ADAPTIVE PREDICTIVE CONTROL OF INTERCONNECTED LIQUID TANKS

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Abstract: This paper is focused on usability of model predictive control (MPC) approach in the area of control of nonlinear systems. A self-tuning predictive controller is introduced and subsequently used for the control of real-time nonlinear system. The controller integrates on-line identification of an ARX model of a controlled system and a predictive control synthesis on base of the identified model. Various control requirements can be fulfilled by tuning controller's parameters. The real-time testing has been carried out by the control of nonlinear laboratory model of interconnected tanks (DTS200 by Amira company).

Keywords: Predictive control, Adaptive control, Autoregressive models, Recursive least squares method, Nonlinear systems, Real-time systems

1. INTRODUCTION

Most of current control algorithms are based on a model of a controlled system. There are two basic approaches of obtaining system's model: mathematical-physical analysis of the system and black box approach. The mathematical-physical analysis of the system and subsequent derivation of the relations between system inputs and outputs provides general model which can be valid for a whole range of system's inputs and states. On the other hand, there is usually a lot of unknown constants and relations when performing mathematic-physical analysis. Therefore, modelling by mathematic-physical analysis is suitable for simple controlled systems with small number of parameters or for obtaining basic information about the system (range of gain, rank of suitable sample time, etc.).

The black box approach to the modelling is based on analysis of input and output signals of the system. The main advantage of this approach lies in the possibility of usage the same identification algorithm for different controlled systems. Also, the knowledge of physical principle of controlled system and solution of possibly complicated set of mathematical equation is not required. On the other hand, model obtained by black box approach is generally valid only for signals it was calculated from. For example, if only low frequency changes of input signals were used to obtain the model, this model need not be usable for high frequency changes of input signals.

This paper deals with black box approach to the identification problem where linear models of the controlled system are used. The identification is based on selection of appropriate linear model structure and subsequent computation of its parameters. The computation of parameters is usually based on Least Squares Method (LSM) (Bobál, 2005). This method can be used in both off-line and on-line identification systems, but this paper focuses to on-line identification only.

When the model of the controlled system is known the problem of selecting an appropriate control synthesis arises. Many successful control techniques have been developed in past decades. One of them is model predictive control (MPC) (Camacho and Bordons, 2004). Contrary to most other approaches,

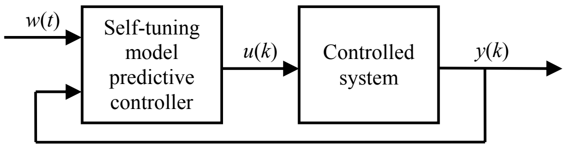

Fig. 1. Control circuit with Self-tuning Model Predictive Controller

MPC uses not only current and previous values of control circuit signals but also future values of reference signal. Future course of reference signal is known in many applications and thus can be used in controller synthesis.

Self-tuning predictive controller encapsulates its two main parts: on-line identification block and model predictive control block. The scheme of a simple control circuit with self-tuning predictive controller is shown in Fig. 1. Note that the reference signal is marked as $w(t)$, This means that the course of reference signal is sent to the controller, not only the current value $w(k)$.

2. THEORETICAL BACKGROUND

Self-tuning control is based on-line identification of controlled process and controller synthesis which uses results from the identification. Thus, each self-tuning predictive controller consists of two relatively stand-alone parts:

- On-line identification
- Model predictive controller

The on-line identification part is responsible for computing estimates of parameters of linear model of controlled system. Model predictive controller computes control signal on base of current model and control, controlled and reference signals. Scheme of the self-tuning predictive controller is shown in Fig. 2. Previous value of reference signal – $u(k\text{-}1)$ acts as an controller input because controller output can be subject of some technological limitations e.g. saturation.

2.1 On-line Identification Methods

Various discrete parametric models are used to describe dynamic behaviour of controlled systems. Overview of these models is given in (Ljung, 2001).

