# FUZZY CONTROL FOR ROBOT'S DRIVINGS

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**Abstract:** The paper presents the possibility to control a robot arm using fuzzy systems. A three-phase induction motor is controlled using a vectorial control and fuzzy logic controllers with three and five linguistic labels. It is realized an analogy between a PI controller and a fuzzy controller. The simulations using such of controllers highlighted the advantages of using a fuzzy controller: increasing responding time of induction motor speed and respectively the statoric current limitation of the used induction motor.

***Key words:*** fuzzy logic controller, typical surface of the controller, control surface**,** Park transformation.

## 1. MATHEMATICAL DRIVING MODEL

Let us consider a three-phase induction motor with $a_s$, $b_s$ and $c_s$ the stator phases and $a_r$, $b_r$ and $c_r$ the rotor phases (fig. 1a). The time variable electrical angle $\alpha$, defines the instantaneous position between magnetic axes of $a_s$ and $a_r$ phases chosen as reference axes, and $d$ axis of the orthogonal axes reference system $d-q$. The angles $\alpha_s$ and $\alpha_r$ are the angles between a stator phase respectively a rotor phase with $d$ axis of the orthogonal reference system $d-q$.

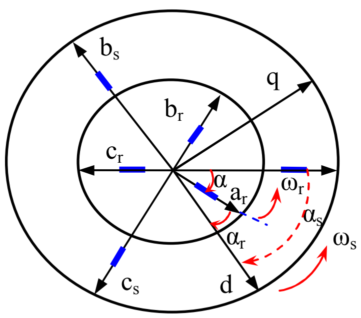



Fig. 1a Explanatory for the position of the system of statoric and rotoric axes for an induction motor

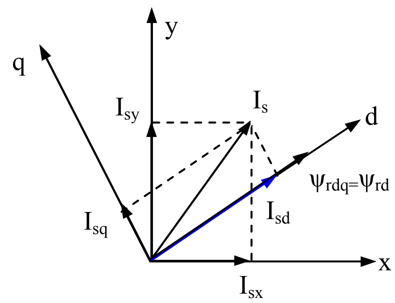



Fig. 1b Explanatory for the position of reference system related to rotor flux

where $(x, y)$ is the axis system related to stator and $(d, q)$ the axis system related to motor rotating field.

The induction motor equations, without saturation are

$$U_{as} = \frac{d\psi_{as}}{dt} - R_s \cdot i_{as}\,; \quad U_{bs} = -\frac{d\psi_{bs}}{dt} - R_s \cdot i_{bs}$$
$$U_{cs} = -\frac{d\psi_{cs}}{dt} - R_s \cdot i_{cs}\,; 0 = \frac{d\psi_{ar}}{dt} + R_r \cdot i_{ar}$$
$$0 = \frac{d\psi_{br}}{dt} + R_r \cdot i_{br}\,; \quad 0 = \frac{d\psi_{cr}}{dt} + R_r \cdot i_{cr} \qquad (1)$$

where $R_s$ and $R_r$ are the resistances for one stator phase respectively one rotor phase; $U_{mn}$, $i_{mn}$, $\psi_{mn}$ are the voltages, the currents and the flux (m index denotes the a, b or c phase and n index denotes the stator or the rotor).

The relations between fluxes and currents are given by the equations (2) (Câmpeanu, 1998) where $L_{as}$, $L_{ar}$ are the inductances for one stator phase respectively one phase inductances rotor; $L_{mas}$, $L_{mar}$ are the mutual inductances between two stator phases respectively rotor phases; $L_{mrs}$ is maximum mutual inductances.

