A CONTROL DESIGN FOR A TYPICAL FLUTTER SUPPRESSION PROBLEM

Adrian Stoica
POLITEHNICA University Bucharest; Faculty of Aerospace Engineering, Polizu 1-6, Bucharest, Romania
e-mail: amstoica@fx.ro

Marius Stoia-Djeska
POLITEHNICA University Bucharest; Faculty of Aerospace Engineering, Polizu 1-6, Bucharest, Romania
and STRAERO S.A., Iuliu Maniu 220, Bucharest
e-mail: mstoia@aero.pub.ro

Achim Ionita
STRAERO S.A., Iuliu Maniu 220, Bucharest, Romania
e-mail: ionita@aero.incas.ro

Abstract: Aeroelastic problems of light weight structures of modern aerospace vehicles are the result of interactions between aerodynamics, structural and inertial forces. The most dangerous and also investigated aeroelastic phenomenon is the flutter, ***i.e.*** a self-excited oscillation of the elastic structure under the action of the aerodynamic loads. Flutter instabilities often exhibit an explosive behavior that causes a sudden change in the stability of the aeroelastic system despite only a small change in flight condition and lead to the catastrophic failure of the structure. Consequently, flutter prevention is an important design task. The aim of this paper is to present the design of a control able to suppress the flutter instability of a typical section with a trailing-edge control flap in a prescribed speed range. The mathematical model of the aeroelastic problem is based on the Lagrange equations of motion for the structural dynamics and on a quasi-steady approach of the generalized unsteady incompressible aerodynamic forces. The state vector includes the angle of attack, the normal displacement and their rates. The purpose of the control law design that is presented in this paper is to augment the damping of the structural modes over a wide range of dynamic pressure. The proposed design procedure uses a polytopic approximation of the aeroelastic system subjected to parametric uncertainty. The robust design problem considered here consists in determining a state-feedback control such that the closed-loop system has the poles in a prescribed domain for a large interval of variation of the dynamic pressure.

Key words: flutter, robust control, parametric uncertainty

INTRODUCTION

Aeroservoelastic problems of aircraft light weight structures are the result of interactions between aerodynamics, structural, inertial, actuation and control system dynamics. The stability properties of the aeroservoelastic dynamics must be investigated to determine a flight envelope that is clear of instabilities for new aircraft designs (Lind 1999). The result of an aeroservoelastic stability analysis is the determination of a flight envelope within which the aerospace vehicle equipped with servo controls may safely operate (Lind 1999). In contradistinction with aeroservoelasticity, the aeroelasticity considers only the interaction of aerodynamic, inertial and structural forces (Dowell 1995, Lind 1999). The aeroelastic phenomena may not only strongly influence the structural dynamics and dynamic flight stability, but also the overall performance and controllability of the aircraft. Undoubtedly, the most important aeroelastic phenomenon is the flutter, i.e. a self-excited oscillation of the elastic structure under the action of the aerodynamic loads (Dowell 1995). Flutter instabilities often exhibit an explosive behavior that causes a sudden change in stability despite only a small change in flight condition. However, we notice here that the mechanisms associated with flutter may be different from those associated with aeroservoelastic instabilities. Until now, several methods for characterizing aeroservoelastic instabilities and/or flutter conditions have been developed and are continually improved.

In this paper we will refer only to aeroelastic phenomena which can be avoided or kept under control by some active measures. Since flutter tends to occur in almost all of the flight regimes and leads to the catastrophic failure of the structure, flutter avoidance is essential in getting certificate for newly designed aerospace vehicles. Further, even the aeroelastic vibrations characterized by weak structural damping are not critical for the structure, they reduce the fatigue life of the structure and thus increase the operational costs and also may lead to catastrophic failures of the aircraft components. Consequently, nowadays the active control is used both for flutter prevention and structural load alleviation.