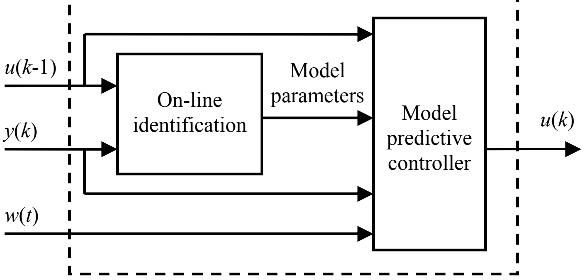

Fig. 2. Self-tuning Model Predictive Controller

Widely used simplification of general input-output model is ARX model. The ARX model for single input single output (SISO) systems has the following form:

$$A\left(z^{-1}\right)y\left(t\right)=B\left(z^{-1}\right)z^{-d}u\left(t\right)+n\left(t\right) \tag{1}$$

Then the transfer function of model of identified system is assumed to be in the following form:

$$G\left(z\right)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}z^{-d}=\frac{b_1z^{-1}+b_2z^{-2}+\ldots+b_mz^{-m}}{1+a_1z^{-1}+a_2z^{-2}+\ldots+a_nz^{-n}}z^{-d} \tag{2}$$

Then it is possible to write an equation for computing the output of the system in k-th step:

$$y\left(k\right)=\mathbf{\Theta}^T\left(k\right)\cdot\mathbf{\Phi}\left(k-1\right)+n\left(k\right) \tag{3}$$

where $n(k)$ represents the influence of an immeasurable disturbances, $\mathbf{\Theta}\left(k\right)$ the vector of the parameters of the controlled system model and $\mathbf{\Phi}\left(k-1\right)$ the data vector.

The identification problem is formulated as a process of finding the $\mathbf{\Theta}\left(k\right)$ vector with respect to some criterion. Exact values of parameters are unknown during the identification process and just the vector of parameter estimations is used.

$$\hat{\mathbf{\Theta}}\left(k\right)=\left[\hat{a}_1,\hat{a}_2,\ldots,\hat{a}_n,\hat{b}_1,\hat{b}_{2,}\ldots,\hat{b}_m\right]^T \tag{4}$$

The aim of the identification process is to compute the estimations $\hat{\mathbf{\Theta}}\left(k\right)$ as close as possible to the actual parameters $\mathbf{\Theta}\left(k\right)$.

Recursive least squares method The recursive least squares method (RLSM) is based on minimization of sum of squares of differences between actual system outputs and outputs estimated on base of system model. If the k-th identification steps is performed and data corresponding to r previous system inputs and outputs are available, the criterion to be minimized can be formulated as follows:

$$J=\frac{1}{2}\left[\mathbf{y}\left(k\right)-\hat{\mathbf{y}}\left(k\right)\right]^T\left[\mathbf{y}\left(k\right)-\hat{\mathbf{y}}\left(k\right)\right] \tag{5}$$

where $\boldsymbol{y}(k)$ is the vector of system outputs, and $\hat{\mathbf{y}}\left(k\right)$ is the vector of system outputs estimations.

The resulting equation for parameter estimations update is:

$$\hat{\mathbf{\Theta}}\left(k\right)=\hat{\mathbf{\Theta}}\left(k-1\right)+\frac{\mathbf{C}\left(k-1\right)\mathbf{\Phi}\left(k-1\right)}{1+\mathbf{\Phi}^T\left(k-1\right)\mathbf{C}\left(k-1\right)\mathbf{\Phi}\left(k-1\right)}\cdot \\ \cdot\left[y\left(k\right)-\mathbf{\Phi}^T\left(k-1\right)\hat{\mathbf{\Theta}}\left(k-1\right)\right] \tag{6}$$