$$
\begin{bmatrix}\psi_{as}\\ \psi_{bs}\\ \psi_{cs}\\ \psi_{ar}\\ \psi_{br}\\ \psi_{cr}\end{bmatrix}=
\begin{bmatrix}
L_{as} & L_{mas} & L_{mas} & L_{mrs}\cdot\cos\alpha & L_{mrs}\cdot\cos(\alpha-\frac{4\pi}{3}) & L_{mrs}\cdot\cos(\alpha-\frac{2\pi}{3})\\
L_{mas} & L_{as} & L_{mas} & L_{mrs}\cdot\cos(\alpha-\frac{2\pi}{3}) & L_{mrs}\cdot\cos\alpha & L_{mrs}\cdot\cos(\alpha-\frac{4\pi}{3})\\
L_{mas} & L_{mas} & L_{as} & L_{mrs}\cdot\cos(\alpha-\frac{4\pi}{3}) & L_{mrs}\cdot\cos(\alpha-\frac{2\pi}{3}) & L_{mrs}\cdot\cos\alpha\\
L_{mrs}\cdot\cos\alpha & L_{mrs}\cdot\cos(\alpha-\frac{2\pi}{3}) & L_{mrs}\cdot\cos(\alpha-\frac{4\pi}{3}) & L_{ar} & L_{mar} & L_{mar}\\
L_{mrs}\cdot\cos(\alpha-\frac{4\pi}{3}) & L_{mrs}\cdot\cos\alpha & L_{mrs}\cdot\cos(\alpha-\frac{2\pi}{3}) & L_{mar} & L_{ar} & L_{mar}\\
L_{mrs}\cdot\cos(\alpha-\frac{2\pi}{3}) & L_{mrs}\cdot\cos(\alpha-\frac{4\pi}{3}) & L_{mrs}\cdot\cos\alpha & L_{mar} & L_{mar} & L_{ar}
\end{bmatrix} \quad (2)
$$

*Park transformation* turns the stator and rotor windings into orthogonal equivalent windings. Thus, the $a_s$, $b_s$ and $c_s$ windings are changed by two equivalent windings $d_s$ and $q_s$, and the rotor windings $a_r$, $b_r$ and $c_r$ by the equivalent windings $d_r$ and $q_r$.

Choosing a reference system related to the rotary field such as

$$\frac{d\alpha_s}{dt}=\omega_s \quad \text{and} \quad \frac{d\alpha_r}{dt}=\omega_r=p_s\cdot\omega \quad (3)$$

where $p_s$ are the number of stator poles pairs and $w_r$, $w_s$ are the mechanical and electrical angular speed, the equations (1) becomes:

$$\frac{d\psi_{sd}}{dt}=\omega_s\cdot\psi_{sq}-R_s\cdot i_{sd}+u_d$$
$$\frac{d\psi_{sq}}{dt}=-\omega_s\cdot\psi_{sd}-R_s\cdot i_{sd}+u_q$$
$$\frac{d\psi_{rd}}{dt}=(\omega_s-p_s\cdot\omega)\cdot\psi_{rd}-R_r\cdot i_{rd}$$
$$\frac{d\psi_{rq}}{dt}=-(\omega_s-p_s.\omega)\cdot\psi_{sq}-R_r\cdot i_{rq} \quad (4)$$

The differential equation of the movement for a rigid coupling is

$$J\cdot\frac{d\omega}{dt}=M_e-M_s=M_e-M_0-K_1\cdot\omega-K_2\cdot\omega^2 \quad (5)$$

where $M_0$ is the constant component part of the static torque $M_s$; $K_1$ and $K_2$ are proportional constants; $M_e$ is the electromagnetic torque ; $\omega$ is the angular speed. With the assumption that the static torque that depend of the square speed is neglected, the equation (5) (Popescu 2002) becomes

$$J\cdot\frac{d\omega}{dt}=M_e-M_s=M_e-M_0-K_1\cdot\omega \quad (6)$$

The electromagnetic torque $M_e$ can be expressed by currents

$$M_e=p_s\cdot L_m(i_{sq}\cdot i_{rd}-i_{sd}\cdot i_{rq}) \quad (7)$$

The relations between fluxes and currents in *Park* model are

$$\psi_{sd}=L_s\cdot i_{sd}+L_m\cdot i_{rd}$$
$$\psi_{sq}=L_s\cdot i_{sq}+L_m\cdot i_{rq}$$
$$\psi_{rd}=L_m\cdot i_{sd}+L_r\cdot i_{rd}$$
$$\psi_{rq}=L_m\cdot i_{sq}+L_r\cdot i_{rq} \quad (8)$$

