VARIABLE STRUCTURE CONTROL – FROM PRINCIPLES TO APPLICATIONS

Andrzej Bartoszewicz

Institute of Automatic Control, Technical University of Łódź,
18/22 Stefanowskiego St., 90-924 Łódź, Poland

Abstract: The theory of variable structure systems (VSS) with sliding modes is currently one of the most significant research topics within the control engineering domain. Moreover, recently a number of important applications of the theory in the field of power electronics, motion control, robotics, bioprocess etc. have also been reported. Therefore, this paper presents a brief introduction to the theory of sliding mode control, then enumerates the most important novel trends in fundamental research in this field and finally gives some examples of successful engineering applications.

Keywords: variable structure systems, sliding mode control, discrete time sliding mode, chattering

1. INTRODUCTION TO VSS

In recent years much of the research in the area of control theory focused on the design of discontinuous feedback which switches the structure of the system according to the evolution of its state vector. This control idea may be illustrated by the following example.

Let us consider the second order system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_2 + u_i \qquad i = 1,2 \end{aligned} \tag{1}$$

where $x_1(t)$ and $x_2(t)$ denote the system state variables, with the following two feedback control laws

$$u_1 = f_1(x_1, x_2) = -x_2 - x_1 \tag{2}$$

$$u_2 = f_2(x_1, x_2) = -x_2 - 4x_1 \tag{3}$$

The performance of system (1) controlled according to (2) is shown in figure 1, and figure 2 presents the behaviour of the same system with feedback control (3). It can be clearly seen from those two figures that each of the feedback control laws (2) and (3) ensures the system stability only in the sense of Lyapunov.

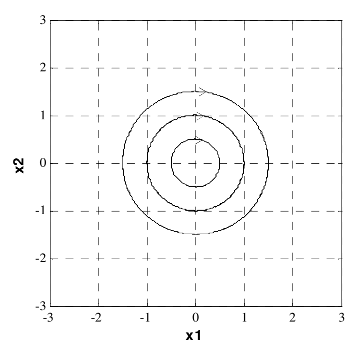

Fig. 1. Phase portrait of system (1) with controller (2).

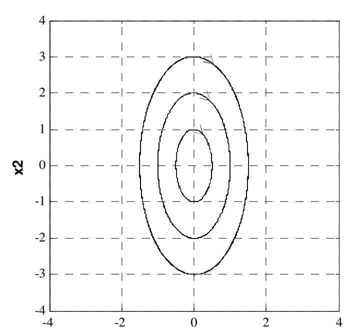

Fig. 2. Phase portrait of system (1) with controller (3).

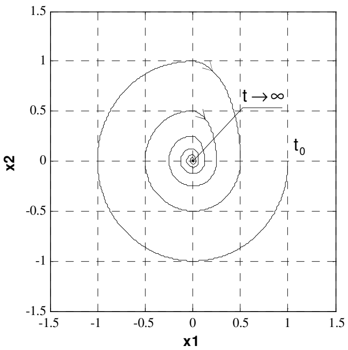

Fig. 3. Phase portrait of system (1) when switching strategy (4) is applied.

However, if the following switching strategy is applied

$$i = \begin{cases} 1 & \text{for} \quad \min\{x_1, x_2\} < 0 \\ 2 & \text{for} \quad \min\{x_1, x_2\} \geq 0 \end{cases} \qquad (4)$$

then the system becomes asymptotically stable. This is illustrated in figure 3. Moreover, it is worth to point out that system (1) with the same feedback control laws may exhibit completely different behaviour (and even become unstable). For example, if the switching strategy (4) is modified as

$$i = \begin{cases} 1 & \text{for} \quad \min\{x_1, x_2\} \geq 0 \\ 2 & \text{for} \quad \min\{x_1, x_2\} < 0 \end{cases} \qquad (5)$$

then the system output increases to infinity. The system dynamic behaviour, in this situation, is illustrated in figure 4.

This example presents the concept of variable structure control (VSC) and stresses that the system dynamics in VSC is determined not only by the applied feedback controllers but also, to a large extent, by the adopted switching strategy.

VSC is inherently a nonlinear technique and as such, it offers a variety of advantages which cannot be achieved using conventional linear controllers. Our

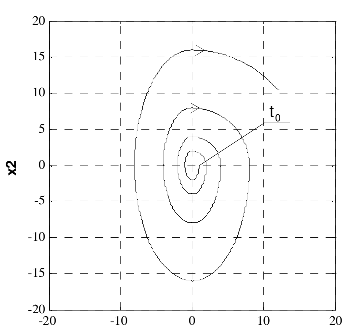

Fig. 4. Phase portrait of system (1) when switching strategy (5) is applied.

