noting that $e[t-i]$ and $e[t-j]$ are uncorrelated if $i \neq j$, results:

$$r_y(0) = Ey[t] \cdot y[t] = = -b \cdot \sum_{j=0}^{\infty} a^j \cdot Ee^2[t-j] = \lambda^2 \cdot \frac{b}{1-a} \qquad (18)$$

where E(.) is the expectation operator. For model (15) the expectation operator is directly applied and follows:

$$r_y(0) = E(c \cdot e[t] - d \cdot e[t-1])^2 = = \lambda^2 \cdot \left(c^2 + d^2\right) \qquad (19)$$

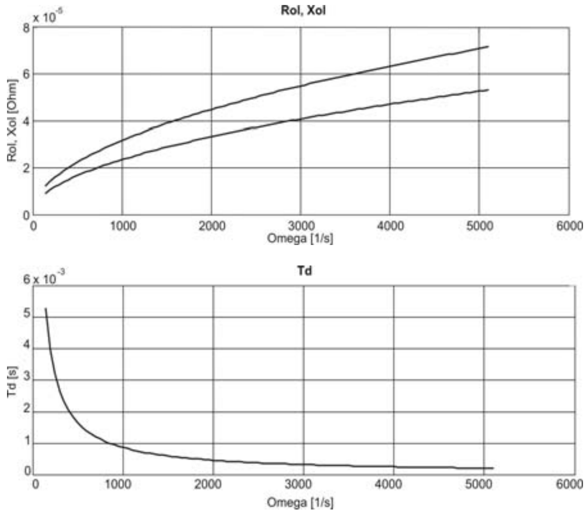

Fig. 2. The graph above: Dependencies of $R_{ol}(\omega)$ - the above curve - and $X_{ol}(\omega)$ – the below curve. The graph below: $T_d(\omega)$ dependency.

Expressions (18) and (19) allow variance computation of two systems outputs depending on the white noise variance and brake electrical parameters.

4. SIMULATION RESULTS

Theoretical results presented above have been verified with available date in (Dănilă, 2006) for a given eddy currents brake operating in stationary regime. Brake's parameters are presented in Table 1.

Table1: Eddy currents brake features.

Denomination	Symbol	U.M.	Value
Rotor radius	R	m	0.1
Ideal induced width	l_i	m	0.1
Number of pair-poles	p	-	20
Poles step	τ	m	0.2
Air gap length	δ	m	0.001
Magnetic coefficient with respect to the Od axis	k_{ad}	-	0.036

Values of equivalent resistance and inductive reactance of the induced have been calculated, with the relations:

$$R_{OL} = \frac{m \cdot \omega \cdot (w \cdot k_w)^2 \cdot \alpha_i \cdot k_f \cdot k_{rOL}}{p} \times \times \underbrace{\frac{\sqrt{2n}}{\sqrt{3n+1}} \cdot \sqrt[4]{\frac{2n}{n+1}}}_{f_1(n)} \cdot \frac{\pi \cdot l_i}{\tau} \cdot \sqrt{\frac{\rho_{OL} \cdot \mu_e}{\omega}} \qquad (20)$$

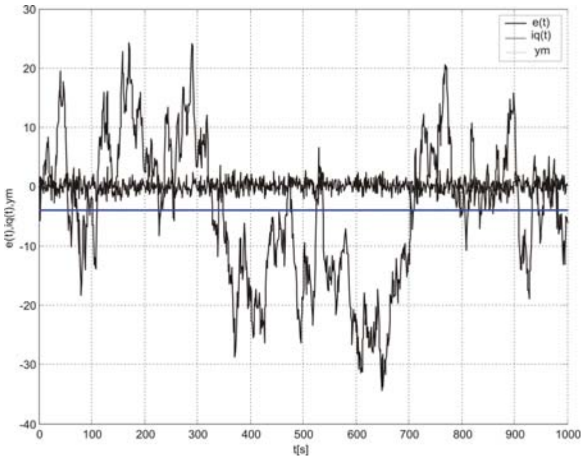

Fig. 3. Noise output of AR(1) process.

