NEW RESULTS IN BACKSTEPPING DESIGN FOR ELECTROHYDRAULIC SERVOS:
ADAPTIVE CONTROL SYNTHESIS

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Abstract: This paper continues recent research of authors, considering the adaptive control synthesis in the presence of parametric uncertainties, with application to electrohydraulic servos actuating primary flight controls. To account for these parametric uncertainties in the model, a parameter adaptation scheme is essential. So, certain parameters are adjusted online; using in synthesis the methods of Control Lyapunov Functions and backstepping, the obtained dynamic update law for parameters, together with the system’s model, would render the closed loop system stable and would achieve the regulation of the output. The work also illustrates how the main theory can be brought or adapted to control design practice as defined by a given mathematical model. Barbalat’s Lemma is used in the proof of control law structure. Numerical simulations are reported from viewpoint of servo time constant performance.

Keywords: nonlinear control synthesis, adaptive backstepping, Control Lyapunov Function, Barbalat’s Lemma, electrohydraulic servo.

1. INTRODUCTION

This paper continues the theme of recent researches of the authors (Ursu and Popescu, 2003; Ursu et al., 2004; I. Ursu and F. Ursu, 2004; Ursu et al., 2005), by detailing procedure of adaptive backstepping control synthesis for the Electrohydraulic Servos (EHSs). This procedure of adaptive backstepping is introduced avant la lettre in Kanellakopoulos et al., (1991), where a characterization of the class of nonlinear systems to which the new adaptive scheme is applicable is achieved. In our paper, a simpler scheme than those described in Kanellakopoulos et al. (1991) is proposed for an electrohydraulic servo with unknown or uncertain parameters. So, this parameter is estimated online, in the framework of a recurrent control law using the methods of Control Lyapunov Functions and backstepping. The estimation error is proved to be asymptotically stable. The obtained control law, including a dynamic update for uncertain parameter, and operating on the system’s model, renders the closed loop stable and ensure the regulation of the desired output.

2. BACKSTEPPING ADAPTIVE CONTROL SYNTHESIS FOR AN ELECTROHYDRAULIC SERVO

The basic backstepping assumes the knowledge of the system parameters. In fact, frequently it may be necessary to identify some of these parameters offline or estimate them using online adaptive schemes. The essence of adaptive control is that, by learning from the past information through this parameter adaptation mechanism, the real parametric uncertainty can be evaded. To illustrate the adaptive backstepping machinery, let consider the mathematical model of electrohydraulic servo (see modelling aspects in Ursu and Ursu, 2002; Halanay et al., 2004):

\[ \dot{x}_1 = x_2, \quad \dot{x}_2 = \frac{1}{m} \left[ -k \dot{x}_1 - f_x + S(x_3 - x_4) \right] \]  (1)
\[ \dot{x}_3 = \frac{B}{V + S x_1} \left( c x_5 \sqrt{p_a - x_3} - S x_2 \right) \]
\[ \dot{x}_4 = \frac{B}{V + S x_1} \left( -c x_5 \sqrt{x_4} + S x_2 \right) \]
\[ \dot{x}_5 = -\frac{x_5 + k_ex}{\tau}, \quad c := c_d w \frac{2}{\sqrt{\rho}}. \]  

These differential equations are reported having as a reference point the hydromechanical servomechanism SMHR included in the aileron control chain of Romanian military jet IAR 99. The state variables are denoted by: \( x_1 [\text{cm}] \) – EHS load displacement; \( x_2 [\text{cm/s}] \) – EHS load velocity; \( x_3, x_4 \) [daN/cm\(^2\)] – pressures in the cylinder chambers; \( x_5 [\text{cm}] \) – valve position; \( u [\text{V}] \) – control variable. The nominal values of the parameters appearing in equations (1) are: \( m = 0.033 \text{ daN s}^2/\text{cm} \) – equivalent inertial load of primary control surface reduced at the EHS’s rod; \( f = 1 \text{ daN s}/\text{cm} \) – equivalent viscous friction force coefficient; \( k_i = 100 \text{ daN cm} \) – equivalent aerodynamic elastic force coefficient; \( S = 10 \text{ cm}\(^2\) \) – EHS’s piston area; \( w = 0.05 \text{ cm} \) – valve-port width; \( B = 6000 \text{ daN cm}/\text{s} \) – bulk modulus of the oil; \( p_s = 210 \text{ daN cm}^2 \) – supply pressure; \( V = 30 \text{ cm}^3 \) – cylinder semivolume; \( \tau = 1/573 \text{ s} \) – time constant of the (servo) valve; \( c_d = 0.63 \) – volumetric flow coefficient of the valve port; \( \rho = 85/(981 \times 10^5) \text{ daN cm}^3/\text{m}^3 \) – volumetric density of oil; \( k_e = 0.0085/(0.05 \times 10^4) \text{ cmV} \) – valve displacement/voltage coefficient.