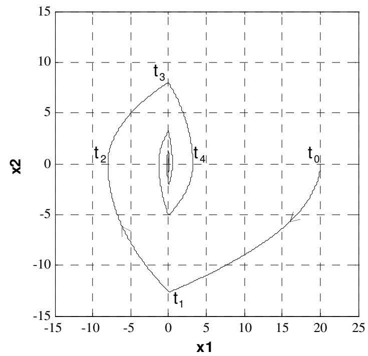

Fig. 5. Phase portrait of system (1) controlled according to (6).

next example shows one of those favourable features – namely it demonstrates that VSC may enable finite time error convergence. In this example, again we consider system (1), however now we apply the following controller

$$u = -x_2 - a\,\mathrm{sgn}(x_1) - b\,\mathrm{sgn}(x_2) \qquad (6)$$

where $a > b > 0$. Closer analysis of the behaviour of system (1) with control law (6) demonstrates that, in this example, the system error converges to zero in finite time which can be expressed as

$$T = \frac{a}{b}\sqrt{2x_{10}}\left(\frac{1}{\sqrt{a-b}} + \frac{1}{\sqrt{a+b}}\right) \qquad (7)$$

where x_{10} and $x_{20} = 0$ represent initial conditions of system (1). Even though the error converges to zero in finite time, the number of oscillations in the system tends to infinity, with the period of the oscillations decreasing to zero. This is illustrated in figures 5 and 6. In the simulation example presented in the figures, the following parameters are used $a = 7$, $b = 3$, $x_{10} = 20$ and $x_{20} = 0$. Consequently, the system error is nullified at the time instant $T = 12.045$ and remains equal to zero for any time greater than T. Clearly these favourable properties are achieved using finite control signal. This controller, due to the way the phase trajectory – shown in figure 5 – is

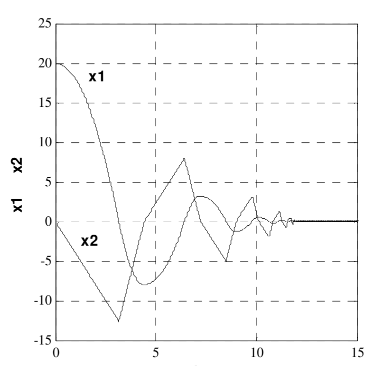

Fig. 6. State variables of system (1) controlled according to (6).

drawn, is usually called "twisting controller". It also serves as a good, simple example of the second order sliding mode controllers.

2. SLIDING MODE CONTROL

The two examples presented up to now demonstrate the principal properties of VSC systems. However, the main advantage of the systems is obtained when the controlled plant exhibits the sliding motion (Utkin, 1977; DeCarlo *et al.*, 1988; Slotine and Li, 1991; Hung *et al.*, 1993). The idea of sliding mode control (SMC) is to employ different feedback controllers acting on the opposite sides of a predetermined surface in the system state space. Each of those controllers pushes the system representative point (RP) towards the surface, so that the RP approaches the surface, and once it hits the surface for the first time it stays on it ever after. The resulting motion of the system is restricted to the surface, which graphically can be interpreted as "sliding" of the system RP along the surface. This idea is illustrated by the following example.

Let us consider another second order plant

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= b\cos(mx_1) + u \qquad |b| < 1 \end{aligned} \tag{8}$$

where b and m are possibly unknown constants. We select the following line in the state space