where

- $\psi_{sd}$, $\psi_{sq}$, $\psi_{rd}$, $\psi_{rq}$ are the stator and rotor fluxes along the axes $d$ and $q$;
- $i_{sd}$, $i_{sq}$, $i_{rd}$, $i_{rq}$ are the stator and rotor currents along the axes $d$ and $q$;
- $L_s$ and $L_r$ are the inductivity of the stator and rotor windings;
- $L_m$ is the periodical mutual inductivity between the stator and the rotor, such as

$$L_s=L_{as}-L_{mas}$$
$$L_r=L_{ar}-L_{mar}$$
$$L_m=3/2\cdot L_{mrs} \quad (9)$$

Chosen a reference position such as the axis d is along the rotor flux vector $\psi_r$ (fig.1b), then the vectorial control will allow the rotor flux regulation by controlling the current $i_{sd}$ and the electromagnetic torque developed by the motor.

If we consider an induction motor with short circuit rotor ($u_r=0$), and if the rotor flux vector is along $d$ axis of the $(d,q)$ axes system, then the $q$ axis flux component part is null.

$$\psi_r=\psi_{rdq}=\psi_{rd} \quad \Rightarrow \psi_{rq}=0\,. \quad (10)$$

and the equations (4) becomes

$$\frac{d\psi_{sd}}{dt}=\omega_s\cdot\psi_{sq}-R_s\cdot i_{sd}+u_d$$
$$\frac{d\psi_{sq}}{dt}=-\omega_s\cdot\psi_{sd}-R_s\cdot i_{sd}+u_q$$
$$\frac{d\psi_{rd}}{dt}=-R_r\cdot i_{rd}$$
$$0=-(\omega_s-p_s.\omega)\cdot\psi_{sq}-R_r\cdot i_{rq} \quad (11)$$

Replacing the fluxes in equations (11) by theirs expression, these becomes

$$u_d=R_s\cdot i_{sd}+L_s\frac{di_{sd}}{dt}-\omega_s\cdot L_s\cdot i_{sq}+L_m\frac{di_{rd}}{dt}-\omega_s\cdot L_m\cdot i_{rq}$$
$$u_q=\omega_s\cdot L_s\cdot i_{sd}+R_s\cdot i_{sq}+L_s\frac{di_{sd}}{dt}+\omega_s\cdot L_m\cdot i_{rd}+L_m\frac{di_{rq}}{dt}$$

$$0 = L_m \frac{di_{sd}}{dt} - (\omega_s - p_s \cdot \omega) L_m \cdot i_{sq} + R_r \cdot i_{rd} + L_r \frac{di_{rd}}{dt} - (\omega_s - p_s \cdot \omega) \cdot L_r \cdot i_{rq}$$

$$0 = (\omega_s - p_s \cdot \omega) L_m \cdot i_{sd} + L_m \frac{di_{sq}}{dt} + (\omega_s - p_s \cdot \omega) \cdot L_r \cdot i_{rd} + R_r \cdot i_{rq} + L_r \frac{di_{rq}}{dt} \quad (12)$$

and the electromagnetic torque is

$$M_e = p_s \cdot \frac{L_m}{L_r} \cdot \psi_{rd} \cdot i_{sq} . \quad (13)$$

The most important perturbance for a driving system is the static torque $M_s$, for rotation movement, respectively static force $F_s$, for translation movement. For driving is highlight a constant component part of static force $F_0$ (bearing dry friction), a speed proportionally component (viscous friction) and another component, square speed proportionally.