In the equations (2) is assumed the uncertainty of the coefficient \( c \) enclosed in the mixed parameter \( Bc \).

\[ \alpha := Bc. \]  

Thus, these equations will be rewritten in the form

\[ \dot{x}_3 = \frac{c x_5 \sqrt{p_a - x_3}}{V + S x_1} - \frac{B S}{V + S x_1} x_2 \]
\[ \dot{x}_4 = \frac{-c x_5 \sqrt{x_4}}{V - S x_1} + \frac{B S}{V - S x_1} x_2 \]

and will be added to the equations of the system (1) and (3) to define a system whose equations belong to a general class of nonlinear systems treated in Kanellopoulos et al., (1991)

\[ \dot{x} = f_0(x) + \sum_{i=1}^{p} f_i(x) + \left( g_0(x) + \sum_{i=1}^{p} g_i(x) \right) u \]  

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R} \) is the control vector, \( f_0, g_0, f_i, g_i \) are smooth vector fields of appropriate dimensions and \( [\theta_1, \ldots, \theta_p]^T \) is the vector of unknown (uncertain) constant parameters. The geometrical approach therein developed was avoided in this paper, instead using a simple, intuitive scheme of adaptive backstepping having as object the system (1), (2), (3), which represents the electrohydraulic servo as a tracking system. Therefore, for this system the aim of control synthesis is to have a good tracking by the state variable \( x_1 \) of the specified \( x_{id} \) desired position references. The closed loop performance of the system can be measured by the actual (realised) servo time constant \( \tau_s \). Thus, a good tracking system is characterised by fast (little) time constant \( \tau_s \). Both servo time constant and position reference signal are in connection with the response of a first order system to step inputs \( x_{ii} \)

\[ x_{id} = x_{ii} \left( 1 - e^{-t/\tau_s} \right). \]  

\( x_{ii} \) stands for stationary value of the state \( x_1 \), and \( \tau_s \) stands for associated desired time constants.

The main result of the paper is given by the following

**Proposition.** Consider the uncertain parameter \( \alpha \) for the EHS mathematical model (1), (2), (3). Let \( k_1 > 0, k_2 > 0, k_3 > 0, \rho_\alpha > 0 \) be tuning parameters and \( \tilde{\alpha} \neq 0 \) the notation for the estimate of the uncertain parameter \( \alpha \). Define the learning error \( \tilde{\alpha} := \alpha - \tilde{\alpha} \). Under the rather physical assumptions \( |x_1| < V/S, 0 < x_3 < p_a, 0 < x_4 < p_a \), the control \( u \) given by

\[ u = \frac{1}{k_v} \left[ x_{5d} + \tau \left( \dot{x}_{5d} - \bar{\alpha} k_2 g_2^e p \right) \right] \]
\[ x_{5d} = -\frac{1}{\alpha g_2} \left( k_v e_p - \dot{p}_d + g_1 \right) \]

\[ \dot{\bar{\alpha}} = \frac{1}{\rho_\alpha} \left( g_2^e x_5 e_p + k_3 \bar{\alpha} \right), \quad \dot{\tilde{\alpha}} = -\frac{1}{\rho_\alpha} \left( g_2^e x_5 e_p + k_3 \tilde{\alpha} \right) \]

\[ e_p := p - \dot{p}_d, \quad p := x_3 - x_4, \]
\[ \dot{p}_d := \frac{k}{S} x_1(t) := \frac{k}{S} x_1 \left( 1 - e^{-t/\tau_1} \right) \]
\[ e_\xi := x_5 - x_{5d} \]
\[ g_1(x_1, x_2) := \frac{-2BSV x_2}{V + S x_1} \]
\[ g_2(x_1, x_3, x_4) := \alpha \frac{\sqrt{p_a - x_3}}{V + S x_1} + \frac{\sqrt{x_4}}{V - S x_1} \]
\[ g_2^2(x_1, x_3, x_4) := \frac{g_2^2(x_1, x_3, x_4)}{\alpha}. \]

when applied to (1), (2'), (3), guarantees asymptotic stability for the learning error \( \tilde{\alpha} \) and the position tracking error \( e_1 := x_1 - x_{id} \); more precisely,
$$\lim_{t \to \infty} \alpha(t) = 0, \quad \lim_{t \to \infty} e_1(t) = 0.$$  