$$s = x_2 + cx_1 = 0 \tag{9}$$

(c = const.) and apply the controller

$$u = -cx_2 - \operatorname{sgn}(s) \tag{10}$$

In this equation sgn(.) function represents the sign of its argument, i.e. $\operatorname{sgn}(s < 0) = -1$ and $\operatorname{sgn}(s > 0) = +1$. With this controller the system representative point moves towards line (9) always when it does not belong to the line. Then, once it hits the line, the controller switches the plant input (in the ideal case) with infinite frequency. Therefore, line (9) is called the switching line. Furthermore, since after reaching the line, the system RP slides along it, then the line is also called the sliding line. This example is illustrated in figure 7. The system parameters used in the presented simulation are $c = 0.5$, $b = 0.75$, $m = 10$ and the simulation is performed for the following initial conditions $x_{10} = 5$ and $x_{20} = 1$. Notice that when the plant remains in the sliding mode, its dynamics is completely determined by the switching line (or in general the switching hypersurface) parameters. This implies that neither model uncertainty nor matched external disturbance affects the plant dynamics (Draženović, 1969) which is a highly desirable system property. This property can also be justified geometrically, if one notices that in our example the slope of line (9) fully governs the plant motion in the sliding mode. Therefore, in SMC systems we usually make the distinction between two phases: the first one – called the reaching phase – lasts until the controlled plant RP hits the switching surface, and the second one – the sliding phase – begins when the RP reaches the surface. In the latter phase the plant insensitivity to a class of modelling inaccuracies and external disturbances is ensured. Let us stress that the system robustness with respect to unmodelled dynamics, parameter uncertainty and external disturbances is guaranteed only in the sliding mode. Therefore, shortening or (if possible) even complete elimination of the reaching phase is an important and timely research issue (Chang and Hurmuzlu, 1992; Chang and Hurmuzlu, 1993; Choi *et al.*, 1994; Bartoszewicz, 1995; Bartoszewicz 1996a; Betin *et al.*, 2002a; Betin *et al.*, 2002b, Tokat *et al.*, 2003; Sivert *et al.*, 2004a; Sivert *et al.*, 2004b; Sivert *et al.*, 2004c; Bartoszewicz and Nowacka, 2005; Bartoszewicz and Nowacka, 2007a; Bartoszewicz and Nowacka, 2007b; Corradini and Orlando, 2007; Pan and Furuta, 2007) in the field of SMC.

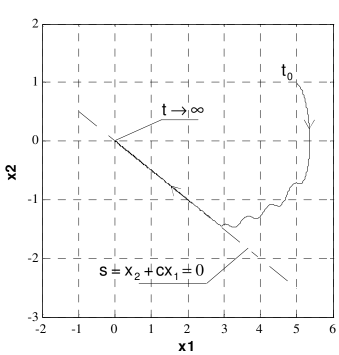

Fig. 7. Phase trajectory of system (1) controlled according to (10).

Another immediate consequence of the fact that in the sliding mode, the system RP is restricted to the switching hypersurface (which is a subset of the state space) is reduction of the system order. If the system of the order n has m independent inputs, then the sliding mode takes place on the intersection of m hypersurfaces and the reduced order of the system equals to the difference $n - m$. To be more precise, in multi-input systems the sliding mode may take place either independently on each switching hypersurface or only on the intersection of the surfaces. In the first case the system RP approaches each surface at any time instant and once it hits any of the surfaces it stays on this surface ever after. In the latter case, however, the system RP does not necessarily approach each of the surfaces, but it always moves towards their intersection. In this case the system RP may hit a surface and move away from it (might possibly cross a switching surface), but once it reaches the intersection of all the surfaces, then the RP never leaves it.

As it has already been mentioned, the switching surface completely determines the plant dynamics in the sliding mode. Therefore, selecting this surface is one of the two major tasks in the process of the SMC system design. In order to stress this issue let us point out that the same controller which has been considered in the last example may result in a very different system performance, if the sliding line slope c is selected in another way. This can be easily noticed if one takes into account any negative c. Then, controller (10) still ensures stability of the sliding motion on line (9), i.e. the system RP still converges to the line, however the system is unstable since both state variables x_1 and x_2 tend to (either plus or minus) infinity while the system RP slides away from the origin of the phase plane along line (9).

The other major task in the SMC system design is the selection of an appropriate control law. This can be achieved either by assuming a certain kind of the control law (usually motivated by some previous engineering experience) and proving that this control satisfies one of the so-called reaching conditions or by applying the reaching law approach. The reaching conditions (Edwards and Spurgeon, 1998) ensure stability of the sliding motion and therefore they are naturally derived using Laypunov stability theory. On the other hand, if the reaching law approach is adopted for the purpose of a sliding mode controller construction (Hung *et al.*, 1993), then a totally different design philosophy is employed. In this case the desired evolution of the switching variable s is specified first, and then a control law ensuring that s changes according to the specification is determined.

Sliding mode controllers guarantee system insensitivity with respect to matched disturbance and model uncertainty, and cause reduction of the plant order. Moreover, they are computationally efficient, and may be applied to a wide range of various, possibly nonlinear and time-varying plants. However, often they also exhibit a serious drawback which essentially hinders their practical applications. This drawback – high frequency oscillations which inevitably appear in any real system whose input is supposed to switch infinitely fast – is usually called chattering. If system (8) exhibits any, even arbitrarily small, delay in the input channel, then control strategy (10), will cause oscillations whose frequency and amplitude depend on the delay. With the decreasing of the delay time, the frequency rises and the amplitude is getting smaller. This is a highly undesirable phenomenon, because it causes serious wear and tear on the actuator components. Therefore, a few methods to eliminate chattering have been proposed. The most popular of them uses function

$$\mathrm{sat}(s) = \begin{cases} -1 & \text{for } s < -\rho \\ \dfrac{1}{\rho} s & \text{for } |s| \le \rho \\ 1 & \text{for } s > \rho \end{cases} \tag{11}$$