One of the main purposes of the control system designed in the present paper is to augment the damping of the structural modes over a wide interval of dynamic pressure. The problem is frequently addressed in the control literature, see ***e.g.*** (Lind 1999, Buschek 1993, Carl et al. 2000) and their references. Its difficulty consists in the complexity of the dependence of aeroelastic model with respect to the uncertain parameters. Although the uncertainty is essentially parametric, some solutions have been derived using the dynamic representation of the modeling uncertainty

(Lind 1999) for which effective design techniques are available. The μ-synthesis methodology is also frequently used, see *e.g.* (Lind 1999, Buschek 1993), due to its capability to handle simultaneously parametric and dynamic uncertainty. The design procedure proposed in the present paper uses a polytopic approximation of the aeroelastic system subjected to parametric uncertainty. The robust design problem considered here consists in determining a state-feedback control such that the closed-loop system has the poles in a prescribed domain for a large interval of variation of the dynamic pressure.

This paper is organized as follows: the dynamic model of the aeroelastic pitch-plunge system is given in the second section. The third section presents the robust stabilization problem and numerical results. Some concluding remarks are given in the fourth section.

EQUATIONS OF MOTION

The aim of the paper is to present the design of a control able to suppress the flutter instability of a typical section with a trailing-edge control flap in a prescribed speed range. Synthesis of controllers requires a good analytical model of the system be available. Therefore, we have adopted in this work a simplified two degrees of freedom representation of an airplane wing as the aeroelastic system for which flutter suppression is desired. The model is the well-known typical section with trailing edge flap (Dowell 1995, Ko 1997, Lindt 1999). The two degrees of freedom are the plunge position h of the elastic center and the leading edge up pitch angle α about this center. The model is presented in Figure 1. These degrees of freedom correspond to the bending and torsion displacements of a high aspect ratio wing under real loads, (Dowell 1995).

Although the flutter phenomenon is a complicated one, the present approach is based on a simple linear aeroelastic model. Thus, we assume that the two displacements are small and that the model parameters do not depend on them. Further, we assume that the stiffness of the flap hinge is very large compared with the constant stiffness k_h of the plunging motion and with the constant stiffness k_α associated with the pitching motion. We also assume that the flap moves instantaneously so that the contribution of the flap motion to the kinetic energy of the typical section will be neglected. Due to this last assumption, the flap can still be used as a control surface but its deflection β does not represent an additional degree of freedom for the stability and remains only a control parameter of the aeroelastic system's dynamics. Finally, the external forces acting on the wing section are due only to the pressure difference distribution on the typical section.

The equations of motion describing the plunge and pitch during an aeroelastic response are derived starting from the Lagrange equations (Dowell 1995):

$$\frac{d}{dt}\left(\frac{\partial(T-U)}{\partial \dot{\underline{q}}}\right)-\frac{\partial(T-U)}{\partial \underline{q}}=\underline{Q}_q \tag{1}$$

where the set of the generalized coordinates is $\underline{q}=\{h,\alpha\}^T$. The corresponding generalized forces are $Q=\{-L,M\}^T$ where L and M are the aerodynamic lift and moment, respectively, and given later below.

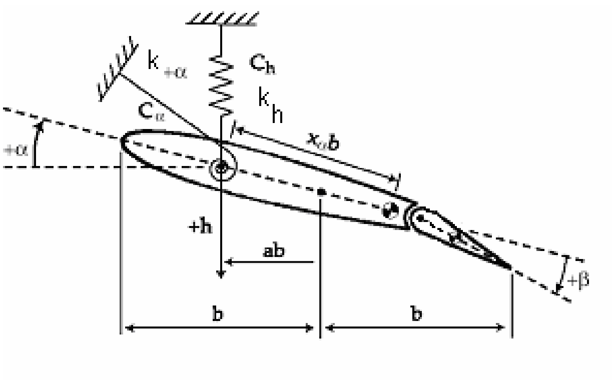

Figure 1: The typical section model

. The kinetic energy is given by:

$$T=\frac{1}{2}m\dot{h}^2+mx_\alpha b\dot{h}\dot{\alpha}+\frac{1}{2}I_\alpha\dot{\alpha}^2 \tag{2}$$

and the potential energy of strain is:

$$U=\frac{1}{2}k_\alpha\alpha^2+\frac{1}{2}k_h h^2 \tag{3}$$

Hence, the governing equations of the aeroelastic system under considerations are:

$$\begin{bmatrix} m & m x_\alpha b \\ m x_\alpha b & I_\alpha \end{bmatrix}\ddot{\underline{q}}+\begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\underline{q}=\begin{Bmatrix} -L \\ M \end{Bmatrix} \tag{4}$$

where m is the mass of the typical section, I_α is the mass moment of inertia about the elastic axis, x_α is the dimensionless distance between the elastic center and center of mass and b is the semichord length. Following (Lind 1999), in the above system on can introduce a structural dissipation through the structural damping coefficients in pitch and plunge, c_h and c_α, so that the equations become:

$$\begin{aligned}\begin{bmatrix} m & m x_\alpha b \\ m x_\alpha b & I_\alpha \end{bmatrix}\ddot{\underline{q}}+\begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\dot{\underline{q}}+ \\ +\begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\underline{q}=\begin{Bmatrix} -L \\ M \end{Bmatrix}\end{aligned} \tag{5}$$

We assume that the aerodynamic lift and moment are determined according to a linear quasi-steady model (Dowell 1995, Lind 1999):

$$L=\rho U^2 b c_z^\alpha\left(\alpha+\frac{\dot{h}}{U}+\left(\frac{1}{2}-a\right)b\frac{\dot{\alpha}}{U}\right)+\rho U^2 b c_z^\beta \beta \tag{6}$$

and

$$M=\rho U^2 b^2 c_m^\alpha\left(\alpha+\frac{\dot{h}}{U}+\left(\frac{1}{2}-a\right)b\frac{\dot{\alpha}}{U}\right)+\rho U^2 b^2 c_m^\beta \beta \tag{7}$$

where ρ is the density of the air, U is the velocity of the free air stream, c_z^α, c_m^α are the lift and moment coefficients per angle of attack and c_z^β, c_m^β are the lift and moment coefficients per flap deflection and a is the dimensionless distance between the midchord and the elastic axis.

After substituting the lift and moment into the equations of motion we obtain the final system of two coupled second order differential equations:

$$\begin{bmatrix} m & m x_a b \\ m x_a b & I_a \end{bmatrix} \ddot{\underline{q}} + \begin{bmatrix} c_h + rUbc_z^a & rUb^2 c_z^a\left(\frac{1}{2} - a\right) \\ rUb^2 c_m^a & c_a - rUb^3 c_z^a\left(\frac{1}{2} - a\right) \end{bmatrix} \dot{\underline{q}} + \begin{bmatrix} k_h & rUb^2 c_z^a \\ 0 & k_a - rU^2 b^2 c_m^a \end{bmatrix} \underline{q} = \begin{Bmatrix} -rU^2 bc_z^b \\ rU^2 b^2 c_m^b \end{Bmatrix} b \qquad (8)$$

The nominal values for the parameters of the aeroelastic system (8) are given in Table 1. These values are taken from literature (Lind 1999) and correspond to a model used for wind tunnel flutter tests.

Table 1. Nominal parameters for the equation of motion

$U = 6\,m/s$
$a = -0.6$
$m = 12.387\,kg$
$b = 0.135\,m$
$I_a = 0.065\,Kg\ m^2$
$x_a = 0.2466$
$k_h = 2844.4\,n/m$
$k_a = 3.525\,Nm/rad$
$c_h = 27.43\,kg/s$
$c_a = 0.036\,Kg\ m^2/s$
$c_z^a = 6.28$
$c_m^a = -0.635$
$c_z^b = 3.358$
$c_m^b = 12.387$

ROBUST STATE-FEEDBACK DESIGN AND NUMERICAL RESULTS

In this section a state-feedback control is designed in order to ensure an augmented damping of the aeroelastic modes for a wide interval of airspeed. The dependence of the eigenvalues to the open-loop system with respect to the airspeed U is shown in Figure 2.