**Proof:** By inspecting the system (1), (2'), (3), it follows that the internal states $x_1$ and $x_2$ are stable; indeed, the roots of the characteristic equation

$$m\lambda^2 + \beta\lambda + k = 0$$  

are stable roots – negative real, or complex with negative real parts – due to the viscous friction force in hydraulic cylinder. Therefore, a special care to stabilise the states $x_1$, $x_2$ is not necessary. Thus, evading the equations (1) in the backstepping procedure, this technique will be applied only with regard to the variables $x_3 - x_4$ and $x_5$. Consider now the Lyapunov like function

$$V_1 = \frac{1}{2} e_p^2 + \frac{1}{2k_2} e_s^2 + \frac{1}{2} \rho_\alpha \dot{\alpha}^2.$$  

Then, its derivative along the system (1), (2'), (3) is

$$\dot{V}_1 = e_p (g_2 x_5 + g_1 - \dot{p}_d) +$$

$$+ \frac{e_5}{k_2} \left( -x_5 + k_4 u - \dot{x}_5 d \right) - \rho_\alpha \dot{\alpha}.$$  

Now, by using (10), (11), (8), (9), we have

$$e_p (g_2 x_5 + g_1 - \dot{p}_d) = e_p (g_2 (x_5 + x_2) + g_1 - \dot{p}_d) =$$

$$= e_p \left( g_2 x_5 + g_1 - \dot{p}_d + \frac{\alpha}{\alpha} (-g_1 + \dot{p}_d - k_1 e_p) \right) =$$

$$= e_p \left[ g_2 x_5 + g_1 - \dot{p}_d + \frac{1 + \frac{\alpha}{\alpha}}{1} (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

$$= e_p \left[ g_2 x_5 + g_1 - \dot{p}_d - \frac{\alpha}{\alpha} (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

$$= -k_1 e_p^2 + g_2 e_p e_5 + \frac{\alpha}{\alpha} \left( \alpha + \frac{\alpha}{\alpha} \right) (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

$$= -k_1 e_p^2 + g_2 e_p e_5 + \frac{\alpha}{\alpha} \left( \alpha + \frac{\alpha}{\alpha} \right) (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

$$= -k_1 e_p^2 + g_2 e_p e_5 + \frac{\alpha}{\alpha} \left( \alpha + \frac{\alpha}{\alpha} \right) (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

$$= -k_1 e_p^2 + g_2 e_p e_5 + \frac{\alpha}{\alpha} \left( \alpha + \frac{\alpha}{\alpha} \right) (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

Thus

$$\dot{V}_1 = -k_1 e_p^2 + g_2 e_p e_5 + \frac{\alpha}{\alpha} \left( \alpha + \frac{\alpha}{\alpha} \right) (-g_1 + \dot{p}_d - k_1 e_p) \right] =$$

$$+ \frac{e_5}{k_2} \left( -x_5 + k_4 u - \dot{x}_5 d \right) - \rho_\alpha \dot{\alpha}.$$  

By substituting (9)–(11), one gets

$$\dot{V}_1 = -k_1 e_p^2 + \frac{1}{k_2^2} e_5^2 +$$

$$+ \alpha \left( \alpha + \frac{\alpha}{\alpha} \right) (-g_1 + \dot{p}_d - k_1 e_p) + g_2 e_p e_5 - \rho_\alpha \dot{\alpha}.$$  

The equations for the errors $e_p$, $e_5$ can be written as

$$\dot{e}_p = -k_1 e_p + g_2 e_5 + \frac{\alpha}{\alpha} (-g_1 + \dot{p}_d - k_1 e_p)$$

$$\dot{e}_5 = -\frac{e_5}{\alpha} - \alpha k_2 g_5 e_p$$

$$\dot{\alpha} = -\frac{1}{\rho_\alpha} (k_3 \alpha + g_2 x_5 e_p)$$

A tedious way to continue the proof is that of checking the asymptotic stability of the errors $e_p, e_5, \alpha$ by using the second method of Lyapunov for systems with variable coefficients (see Kalman and Bertram, 1960, Theorem 1). An alternative and very efficient procedure will be shown bellow: that of using Barbalat’s Lemma (Popov, 1973). The reasoning is as follows. Making use of the definitions (9)–(11) for $e_p, e_5$ and $\alpha$, we have $V_1(0) > 0$ when $t \to 0$ (see $x_{5d} \neq 0$). Since $V_1 \leq 0$, it is obvious that $0 \leq V_1(t) \leq V_1(0), (\forall) t > 0$, hence the positive function $V_1(t)$ is bounded and consequently $e_p, e_5$ and $\alpha$ are bounded; so, $p = p(t)$ is also bounded in the interval $\mathbb{R}_+ = [0, \infty)$. Now, taking the derivative of (14) yields