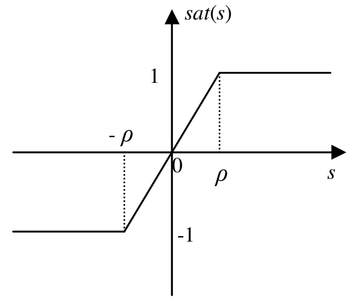

Fig. 8. Function sat(s).

instead of sgn(s) in the definition of the discontinuous control term. With this modification the term becomes continuous and the switching variable does not converge to zero but to the closed interval $[-\rho, \rho]$. Consequently, the system RP after the reaching phase termination, belongs to a layer around the switching hyperplane and therefore this strategy is called boundary layer controller (Slotine and Li, 1991).

Other approaches to the chattering elimination include:

a) introduction of other nonlinear approximations of the discontinuous control term, for example the so called fractional approximation defined as

$$\mathrm{approx}(s) = \frac{s}{\delta + |s|} \tag{12}$$

where δ is a small positive constant (Ambrosino *et al.*, 1984; Burton and Zinober, 1986; Spurgeon and Davies, 1993; Yu and Lloyd, 1997);

b) replacing the boundary layer with a sliding sector (Shyu *et al.*, 1992; Xu *et al.*, 1996);

c) using dynamic sliding mode controllers (Sira-Ramirez, 1993a; Sira-Ramirez, 1993b; Sira-Ramirez and Llanes-Santiago, 1994; Zlateva, 1996; Bartolini and Pydynowski, 1996; Rios-Bolivar *et al.*, 1997; Spurgeon and Lu, 1997; Lu and Spurgeon, 1998; Selisteanu *et al.*, 2007);

d) using fuzzy sliding mode controllers (Palm, 1994; Palm *et al.* 1997; Choi and Kim, 1997);

e) using second (or higher) order sliding mode controllers (Levant, 1993; Bartolini *et al.*, 1997a; Bartolini *et al.*, 1997b; Bartolini *et al.*, 1998; Levant, 2003).

The phenomenon of chattering has been extensively analysed in many papers using describing function method and various stability criteria (Shtessel and Lee, 1996; Chung and. Lin, 1999; Bartoszewicz, 2000).

3. DISCRETE-TIME SMC

In sections 1 and 2 of this paper we focused our attention on continuous time systems, which can change the plant input at any instant of time t. However, nowadays controllers are usually built as microprocessor systems and their output signal cannot change at arbitrarily selected moments, but only at the predetermined, discrete instants of time $t = kT$, where k is a natural number and T denotes the sampling (or discretisation) period. Therefore, digitally implemented SMC systems often exhibit essentially different properties from the same systems realized using only analogue devices. Those digitally implemented systems are usually called discrete time sliding mode control (DSMC) systems and they no longer ensure complete insensitivity of the controlled plant with reference to external disturbances and model uncertainty. However, they still offer some degree of robustness measured in a strictly defined sense, most often measured in terms of the switching variable magnitude.

There are two major approaches to the DSMC system design. One of them refers to the notion of the sliding hypersurface (or typically hyperplane) and the other one uses the concept of the sliding sector. The first approach is a natural generalization of the continuous time SMC idea and has been developed gradually by many authors working for various research institutions (Milosavljevic, 1984; Sarpturk *et al.* 1987; Kotta, 1989; Spurgeon, 1992; Gao *et al.*, 1995; Bartoszewicz, 1998). In general, the idea is quite similar to that described in section 2 and it comprises of selecting an appropriate sliding hyperplane $s(kT) = 0$ and choosing such a control law $u(kT)$ which ensures that $s[(k + 1)T)] = 0$. We will illustrate this idea with the following, very standard example. Let us consider a single input, linear discrete time system

$$\boldsymbol{x}[(k+1)T] = \boldsymbol{A}\boldsymbol{x}(kT) + \boldsymbol{b}u(kT) \tag{13}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{A}$ is an n by n state matrix and $\boldsymbol{b}$ is an n by 1 input vector. Then we choose the following sliding hyperplane

$$s(kT) = \boldsymbol{c}^T \boldsymbol{x}(kT) = 0 \tag{14}$$

in the n-dimensional state space $\Re^n$, and in order to obtain $s[(k + 1)T)] = 0$ we select

$$u(kT) = -(\boldsymbol{c}^T \boldsymbol{b})^{-1} \boldsymbol{A}\boldsymbol{x}(kT) \tag{15}$$