The covariance matrix $\boldsymbol{C}$ is updated in each sample time according to the following equation:

$$\mathbf{C}(k)=\mathbf{C}(k-1)-\frac{\mathbf{C}(k-1)\boldsymbol{\Phi}(k-1)\boldsymbol{\Phi}^T(k-1)\mathbf{C}(k-1)}{1+\boldsymbol{\Phi}^T(k-1)\mathbf{C}(k-1)\boldsymbol{\Phi}(k-1)} \quad (7)$$

The covariance matrix $\boldsymbol{C}$ is usually initialized as a diagonal matrix with elements 10^3 on the main diagonal (Hang, *et al.*,1993),. The main diagonal of covariance matrix $\boldsymbol{C}$ contains dispersions of identified parameters and thus if the initial parameter estimations are known to be close to the actual values, the initial values of elements on the main diagonal are to be smaller.

Recursive least squares method with exponential forgetting When using the least squares method, the influence of all pairs of identified system inputs and outputs to the parameters estimations is the same. This property can be inconvenient for example when identifying the system with time-varying parameters or non-linear system. In this case, it is better to use least squares method with exponential forgetting where the influence of newer data to the parameters estimations is greater then the influence of older data. The criterion to be minimized is in the following form:

$$J=\frac{1}{2}\left[\mathbf{y}(k)-\hat{\mathbf{y}}(k)\right]^T \mathbf{W}\left[\mathbf{y}(k)-\hat{\mathbf{y}}(k)\right] \quad (8)$$

where $\boldsymbol{W}$ is a diagonal weight matrix:

$$\mathbf{W}=\begin{bmatrix}1 & 0 & 0 & \cdots & 0\\ 0 & \varphi & 0 & \dots & 0\\ 0 & 0 & \varphi^2 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & 0\\ 0 & 0 & 0 & 0 & \varphi^{r-1}\end{bmatrix} \quad (9)$$

and the φ is a forgetting coefficient which is assumed to be in range $0<\varphi\leq 1$. The RLSM with exponential forgetting can be transferred to pure RLSM by selecting φ=1. The lower value of φ denotes more rapid forgetting of older data and thus smaller influence of older data to resulting parameter estimations. The choice of coefficient φ is individual and depends on the relation between identification sample time and speed of identified system but usually is taken from range $\langle 0.90, 0.99\rangle$.

Recursive least squares method with adaptive directional forgetting The exponential forgetting method can be further improved by adaptive directional forgetting (Kulhavý, 1987) which changes forgetting coefficient with respect to changes of input and output signals of identified system. Parameter estimations are updated using recursive equation (6) and covariance matrix $\boldsymbol{C}$ is updated according to equations (10):

$$\begin{aligned}\mathbf{C}(k)&=\mathbf{C}(k-1)-\frac{\mathbf{C}(k-1)\boldsymbol{\Phi}(k-1)\boldsymbol{\Phi}^T(k-1)\mathbf{C}(k-1)}{\varepsilon^{-1}+\xi}\\ \varepsilon&=\varphi(k-1)-\frac{1-\varphi(k-1)}{\xi}\\ \xi&=\boldsymbol{\Phi}^T(k-1)\mathbf{C}(k-1)\boldsymbol{\Phi}(k-1)\end{aligned} \quad (10)$$

The forgetting coefficient is adapted with respect to courses of input and output signals according to following equation:

$$\varphi(k)=\frac{1}{1+(1+\rho)\left\{\ln(1+\xi)+\left[\eta\frac{\upsilon(k)+1}{1+\xi+\eta}-1\right]\frac{\xi}{1+\xi}\right\}} \quad (11)$$

and the scalars $\upsilon(k)$, $\lambda(k)$ and η are defined in the following way:

$$\begin{aligned}\upsilon(k)&=\varphi(k-1)\left[\upsilon(k-1)+1\right];\\ \eta&=\frac{\left[y(k)-\hat{\boldsymbol{\Theta}}^T(k-1)\boldsymbol{\Phi}(k-1)\right]^2}{\lambda(k)}\\ \lambda(k)&=\varphi(k-1)\left\{\lambda(k-1)+\frac{\left[y(k)-\hat{\boldsymbol{\Theta}}^T(k-1)\boldsymbol{\Phi}(k-1)\right]^2}{1+\xi}\right\}\end{aligned} \quad (12)$$