Static force for driving is

$$F_s = F_0 + K_1 \cdot v + K_2 \cdot v^2 . \quad (14)$$

General equation for rotation movement, respectively for translation movement (for speed value different from zero) is

$$M_e = M_s + J \frac{d\omega}{dt} \quad (15) \qquad F_e = F_s + m \frac{dv}{dt} \quad (16)$$

where $M_e$ and $F_e$ are electromagnetic torque, respectively electromagnetic force.

## 2. FUZZY LOGIC CONTROLLER

For the robots control based upon driving motor speed regulation, for example the robot arm, starting from movement equation (6) and the electromagnetic torque equation (7) it could be implemented a cvasi-PI standard fuzzy controller (fig.2).

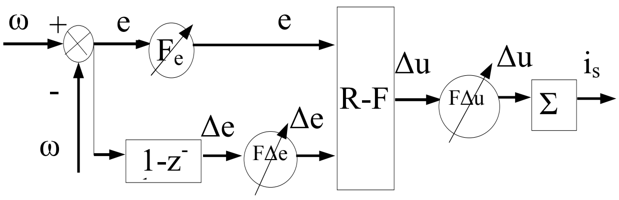



Fig.2 Fuzzy logic controller

A position transducer placed on the robot offers informations about the reference angular speed $\omega_p$. The output $u$ of the speed controller will act on the reference electromagnetic torque, which is proportionally with the current $i_{sq}$ when the flux is invariable in time. The amplifiers of the input and output variables $F_e$, $F \cdot \Delta e$, $F \cdot \Delta u$ situated on the input and output of the fuzzy logic controller are called scale factors. They allow to change the controller region of operation without modify its structure.

The variables with an index n, are the normalized variables of the controller. Denotes with $T_e$ the sampling period, with $e$ the speed error defined by the difference

$$e(k \cdot T_e) = \omega_p(k \cdot T_e) - \omega(k \cdot T_e) \quad (17)$$

and with $\Delta e$ the change of the error, then the output of the controller is

$$i_{sq}(k \cdot T_e) = i_{sq}[(k-1) \cdot T_e] + \Delta u(k \cdot T_e)$$

The inference rules establish the behaviour of the controller and by defuzzification is selected one representative crisp element.

Using in fuzzy description of each input variable the following three linguistic labels: N-negative, Z-zero, P-positive, defined by triangular and trapezoidal membership functions, is easy to implement the fuzzy stage (fig. 3 a, b).

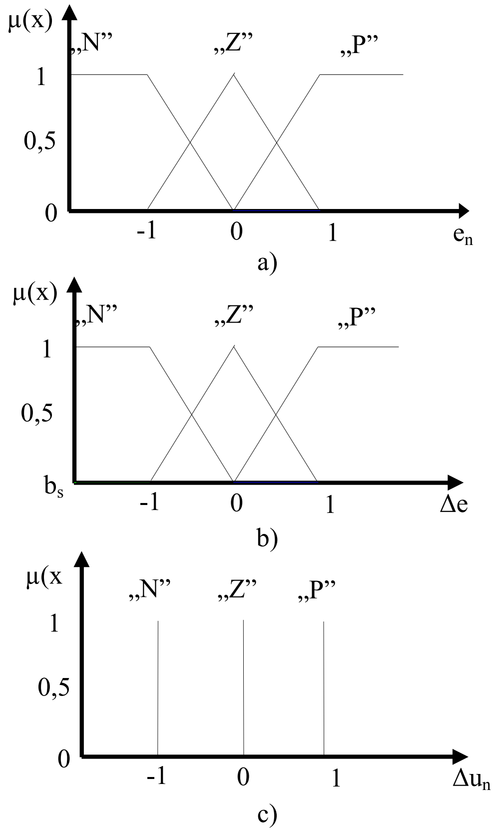



Fig. 3 Membership functions of input and output variables of fuzzy logic controller

The same linguistic labels defined by singleton membership functions are chosen for the output variable, $\Delta u_n$, (fig. 3c). The rule base is created by taking account the pseudo-fuzzy properties of the PI quasi-continual controller.