Clearly vector $\boldsymbol{c}$ has to be chosen in such a way that $\boldsymbol{c}^T\boldsymbol{b} \neq 0$. Substituting equations (13) and (15) into (14) one can easily notice that this controller actually brings the system RP on line (14) at the time instant $k + 1$. Thus the ideal sliding motion without chattering takes place in the system. Moreover, when control strategy (15) is applied, then the closed-loop system state matrix has the following form $\boldsymbol{A_c} = [\boldsymbol{I_n} - \boldsymbol{b}(\boldsymbol{c}^T\boldsymbol{b})^{-1}\boldsymbol{c}^T]\boldsymbol{A}$, where $\boldsymbol{I_n} = \mathrm{diag}\{1, 1, ..., 1\}$ is an n by n identity matrix. This shows that the system dynamic performance may be tuned as desired, and various design objectives might be achieved. Those may include but are not limited to: finite time error convergence (time optimal performance), obtained when the closed loop system characteristic polynomial has the form

$$\det(z\boldsymbol{I}_n - \boldsymbol{A}_c) = z^n, \tag{16}$$

linear-quadratic (LQ) optimal, integral absolute error (IAE) optimal, integral time multiplied by the absolute error (ITAE) optimal performance, etc.

However, as opposed to the continuous SMC systems, DSMC considered in this section, ensures that the switching variable is equal to zero only at the sampling instants, while at any time t which is not an integer multiple of the discretisation period the switching variable may attain certain values different from zero. In other words, intersampling behaviour of this variable is not determined. Furthermore, if system (13) is subject to external disturbance $\boldsymbol{d}(kT)$, i.e. it is described as

$$\boldsymbol{x}[(k+1)T] = \boldsymbol{A}\boldsymbol{x}(kT) + \boldsymbol{b}u(kT) + \boldsymbol{d}(kT) \tag{17}$$

where the absolute value of the ith component $d_i(kT)$ of vector $\boldsymbol{d}(kT)$ is always upper bounded by a known, non-negative value d_{imax} ($|d_i| \leq d_{imax}$), then the system RP no longer stays on line (14) even at the discrete sampling instants, but remains in a vicinity of the line

$$|s[(k+1)T]| \leq \sum_{i=1}^{n} |c_i| d_{i\max} \tag{18}$$

instead. In other words the system RP remains in a band around the sliding hyperplane in the state space, and therefore this type of motion is sometimes called a quasi-sliding mode or a non-ideal sliding motion (Gao *et al.*, 1995; Bartoszewicz, 1996b; Bartoszewicz, 1998). Clearly, the thinner the band (smaller the band width), the better system robustness with respect to the considered disturbance is achieved.

Let us also point out that the example which has been presented in this section shows again that the notions of SMC and VSC are not equivalent. In section 1 of this paper some examples of continuous time VSS without sliding mode were presented, and this example gives an idea of a SMC system which actually is not a VSS.

The other approach to the DSMC system design exploits the notion of the sliding sector (Furuta, 1990; Chan, 1991; Furuta, 1993; Pan and Furuta, 1997; Hara *et al.*, 1998; Pan and Furuta, 2007). The sector (cone like region in the state space) is selected in such a way that if the system RP belongs to it, then the RP always approaches the state space origin.

4. NOVEL TRENDS IN SMC

SMC is currently one of the most significant research topics within the control engineering domain.

Therefore, in this section we are able to point out only a few, arbitrarily selected issues, which we believe to be the most promising and up to date trends in the field. These include, but by no means are limited to:

- design and implementation of sliding mode observers for systems with unknown inputs (Floquet *et al.*, 2007);
- application of sliding mode observers for fault detection and isolation (Yan and Edwards, 2007);
- higher order sliding modes (Levant and Alelishvili, 2007);
- neural network based sliding mode controllers (Topalov *et al.*, 2007a; Topalov *et al.*, 2007b);
- sliding mode control of dynamic nonlinear systems by output-feedback (Oliveira *et al.*, 2007);
- variable structure control without the reaching phase (Bartoszewicz and Nowacka, 2007a; Bartoszewicz and Nowacka, 2007b; Pan and Furuta, 2007);
- sliding mode control application in robotics and complex motion steering systems (Ferrara and Lombardi, 2007);
- sliding mode control in power electronics and control of electric drives and actuators (Betin and Capolino, 2007; Topalov *et al.*, 2007b);
- sliding mode control of systems in interaction with their environment (Sabanovic, 2007);
- sliding mode control of biotechnological reactors (Selisteanu *et al.*, 2007).

There are many successful applications of SMC systems reported in literature. Therefore, the examples mentioned above are by no means exhaustive and they may only present a good starting point for further analysis. Most of the examples cited in this section come from the recent special issue of International Journal of Adaptive Control and Signal Processing on Sliding Mode Control (Bartoszewicz and Patton, 2007).