Recommended initial values (Bobál, 2005) of identification variables are: $\varphi(0)=1$, $\lambda(0)=0.001$, $\upsilon(0)=10^{-6}$. Initial value of matrix $\boldsymbol{C}$ should be chosen as a diagonal matrix with elements 10^3 on the main diagonal. Parameter ρ states the dynamics of identification process. Reasonable values of ρ are from -1 which corresponds to pure least squares method.

2.2 Model Predictive Controller

Generally, the computation of control signal of model predictive controller is based on minimization of particular criterion (Kwon and Han, 2005). Usually a quadratic criterion is used (Sunan *et al.*, 2002).. For single input single output (SISO) systems the criterion can be written in general form:

$$J(k)=\mathbf{e}^T\mathbf{Q}(k)\mathbf{e}+\boldsymbol{\Delta}\mathbf{u}^T\mathbf{R}(k)\boldsymbol{\Delta}\mathbf{u} \quad (13)$$

where **e** is a vector of predicted control errors, **Δu** is a vector of future differences of control signal samples and square matrixes **Q** and **R** allows to set weighting of individual vector elements. Future outputs of the controlled system, and consequently control errors, are computed on base of its model. Control sequence is obtained by minimizing criterion (13). The receding horizon is usually used: only finite number of future values is used in criterion and only the first element of the obtained control sequence is applied to the controlled system.

Proposed self-tuning predictive controller simplifies criterion (13) by assigning the same weight to all future outputs – matrix **Q** is diagonal and all the elements on the main diagonal are the same. Moreover, similar approach is used for differences of control signal samples – matrix **R** is diagonal and all the elements on the main diagonal are the same. If the model without transport delay is used, the criterion (13) can be rewritten into the following form:

$$J\left(k\right)=\sum_{j=1}^{N}e\left(k+j\right)^{2}+\lambda\cdot\sum_{j=1}^{N_c}\Delta u\left(k+j\right)^{2} \qquad (14)$$

where N is prediction horizon, N_c stands for control horizon and λ states ratio between weights of control errors and differences of control samples. It must hold $N \geq N_c$ and in case of inequality, u is taken as a steady for the rest of predictive horizon.

Process of minimizing of the criterion (13) or (14) can be rewritten to a quadratic programming problem:

$$J\left(k\right)=\mathbf{u}^{\mathrm{T}}\mathbf{H}\left(k\right)\mathbf{u}+\mathbf{p}\left(k\right)\mathbf{u} \qquad (15)$$

where **u** is a vector of future control signal samples to be computed. **H** and **p** are matrix and vector derived from λ and model parameters. Quadratic programming problem is usually solved numerically. This allows further constraints to be applied to vector **u**.

3. INTERCONNECTED TANKS

Proposed dual approach was verified used to control liquid level in the interconnected cylindrical tanks.

3.1 Mathematical model

A scheme of two interconnected tanks is presented in the Fig. 3. The system consists of two interconnected cylindrical tanks T_1 and T_2 and a pump P which is responsible for inflow to the tank T_1. The liquid level heights in the tanks T_1 and T_2 are h_1 and h_2 respectively. The inflow produced by the pump is q_{in}, flow between tanks is $q1$ and the outflow is q_2. The pipe between tanks and the outflow pipe are described by constants $k1$ and k_2 respectively. The