For calculation of the crisp $i_{sq}$ value (the defuzzification of the vague information), is not applied the Mean of Maxima method used when the membership functions of output variables are singleton. In this case is used for defuzzification the Center of Area (COA) method. The rule-base is defined by the matrix (Yager et al. 1994).

| $\Delta u_n$ | | $\Delta e_n$ | | |
|---|---|---|---|---|
| | | N | Z | P |
| $e_n$ | N | N | N | Z |
| | Z | N | Z | P |
| | P | Z | P | P |

with graphical representation presented in fig. 4a, if it is consider a Mamdani controller and for the scale factors Fe, $F \cdot \Delta e$ and $F \cdot \Delta u$ , the values 100, 10 and 5.

The typical surface of the controller corresponding to some real outputs $\Delta u$ , function of the inputs e and $\Delta e$ , when these cover the base set, has two areas:

- A peripherical, smooth parallel area with e and $\Delta e$ , (fig.4a) axes that corresponds to a range where one of the input variables are saturated.

In this area the inference motor evaluates just one or maximum two rules.

- A second central area (in normalized coordinates- fig.4b) $e_n$ and $\Delta e_n \in [-1,1]$, that could be divided in 4 dials and represents the proper central area of the controller.

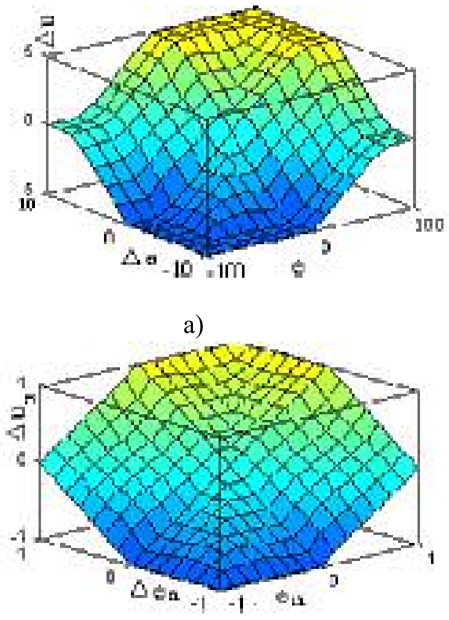



Fig. 4 The control surface of fuzzy logic controller: a) in actual values; b) normalized

Chosen a bigger number of linguistic labels (for example five) the base-rule will be greater and the rating time will increase. If for controller with three linguistic labels, the control is made by the factors scale, for a controller with 5 linguistic labels, this is realized by intuition, by modifying the form and the distribution of the input or output membership functions on the base set.

It is possible that, using an optimize proceeding by Taguchi method, (this consists in identifying the system, starting from its ramp response in open circuit), to appreciably reduce the number of the attempts for establish the best control (Popescu 2000). For input variables $e_n$ and $\Delta e_n$ will be chosen a uniformly distribution of membership functions on base set (fig.5) and for the output $\Delta u_n$ , the membership functions that is corresponding to Ns (negative small) and Ps (positive small) linguistic labels are close to the membership function of Z linguistic label.

The control will be realized by intuition, known that in such of cases the real output of the controller will reach a limiting value (of saturation). When the output variable is very close of specified speed, the linguistic labels Ns and Ps will be requested and because their membership functions are very close to Z (zero) linguistic label, the response of the controller will be slower. This thing needs the multiply with two of the output variable regarding its value obtained in the case of the controller with three linguistic labels.