5. CONCLUSION

The paper gives a brief tutorial introduction to the most fundamental issues in the field of variable structure and sliding mode control systems. Moreover, it presents selected novel trends and successful engineering applications in the field. Both continuous and discrete time sliding mode controllers are discussed. Chattering elimination and the issue of ensuring appropriate system performance are presented.

REFERENCES

Ambrosino, G. Celentano, G. and Garofalo, F. (1984). Variable structure model reference adaptive control systems. *International Journal of Control*, Vol. **39**, pp. 1339–1349.

Bartolini, G. Ferrara, A. and Usai, E. (1998). Chattering avoidance by second-order sliding mode control. *IEEE Transactions on Automatic Control*, Vol. **43**, pp. 241–246.

Bartolini, G. Ferrara, A. and Usai, E. (1997a). Output tracking control of uncertain non linear second order systems. *Automatica*, Vol. **33**, pp. 2203–2212.

Bartolini, G. Ferrara, A. and Usai, E. (1997b). Applications of a sub-optimal discontinuous control algorithm for uncertain second order systems. *International Journal of Robust and Nonlinear Control*, Vol. **7**, pp. 299–319.

Bartolini, G. Ferrara, A. and Spurgeon, S. editors (1997c). New trends in sliding mode control. *special issue of International Journal of Robust and Nonlinear Control*, Vol. **7**, pp. 297–427.

Bartolini, G. and Pydynowski, P. (1996). An improved, chattering free, V.S.C. scheme for uncertain dynamical systems. *IEEE Transactions on Automatic Control*, Vol. **41**, pp. 1220–1226.

Bartoszewicz, A. (2000). Chattering attenuation in sliding mode control systems. *Control and Cybernetics*, Vol. **29**, pp. 585–594.

Bartoszewicz, A. (1998). Discrete time quasi-sliding mode control strategies. *IEEE Transactions on Industrial Electronics*, Vol. **45**, pp. 633–637.

Bartoszewicz, A. (1996a). Time-varying sliding modes for second-order systems. *IEE Proceedings on Control Theory and Applications*, Vol. **143**, pp. 455–462.

Bartoszewicz, A. (1996b). Remarks on "Discrete-time variable structure control systems". *IEEE Transactions on Industrial Electronics*, Vol. **43**, pp. 235–238.

Bartoszewicz, A. (1995). A Comment on 'A time-varying sliding surface for fast and robust tracking control of second-order uncertain systems'. *Automatica*, Vol. **31**, pp. 1893–1895.

Bartoszewicz, A. and Nowacka, A. (2007a). Sliding mode control of the third-order system subject to velocity, acceleration and input signal constraints. *Proceedings of the IET Part D: Control Theory and Applications,* Vol. **1**, pp. 1461–1470.

Bartoszewicz, A. and Nowacka, A. (2007b). Sliding mode control of the third-order system subject to velocity, acceleration and input signal constraints. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Bartoszewicz, A. and Nowacka, A. (2005). Switching plane design for the sliding mode control of systems with elastic input constraints. *Proceedings of Institution of Mechanical Engineers, Part I, Journal of Systems & Control Engineering*, Vol. **219**, pp. 393–403.

Bartoszewicz, A. and Patton, R. editors (2007). Sliding mode control. *Special issue of International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Betin, F. and Capolino, G. (2007). Sliding mode control for electrical machines submitted to large variations of the mechanical configuration.

International Journal of Adaptive Control and Signal Processing, Vol. **21**.

Betin, F. Pinchon, D. and Capolino, G. (2002a). A time-varying sliding surface for robust position control of a DC motor drive. *IEEE Transactions on Industrial Electronics*, Vol. **49**, pp. 462 – 473.

Betin, F. Sivert, A. and Pinchon, D. (2002b). Time-varying sliding modes for a DC motor drive. *Proceedings of the International Symposium on Industrial Electronics,* L'Aquila, Italy, pp. 378–382.

Burton, J. and Zinober, A. (1986). Continuous approximation of variable structure control. *International Journal of Systems Science*, Vol. **17**, pp. 875–885.

Chan, C. Y. (1991). Servo-systems with discrete-variable structure control. *Systems and Control Letters*, Vol. **17**, pp. 321–325.

Chang, T. H. and Hurmuzlu, Y. (1993). Sliding control without reaching phase and its application to bipedal locomotion. *Transactions of the ASME – Journal of Dynamic Systems, Measurement, and Control*, Vol. **115**, pp. 447–455.

Chang, T. H. and Hurmuzlu, Y. (1992). Trajectory tracking in robotic systems using variable structure control without a reaching phase. *Proceedings of the American Control Conference*, pp. 1505–1509.