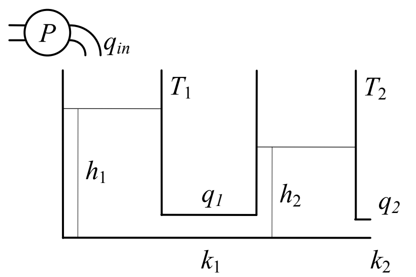

Fig 3. Scheme of two interconnected tanks

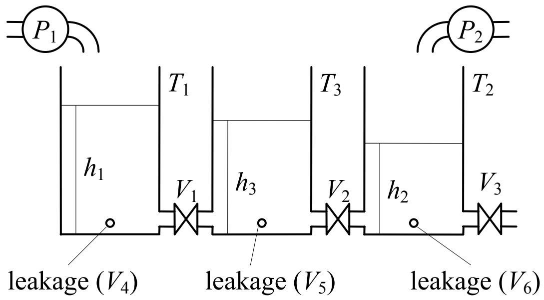

Fig 4. Scheme of Amira DTS200

model can be described by the following system of nonlinear differential equations:

$$q_{in}=q_1+\frac{dV_1}{dt} \qquad q_1=q_2+\frac{dV_2}{dt}$$
$$q_1=k_1\sqrt{\left|h_1-h_2\right|}\cdot\mathrm{sign}\left(h_1-h_2\right) \qquad q_2=k_2\sqrt{h_2} \qquad (16)$$

where V_1 and V_2 are capacities of liquid in the tanks T_1 and T_2.

The system can be considered as a single input single output system (SISO) where the input is inflow q_{in} and output is liquid level h_2. This configuration was used in the experiments described in the following chapters.

3.2 Real-time laboratory plant DTS200

Control experiments were performed using real-time laboratory plant Amira DTS200 – Three Tank System. The scheme of this model is shown in Fig. 4.

The plant consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, measurement of liquid levels and other elements. Valves V_2 and V_4 were fully closed during the experiments, valve V_1 was fully opened and valve V_5 was partially opened. The valve positions did not change during the experiments. This configuration leads to the same model as described in the previous chapter. The controlled signal (y) was the height of the liquid level in the middle tank ($y = h_3$). This level was controlled by the control voltage of the pump P_1 (u).

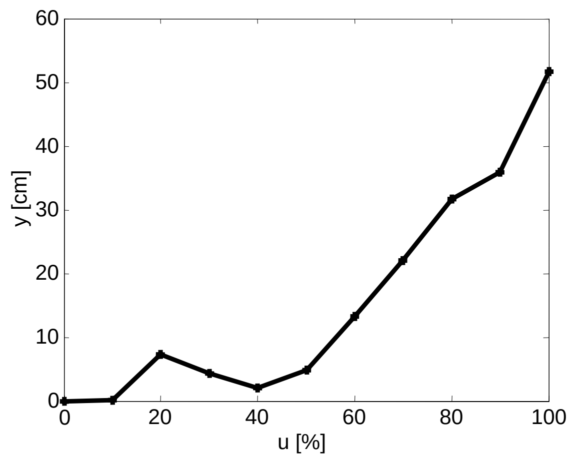

Fig 5. Static characteristics of the controlled system

Due to the characteristics of the valves, pipes and pump, the system behaviour contains more nonlinearities than the mathematical model described by equations (16). This can be seen from the static characteristics shown in Fig. 5.

4. CONTROL OF DTS200 PLANT

Several different control algorithms were used to control the DTS200 plant, which was configured as described in previous chapter. The results of typical control courses are presented later in this chapter. All adaptive controllers used a second order discrete ARX model of the controlled process. No a priori information was used to set initial parameter estimations. Discrete PI controllers were also used to control the plant and their performance was used to compare advantages and disadvantages of adaptive MPC.

The results of control courses obtained using sample time of T_0 = 10 s are presented in this chapter. First adaptive controller used least square method of on-line identification of the model of the controlled system. Other parameters presented in equation (14) were set as follows: N=30, N_c=30 and λ=1. This controller is further referenced as *lsm*. Control performance of *lsm* controller is presented in Fig. 6.