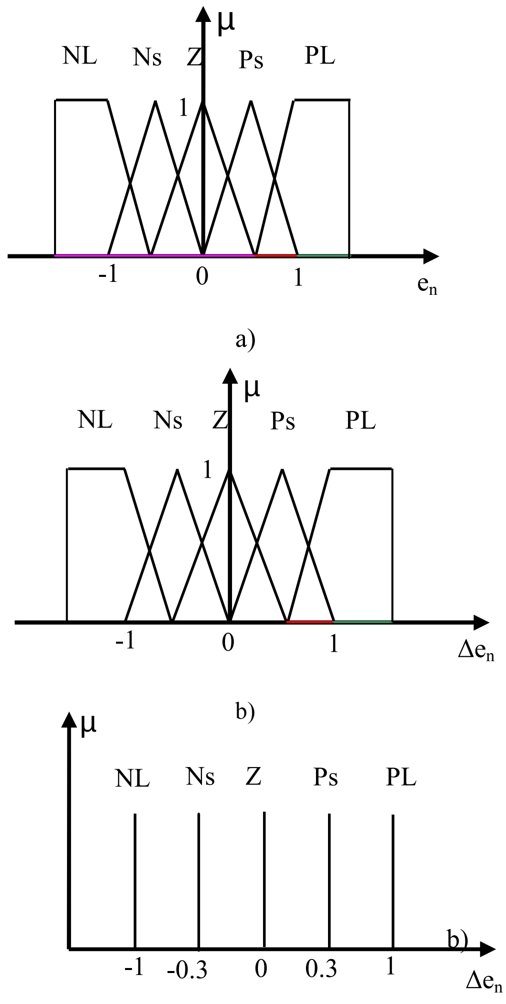
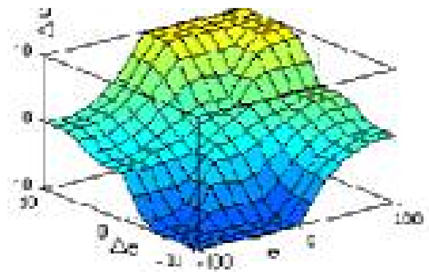



Fig. 5 The graph of the membership functions adjacent to input and output variables

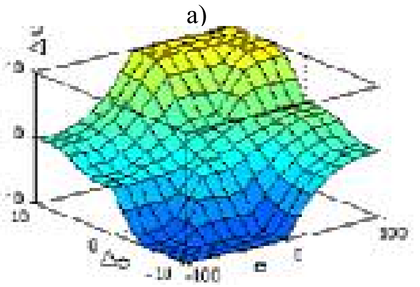
The control surface is presented in fig.6a (considering a *Mamdani controller*), on the base of the inference matrix.

| $\Delta u_n$ | | $\Delta e_n$ | | | | |
|---|---|---|---|---|---|---|
| | | NL | Ns | Z | Ps | PL |
| $e_n$ | NL | NL | NL | Ns | Ns | Z |
| | Ns | NL | Ns | Ns | Z | Ps |
| | Z | NL | Ns | Z | Ps | PL |
| | Ps | Ps | Z | Ps | Ps | PL |
| | NL | Z | Ps | Ps | NL | PL |

The characteristic surface of the controller, in normalized coordinates, is presented in figure 6b. By analogy to the controller with the base set consisted of three linguistic labels, there are an important number of surfaces and those surfaces corresponding to two and four dials they are not smooth any more.



a)



b)

Fig. 6 Control surfaces of fuzzy logic controller
a) in actual values; b) normalized

## 3. CONTROL DESIGN

Stuctural diagram of speed control presented in fig.2, in MATLAB/Simulink is like in fig. 7. Induction driving will be describe by movement equation and electromagnetic torque equation, with a block structural diagram like in fig. 8, considering the fan static torque. Known that the control speed is realized actioning on the scale factors, for the synthesis of the controller will be chosen those scale factors witch offer a fast speed responding time and an accepted starting current.