Choi, S. B. and Kim, J. S. (1997). A fuzzy-sliding mode controller for robust tracking of robotic manipulators. *Mechatronics*, Vol. **7**, pp. 199–216.

Choi, S. B. Park, D. W. and Jayasuriya, S. (1994). A time-varying sliding surface for fast and robust tracking control of second-order uncertain systems. *Automatica*, Vol. **30**, pp. 899–904.

Chung, S. C. and Lin C. L. (1999). A transformed Luré problem for sliding mode control and chattering reduction. *IEEE Transactions on Automatic Control*, Vol. **44**, pp. 563–568.

Corradini, M. L. and Orlando, G. (2007). Linear unstable plants with saturating actuators: robust stabilization by a time varying sliding surface. *Automatica*, Vol. **43**, pp. 88–94.

DeCarlo, R.S., Żak, S. and Mathews, G. (1988). Variable structure control of nonlinear multivariable systems: a tutorial. *Proceedings of IEEE*, Vol. **76**, pp. 212–232.

Draženović, B. (1969). The invariance conditions in variable structure systems. *Automatica*, Vol. **5**, pp. 287–295.

Edwards, C. and Spurgeon, S. (1998). Sliding Mode Control: Theory and Application. Taylor & Francis, London.

Ferrara, A. and Lombardi, C. (2007). Interaction control of robotic manipulators via second order sliding modes. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Floquet, T. Edwards, C. and Spurgeon, K. (2007). On sliding mode observers for systems with unknown inputs. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Furuta, K. (1993). VSS type self-tuning control. *IEEE Transactions on Industrial Electronics*, Vol. **40**, pp. 37–44.

Furuta, K. (1990). Sliding mode control of a discrete system. *Systems & Control Letters*, Vol. **14**, pp. 145–152.

Gao, W. editor (1993). Variable structure control. *special section in IEEE Transactions on Industrial Electronics*, Vol. **40**, pp. 1–88.

Gao, W. Wang, Y. and Homaifa, A. (1995). Discrete-time variable structure control systems. *IEEE Transactions on Industrial Electronics*, Vol. **42**, pp. 117–122.

Hara, M. Furuta, K. Pan, Y. and Hoshino, T. (1998). Evaluation of discrete-time VSC on an inverted pendulum apparatus with additional dynamics. *Applied Mathematics and Computer Science*, Vol. **8**, pp. 159–181.

Hung, J.Y. Gao, W. and Hung, J.C. (1993). Variable structure control: a survey. *IEEE Transactions on Industrial Electronics*, Vol. **40**, pp. 2–22.

Kotta, U. (1989). Comments on ''On the stability of discrete-time sliding mode control systems''. *IEEE Transactions on Automatic Control*, Vol. **34**, pp. 1021–1022.

Levant, A. (1993). Sliding order and sliding accuracy in sliding mode control. *International Journal of control*, Vol. **58**, pp. 1247–1263.

Levant, A. (2003). Higher-order sliding modes, differentiation and output feedback control. *International Journal of Control*, Vol. **76**, pp. 924–941.

Levant, A. and Alelishvili, L. (2007). Integral High-Order Sliding Modes. *IEEE Transactions on Automatic Control*, Vol. **52**, pp. 1278–1282.

Lu, X. Y. and Spurgeon, S. (1998). A new sliding mode approach to asymptotic feedback linearisation with application to the control of non-flat systems. *Applied Mathematics and Computer Science*, Vol. **8**, pp. 21–37.

Милосавлевич, Ч. (1985). Общие условия существования квазискользящего режима относительно гиперплоскости переключения в дискретных спс. *Автоматика и Телемеханика*, Vol. **46**, pp. 36–44.

Misawa, E. and Utkin, V. editors (2000). Variable structure systems. *special issue of Transactions of the ASME – Journal of Dynamic Systems, Measurement, and Control*, Vol. **122**, pp. 585–819.

Oliveira, T. Peixoto, A. Nunes, E. and Hsu, L. (2007). Control of uncertain nonlinear systems with arbitrary relative degree and unknown control direction using sliding modes. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Palm, R. (1994). Robust control by fuzzy sliding mode. *Automatica*, Vol. **30**, pp. 1429–1437.

Palm, R. Driankov, D. and Hellendoorn, H. (1997). Model based fuzzy control. Springer, Berlin.

Pan, Y. and Furuta, K. (2007). Variable structure control with sliding sector based on hybrid switching law. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Pan, Y. and Furuta, K. (1997). Discrete-time VSS controller design. *International Journal of Robust and Nonlinear Control*, Vol. **7**, pp. 373–386.