The course of reference signal was constant at 10 cm from the beginning to 900 s then a step change to the constant value of 20 cm was applied. Reference signal continued with decreasing ramp from 20 cm to 10 cm in the time range from 1500 s to 2100 s and remained constant at 10 cm for the rest of the control course. The same course of the reference signal was used for all control experiments presented in this paper.

An unexpected sudden decrease of plant output can be observed after approximately 550 s from the beginning of the control course. This behaviour is caused by air bubbles which are present in the system at the beginning. A bubble had been washed out from a pipe system at the mentioned time and thus characteristics of the flow changed.

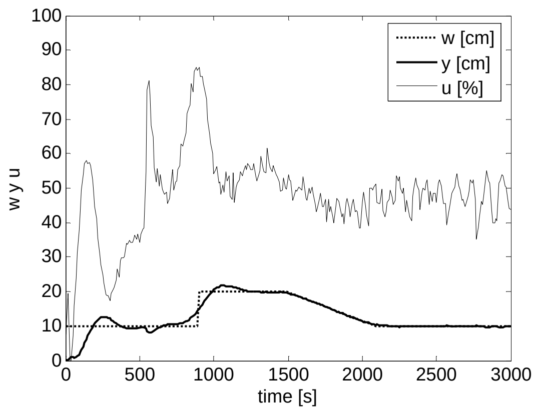

Fig. 6. Control performance of *lsm* controller

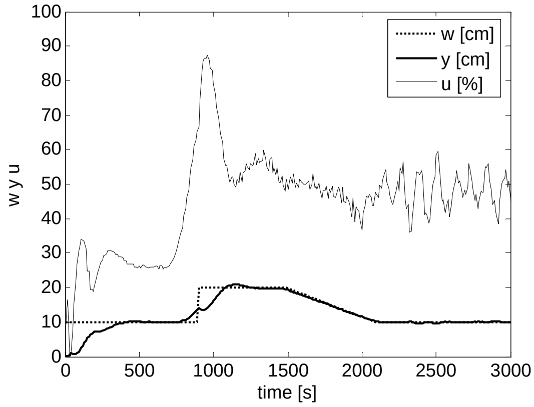

Fig. 7. Control performance of *ef* controller

The second controller was based on on-line identification with exponential forgetting where forgetting coefficient was set to φ=0.99. Parameters of the model predictive part of the controller were set as follows: N=30, N_c=30 and λ=1. This controller is further referenced as *ef* and its performance can be investigated from the Fig. 7.

The course of the controlled signal is smoother compared to *lsm* control but the effect of washing out of an air bubbles can be observed too. The control signal of approximately 30% led to steady output of 10 cm in the first part of the control process whilst control signal of almost 50% was needed to obtain the same output in the last part.

The third controller was using on-line identification with adaptive directional forgetting where parameter ρ=-0.5. Parameters of the model predictive part of the controller were the same as used for previous controllers: N=30, N_c=30 and λ=1. This controller is further referenced as *adf* and its performance can be investigated from the Fig. 8.

A discrete PI controller was used to be compared with adaptive model predictive controllers. Inputs and outputs, which were obtained in the first 900 s of *ef* control, were used to compute a second order ARX model. This model was then used to tune PI controller to minimize criterion (17):