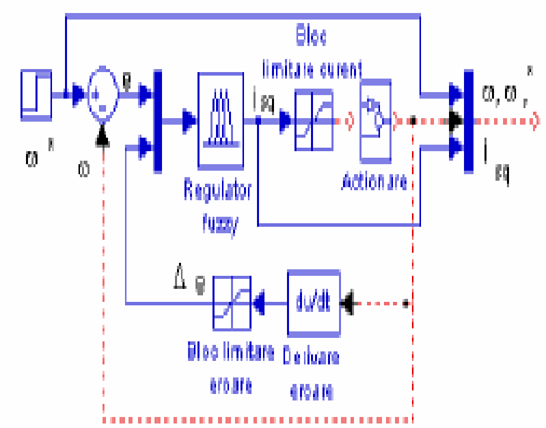



Fig. 7 Structural diagram of driving speed control with fuzzy logic controller, in Simulink

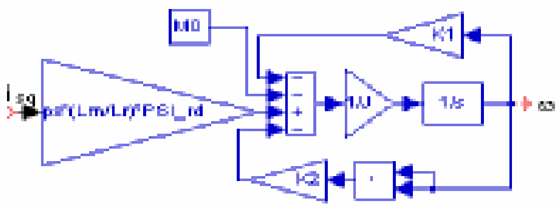



Fig. 8 Driving structural diagram, in Simulink with static torque linear depending of speed and also with square of speed

For the driving with the dates: $L_m$=0,1126H, $L_r$=0,1154H, $p_s$=2, J=4.10$^{-3}$kgm$^2$, $\psi_{rd} = 0,3\text{Wb}$, $K_1$=2,5.10$^{-4}$ Nms (Popescu 2002) using a fuzzy logic controller with three linguistic labels and the scale factors Fe, $F \cdot \Delta e$, $F \cdot \Delta u$ having the values 100, 10 and 5, and considering the sampling period $T_e$=10$^{-3}$s, the only values that are obtained for an unload start-up are presented in fig. 9.

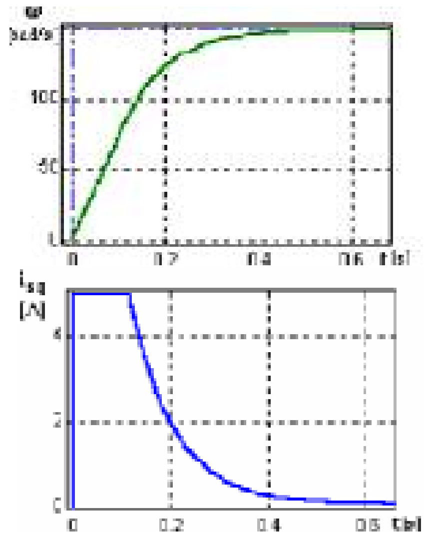


Fig. 9 Driving unload start-up: up to $\omega_* = 150\text{rad} \cdot \text{s}^{-1}$ :
a) speed shape; b) stator current shape

For the previous values of the scale factors and considering a type fan torque, then, for a start, the speed of the drive is a little bellow prescribed value and the current is 8A (fig.10). An increase of scale factor $F \cdot \Delta u$ of the controller does not improve the performances (the current will increase over the prescribed value). Same time, the sensivity of the controller related to external perturbances will increase, which make it practically useless.

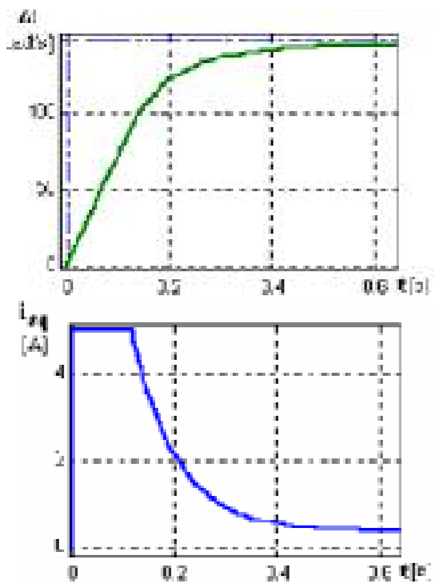
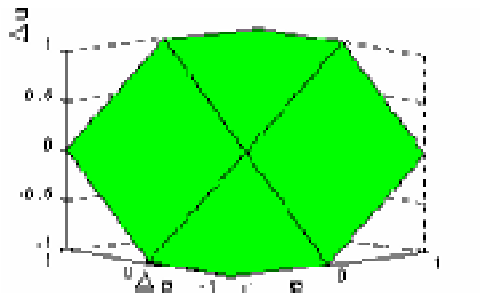