Rios-Bolivar, M. Zinober, A. and Sira-Ramirez, H. (1997). Dynamical adaptive sliding mode output tracking control of a class of nonlinear systems. *International Journal of Robust and Nonlinear Control*, Vol. **7**, pp. 387–405.

Sabanovic, A. (2007). SMC framework in motion control systems. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Sarpturk, S. Istefanopulos, Y. and Kaynak, O. (1987). On the stability of discrete-time sliding mode control systems. *IEEE Transactions on Automatic Control*, Vol. **32**, pp. 930–932.

Selisteanu, D. Petre, E. and Rasvan, V. (2007). Sliding mode and adaptive sliding mode control of a class of nonlinear bioprocesses. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Shtessel, Y. and Lee, Y. J. (1996). New approach to chattering analysis in systems with sliding modes. *Proceedings of the 35th Conference on Decision and Control*, Kobe, Japan, pp. 4014–4019.

Shyu, K. Tsai, Y. and Yung, C. (1992). A modified variable structure controller. *Automatica*, Vol. **28**, pp. 1209–1213.

Sira-Ramirez, H. (1993a). A dynamical variable structure control strategy in asymptotic output tracking problems. *IEEE Transactions on Automatic Control*, Vol. **38**, pp. 615–620.

Sira-Ramirez, H. (1993b). On the dynamical sliding mode control of nonlinear systems. *International Journal of Control*, Vol. **57**, pp. 1039–1061.

Sira-Ramirez, H. and Llanes-Santiago, O. (1994). Dynamical discontinuous feedback strategies in the regulation of nonlinear chemical processes. *IEEE Transactions on Control Systems Technology*, Vol. **2**, pp. 11–21.

Sivert, A. Betin, F. Faqir, A. and Capolino, G. (2004a). Robust control of an induction machine drive using a time-varying sliding surface. *Proceedings of the IEEE International Symposium on Industrial Electronics*, pp. 1369–1374.

Sivert, A. Betin, F. Faqir, A. and Capolino, G. (2004b). Time-varying sliding surface for position control of an induction machine drive. *Proceedings of the 16th International Conference on Electrical Machines*, Cracow, Poland.

Sivert, A. Betin, F. Faqir, A. and Capolino, G. (2004c). Moving switching surfaces for high precision control of electrical drives. *Proceedings of the IEEE International Conference on Industrial Technology*, pp. 175–180.

Slotine, J.J. and Li, W. (1991). *Applied Nonlinear Control*. Prentice-Hall International Editions.

Spurgeon, S. (1992). Hyperplane design techniques for discrete-time variable structure control systems. *International Journal of Control*, Vol. **55**, pp. 445–456.

Spurgeon, S. and Davies, R. (1993). A nonlinear control strategy for robust sliding mode performance in the presence of unmatched uncertainty. *International Journal of Control*, Vol. **57**, pp. 1107–1123.

Spurgeon, S. and Lu, X. Y. (1997). Output tracking using dynamic sliding mode techniques. *International Journal of Robust and Nonlinear Control*, Vol. **7**, pp. 407–427.

Tokat, S. Eksin, I. Guzelkaya, M. and Soylemez, M. (2003). Design of a sliding mode controller with a nonlinear time-varying sliding surface. *Transactions of the Institute of Measurement and Control*, Vol. **25**, pp. 145–162.

Topalov, A. Kaynak, O. and Aydin G. (2007a). Neuro–adaptive sliding-mode tracking control of robot manipulators. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Topalov, A. Cascella, G. Giordano, V. Cupertino, F. and Kaynak, O. (2007b). Sliding Mode Neuro-Adaptive Control of Electric Drives. *IEEE Transactions on Industrial Electronics*, Vol. **54**, pp. 671–679.

Utkin, V. (1977). Variable structure systems with sliding modes. *IEEE Transactions on Automatic Control*, Vol. **22**, pp. 212–222.

Utkin, V. editor (1993). Sliding mode control. *special issue of International Journal of Control,* Vol. **57**, pp. 1003–1259.

Xu, J. Lee, T. Wang, M. and Yu, X. (1996). Design of variable structure controllers with continuous switching control. *International Journal of Control*, Vol. **65**, pp. 409–431.

Yan, X. G. and Edwards, C. (2007). Sensor fault detection and isolation for nonlinear systems based on a sliding mode observer. *International Journal of Adaptive Control and Signal Processing,* Vol. **21**.

Yu, H. and Lloyd, S. (1997). Variable structure adaptive control of robot manipulators. *Proceedings of the IEE Part D: Control Theory and Applications*, Vol. **144**, pp. 167–176.

Zlateva, P. (1996). Variable-structure control of nonlinear systems. *Control Engineering Practice*, Vol. **4**, pp. 1023–1028.