$$X_{OL} = \frac{m \cdot \omega \cdot (w \cdot k_w)^2 \cdot \alpha_i \cdot k_f \cdot k_{rOL}}{p} \times \\ \times \underbrace{\frac{\sqrt{n+1}}{\sqrt{3n+1}} \cdot \sqrt[4]{\frac{2n}{n+1}}}_{f_2(n)} \cdot \frac{\pi \cdot l_i}{\tau} \cdot \sqrt{\frac{\rho_{OL} \cdot \mu_e}{\omega}} \quad (21)$$

where:

m, w, k_w are the phases number, number of turns per one phase and the winding factor for the equivalent phase of the induced;

α_i, k_f, k_{rOL} are the pole factor, shape coefficient of the progressive wave and the increasing factor of resistance due to the transversal boundary effect;

ρ_{OL}, μ_e, n are the induced resistivity, magnetic permeability to the surface of induced and magnetic saturation coefficient of the induced material.

Angular frequency dependencies of parameters R_{ol}, X_{ol} and T_d are depicted in Fig.2. Parameters of two systems over the whole angular frequency range have been computed in the MatLab environment. Three representative values are given in Table 2. The sampling period is $T_e = 10^{-5}$s.

Table 2. Systems' parameters dependencies with respect to angular frequency.

ω	a	b	c	d
600	0.9928	0.0006	575.10^{-7}	701.10^{-8}
2600	0.9727	0.0051	741.10^{-8}	567.10^{-8}
4600	0.9556	0.0110	347.10^{-8}	313.10^{-8}

Notice that values of parameter a are very close to the unit instead the values of parameter b are almost zero. Variance of process AR(1) strongly depends on both angular frequency and on the number of samples within a realization as shown in Table 3. Additionally, notice that the estimate is biased.

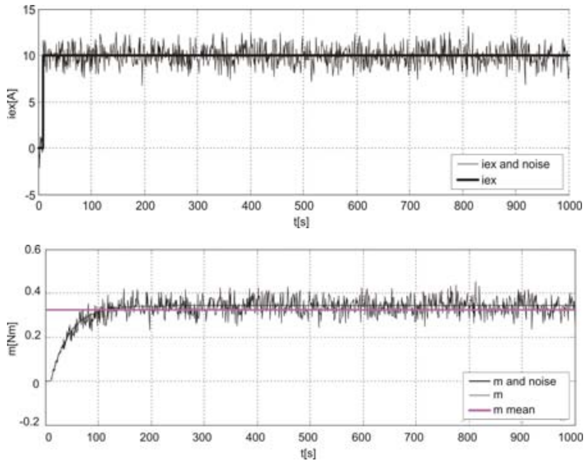

Fig. 4. The above plot: noisy input to the torque observer. The plot below: torque estimate for the given input realization.

Table 3. Mean and variance dependencies with respect to the angular frequency for the AR(1) process.

ω	μ_{iq}	σ_{iq}	$\sigma_{iq,calc}$
600	-2,0697	4,53	44,44
2600	2,4597	106,19	93,4066
4600	-0,2828	386,71	123,874

The graphical representation of the AR(1) for a realization with parameters corresponding to ω = 2600 [1/s] is depicted in Fig. 3. Computations prove that the measurement noise strongly influence current estimation especially at high values of the angular frequency and the variance values differ depending on the sample number within each realization and in comparison to the theoretical value.

In Fig. 4 and Fig. 5 are depicted the output signal, i.e. the estimated torque when the deterministic signal and a 10% magnitude normal distributed, zero mean and unit variance white noise are superposed.

5. CONCLUSIONS

The analysis presented above proves that torque estimation based on the algorithm 10 a – c and on angular speed and excitation current direct measurements does not give a consistent estimate. This is prior due to the large ratio between the number of turns of the excitation windings and the equivalent phase number of turns of the induced that augments noise effect on the estimate. To avoid the multiplication effect of windings ratio on variables measured to the excitation winding of the brake torque estimation has to be performed starting from variables direct related to the induced. This is the case of speed estimation in the induction machine. A possible solution – to be performed in further researches - is to estimate the torque based on the flux linkage direct measurements.

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