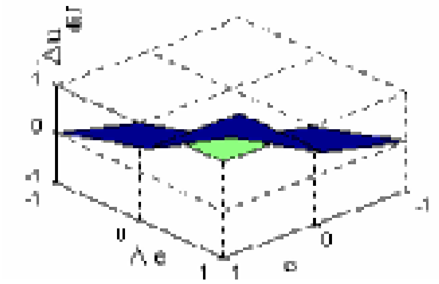
Fig. 10 Driving start-up ($K_2$=0,1.10$^{-4}$ Nms)
a) speed shape; b) stator current shape

A carrefull analysis in tri-dimensional space of the control surface of fuzzy controller and PI conventional controller (fig. 11a), shows that in dials 2 and 4 the difference between controls $\Delta u_{dif}$ (or between control surfaces corresponding to the controllers) is practically null (fig. 11b). The normalized control surfaces of fuzzy logic controller corresponding to four dials are not all of them smooth as in case of conventional PI controller but with peaks in dials 1 and 3.

Thus, the fuzzy logic controller with three linguistic labels will behave like a conventional PI controller if it is not used in the two dials (1 and 3).



a)

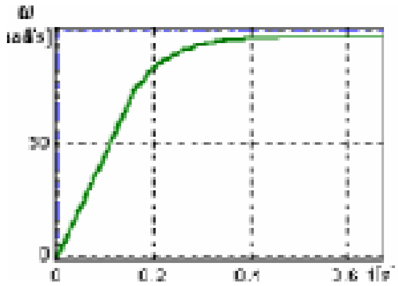


b)

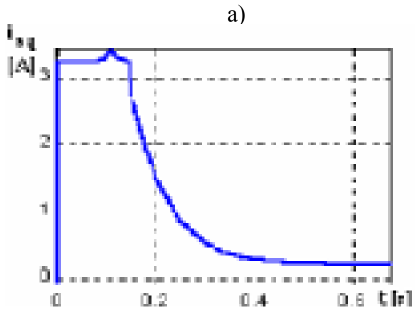
Fig. 11 Control surfaces:
a) corresponding to PI controller;
b) corresponding to difference between control surface of fuzzy logic controller and PI controller

It can be developed a Sugeno fuzzy logic controller (called also procedural controller sincethe antecedent is symbolic and the consequent is a real constant or a polynomial expression), which offers the same response as the conventional PI controller, in so called modal points (corresponding to triangular membership functions). The inference matrix will be a symmetrical matrix to its diagonal.

In case of Mamdani fuzzy logic controller, if membership functions symetrical with respect to modal values would be chosen as for output variable, rectangular and identical as width then it would be obtained a linear interpolation between these modal values.



a)



**b)**

Fig. 12 Driving on-load start-up:
a) speed shape; b) stator current shape

## 4. CONCLUSIONS

The advantage of fuzzy logic controller will disappear when comparing to a wind-up PI controller, knowing that this is working in a linear area.

On the other hand, an wind-up PI-controller does not make any problems when the output variable reach the saturation value since the signal corresponding to the difference between limited output and unlimited output is once more fed to the controller for desaturation.

When the input signal into controller is small (low amplitude step signal or small prescribed speed-for example 100 rad.s$^{-1}$), the performances of the controller are quite affected (the current decrease- fig.12b). The controller saturates which allow a quicker response to step signals.

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