$$\dot{V}_1 = \frac{2}{k_2} \left( k_1^2 e_p^2 + \frac{1}{k_2^2} e_5^2 \right) - 2k_2 g_2 e_p e_5 + \frac{2}{\alpha} g_2 e_p e_5 +$$

$$+ \frac{2k_2 \alpha}{\alpha} e_p (k_1 e_p - \dot{p}_d + g_1) +$$

$$+ \frac{2k_3 \alpha}{\alpha} (g_2 e_p e_5 + k_3 \alpha).$$

Furthermore, $\dot{V}_1$ is bounded, provided that $g_1, g_2, \alpha$ remain bounded during the dynamical process; this condition holds, having in view the assumptions involving the variables $x_1, x_3, x_4$. So, $V_1$ is uniformly continuous (as having a bounded derivative). Let us now consider Barbalat’s Lemma:

*If the function $f(t)$ is differentiable and has a finite limit $\lim_{t \to \infty} f(t)$, and if $f$ is uniformly continuous, then $\lim_{t \to \infty} f(t) = 0$.**

Thus, Barbalat’s Lemma will be applied to show that the errors $e_p$ and $e_5$ tend to zero as time tends to infinity. Indeed, applying Barbalat’s Lemma, $\dot{V}_1 \to 0$. Hence, $e_p$ and $e_5$ tend to zero. Now, let’s look at the equations in (1), which can be rewritten as follows

$$\ddot{x}_1 + 2h \dot{x}_1 + x^2_1 = p_1$$

$$h := \frac{f}{2m}, \quad r := \frac{k}{m}, \quad p_1 = \frac{S}{m}$$
and $p$ is now seen as a bounded function of $t$, $p = p(t) = x_3(t) - x_4(t)$. The most usual case is that of complex roots with negative real parts, considered in the precedent works (Ursu and Popescu, 2003, Ursu et al., 2004, Ursu and Ursu, 2004). But, the occurrence of negative real roots is not excluded. With initial conditions $x_1(0) = \bar{x}_1(0) = 0$, the solution of (16) in this aperiodic case is

$$x_1(t) = \frac{1}{2q} e^{(q-h)t} \int_0^t e^{-h+q} \mu_1(u) du - \frac{1}{2q} e^{-(q+h)t} \int_0^t e^{-h+q} \mu_1(u) du$$

with $q^2 = r^2 - h^2 > 0$, $q$, $h$, $r$, $h-q$ positive. This variant is inherent to hydraulic servo systems owing to small viscous friction in cylinder. Define

$$p_{id}(t) := \frac{S}{m} p_d(t)$$

and let us also consider

$$\bar{x}_{id}(t) := \frac{1}{2q} e^{(q-h)t} \int_0^t e^{-h+q} \mu_{id}(u) du - \frac{1}{2q} e^{-(q+h)t} \int_0^t e^{-h+q} \mu_{id}(u) du.$$ 

(17)

Since $e_p \to 0$ when $t \to \infty$, it is clear that $p_1(t) \to p_{id}(t)$, as $t \to \infty$; this means $(\forall) \varepsilon > 0$, ($\exists$) $\delta(\varepsilon)$ such that for $t > \delta(\varepsilon)$ we have $|p_1(t) - p_{id}(t)| < \varepsilon$. Then, if $t > \delta(\varepsilon)$

$$|x_1(t) - \bar{x}_{id}(t)| \leq \frac{1}{2q} e^{(q-h)t} \int_0^t e^{-h+q} \left|p_1(t) - p_{id}(t)\right| du + \frac{1}{2q} e^{-q+ht} \int_0^t e^{-h+q} \left|p_1(t) - p_{id}(t)\right| du \leq \frac{\varepsilon}{2q} \left(e^{(q-h)t} - e^{-(q+ht)} \right) = \frac{\varepsilon}{2q} \left(1 - \frac{e^{(q-h)t}}{h-q} + \frac{1 - e^{-(q+ht)}}{h+q}\right).$$

Therefore

$$x_1(t) \to \bar{x}_{id}(t) \text{ as } t \to \infty.$$ 

(19)