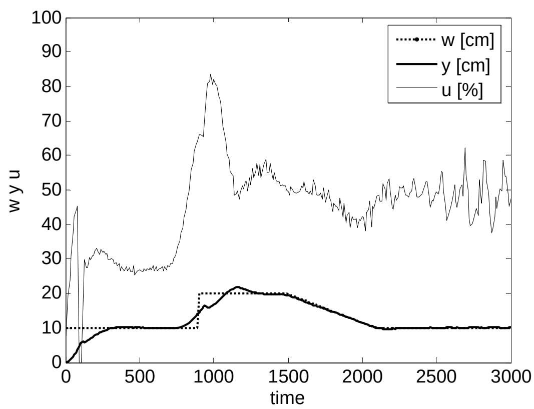

Fig. 8. Control performance *adf* controller

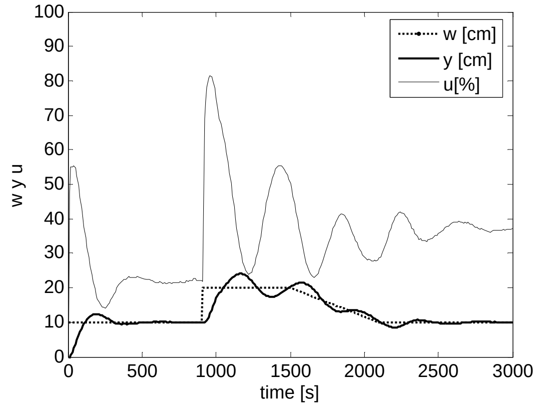

Fig. 9. Control performance *pi* controller

$$J_{PI} = \sum_{j=a}^{b} e(j)^2 + \lambda \cdot \sum_{j=a}^{b-1} \Delta u(j)^2 \tag{17}$$

The reference signal was the same as used in real-time control, sample time T_0=10s was used, parameters *a* and *b* were set to cover whole control courses and weight parameter λ=1. The control law of the PI controller is defined by the following equation:

$$u(k) = P \cdot e(k) + I \cdot T_0 \cdot e(k) + u(k-1) \tag{18}$$

Optimal parameters of *P*=4.7 and *I*=0.05 were found. This controller is referenced as *pi* and corresponding control courses of the control of DTS200 plant are shown in Fig 9. It can be observed that the *pi1* controller was not able to cope with step change of reference signal from 10 cm to 20 cm.

5. COMPARISON OF CONTROL PERFORMANCE USING SUMMING CRITERIA

The performances of individual controllers were compared not only by investigating graphs of performance of controller and process output signal, but also by mathematical criteria. Four criteria were used to compare control courses obtained by individual controllers:

$$\begin{aligned} S_{e2} &= \frac{1}{b-a+1}\sum_{k=a}^{b}\left[w(k)-y(k)\right]^2 \\ S_{ea} &= \frac{1}{b-a+1}\sum_{k=a}^{b}\left|w(k)-y(k)\right| \\ S_{u2} &= \frac{1}{b-a}\sum_{k=a}^{b-1}\left[\Delta u(k)\right]^2 \\ S_{ua} &= \frac{1}{b-a}\sum_{k=a}^{b-1}\left|\Delta u(k)\right| \end{aligned} \tag{19}$$

Values of individual criteria for *lsm*, *ef*, *adf* and *pi* controllers are summarised in table 1.

Table 1. Values of criteria for the control courses of *lsm*, *ef*, *adf* and *pi* controllers

controller	S_{e2}	S_{ea}	S_{u2}	S_{ua}
lsm	4.44	0.94	16.18	2.89
ef	5.63	1.04	9.28	2.19
adf	3.87	0.87	16.82	2.33
pi	5.36	1.29	16.68	1.22

Criteria S_{e2} and S_{ea} are based on control error. And represent accuracy of control process. Criteria S_{u2} and S_{ua} are based on changes control signal and represent demands for actuators. Values of *a* and *b* in (19) were selected to cover whole control process.

6. CONCLUSIONS

Model predictive adaptive controller was proposed and verified by control of nonlinear time varying system. The controller is based on self-tuning approach and several methods of on-line identification were discussed. The DTS200 plant was used to verify and compare different setting of the controller. Parameters of this plant are not constant in time especially due to air bubbles, which are present in the tubes and valves. Thus comparison of individual control courses has to be performed with respect to these changes. Nevertheless, the accuracy of all adaptive MPC control process was significantly better compared to “optimal” PI controller.

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