(20)

Then:

$$\bar{x}_{id} = \frac{k x_4}{2 q m} \left[e^{(q-h)t} \int_0^t e^{-h+q} \left(1 - \frac{e^{-h/q}}{1 - e^{-h/q}}\right) du - \frac{1}{2q} e^{-(q+h)t} \int_0^t e^{-h+q} \left(1 - \frac{e^{-h/q}}{1 - e^{-h/q}}\right) du\right]$$

and, based on definition relations

$$q^2 = h^2 - r^2, \quad h = \sqrt{2m}$$

simple, successive calculations finally give

$$\bar{x}_{id}(t) \to x_4 \text{ as } t \to \infty.$$ 

(21)

Thus, from (20) and (21), a standard proceeding gives

$$x_1(t) \to x_{s1} \text{ as } t \to \infty$$ 

(22)

and so ends the proof. □

Let notice that if $\bar{a} = 0$, the control (7) is identical with the “nonadaptive” control given in Ursu et al., (2004), Ursu and Ursu, (2004).

3. SIMULATION RESULTS

Fourth order Runge-Kutta integrations were performed with integration step 0.003 s. Numerical data are those already given in Section 2. For the simulations, the following values of the tuning parameters were selected as suitable: $p_a = 0.0001$, $k_1 = 0.005$, $k_2 = 0.0005$, $k_3 = 1$.

Reference signal parameters were $x_{1s} = 0.255$ cm and $t_{fr} = 0.001$ s (Fig. 1), $t_{fr} = 0.001$ s (Fig. 2). Number of integration steps in Figures is 50. Thus, the presented plots exhibit the fact that proposed adaptive backstepping controller achieves good tracking performance, when compared with the ideal, nonadaptive case (Ursu and Popescu, 2003, Ursu et al., 2004, Ursu and Ursu, 2004). Indeed, the servo time constants for the system with uncertain or completely unknown parameter $c$ are very closely to the measured SMHR time constant, $\tau_c = 0.037$ s (Ursu, 1984), and to the servo time constant of the ideal, “nonadaptive” system, with total parameter knowledge, $\tau_c = 0.037$ s. The Figures show that the estimation error converges very quickly to zero. Performing various numerical simulations has demonstrated that the initial parameter estimate $\hat{c}(0)$ doesn’t affect the estimation process. Taking into account state and control limitations, the global asymptotic stability of tracking errors in Proposition cannot be stipulated. Moreover, suitable $x_{id}$ and tuning parameters $k_1 > 0$, $k_2 > 0$, $k_3 > 0$, $p_a > 0$, must be chosen to preserve these constraints. Certainly, an increased $x_{s1}$ requires a decreased claim on desired time constant $t_{fr}$.
Fig. 1. Response to position reference signal; reference signal parameters: \(x_{1s} = 0.255 \text{ cm}, \quad t_{tr} = 0.0001 \text{ s}\). Plots of variables \(x_1, u\) and \(\tilde{\alpha}/B, \quad \hat{\alpha}(0) = 1.5Bc, \quad \tau_s \approx 0.036 \text{ s}\).

Fig. 2. Response to a slower position reference signal; reference signal parameters: \(x_{1s} = 0.255 \text{ cm}, \quad t_{tr} = 0.001 \text{ s}\). Plots of variables \(x_1, u\) and \(\tilde{\alpha}/B, \quad \hat{\alpha}(0) = 0.5Bc, \quad \tau_s \approx 0.038 \text{ s}\).
4. CONCLUSIONS

In the paper, adaptive backstepping machinery is proposed to providing the control law in the case of an EHS five-dimensional mathematical model having uncertain or unknown parameters. Our main contribution consists in developing a relatively simple, intuitive scheme of adaptive backstepping, having as object the EHS mathematical model. Thus, the designer of control law is able to avoid a complicated geometrical approach described in Kanellakopoulos et al. (1991). Worthy noting, the law of error estimation is proven to be assimototically one. The full state information was considered available.

Other contributions of the paper are: 1) illustrating how the main theory can be brought or adapted to design practice as defined by a given mathematical model and showing that the backstepping controllers are able to work with a complex plant such as EHS; 2) CLFs synthesis by using of Barbalat's Lemma; 3) developing the idea of partitioning the state system into two subsystems: a first one stable, and a second one taken as framework of control synthesis

The simulation studies attest good tracking performance in the presence of step signals. Furthermore, a close correspondence between the theoretical predictions and the experimental result (Ursu, 1984) has been found.

REFERENCES