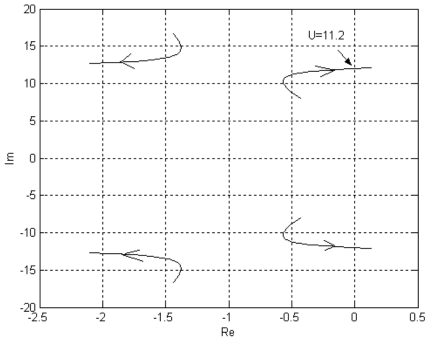

Figure 2: The open-loop eigenvalues for $U \geq 6\,m/s$

The eigenvalues locus for airspeeds larger than the nominal value $U = 6\,m/s$ indicate that the critical airspeed at which the open-loop system becomes unstable is $U_{crt} = 11.2\,m/s$. This is an acceptable upper limit of the airspeed for the considered case study. The time responses of the plunge position h corresponding to the uncontrolled system (8), namely for $\beta = 0$ at $U = 6\,m/s$ and at $U = 11\,m/s$ and with the arbitrary chosen initial conditions $h(0) = 0\,m$, $\dot{h}(0) = 0\,m/s$, and $\alpha(0) = 0.1\,rad$, $\dot{\alpha}(0) = 0\,rad/s$ are illustrated in Figures 3 and 4, respectively.

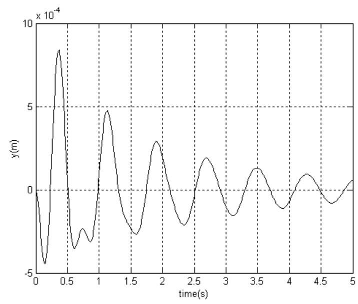

Figure 3: Time response of $h(t)$ in the open-loop case at $U = 6\,m/s$

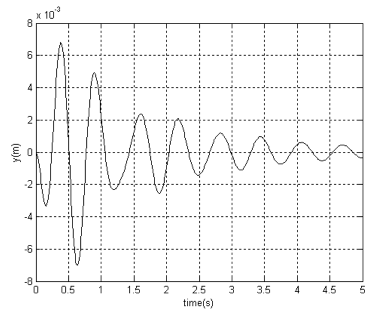

Figure 4: Time response of $h(t)$ in the open-loop case at $U = 11\,m/s$

The above plots show a weak damping on the whole interval of variation of the airspeed. A similar behavior can be noticed for the pitch angle α. In order to augment the damping of the considered flexible structure over the whole range of variation of the airspeed one can use a state-feedback control law, namely

$$b = k_1 h + k_2 \dot{h} + k_3 a + k_4 \dot{a} \qquad (9)$$

The gains $k_i, i = 1, \ldots, 4$ are determined such that the closed-loop system is stable at all operating conditions between the nominal and the critical airspeed and such that the damping of modes is above 0.05. Numerical simulations showed that these specifications can be accomplished on a wide range of independent variation of U and ρ but with high gain control laws. In order to prevent the control saturation, the stability domain $\mathbf{D}$ for the pole placement to the resulting system has been chosen like showed in Figure 5.

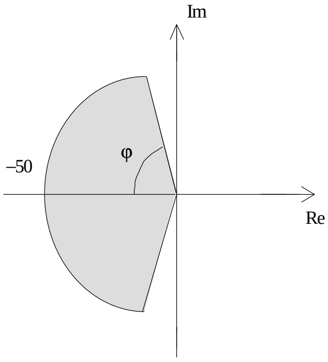

Figure 5: The stability domain $\mathbf{D}$

The angle φ in the above figure equals $\tan^{-1}\left(\sqrt{1-0.05^2}/0.05\right)$. Four different operating conditions have been considered, namely the ones determined by the airspeed and the air density with the values $U(1\pm k)$ and $\rho(1\pm k)$, where $k>0$ is a free parameter. Then the convex polytope determined by these four operating conditions has been considered to approximate the parametric uncertainty of the pitch-plunge aeroelastic system. A static feedback gain that quadratically $\mathbf{D}$-stabilizes this polytopic plant was further determined (Boyd 1994). Although the quadratic stabilization method (Petersen 1986) guarantees the stability of any system inside the polytope, it may provide conservative results. This means that in the considered case study the admissible intervals of variation for U and ρ such that the resulting system is $\mathbf{D}$-stable can be larger than the ones obtained based on the feasibility condition of the quadratic stabilization problem. The state-feedback gain obtained by quadratic $\mathbf{D}$-stabilization of the polytopic aeroelastic system is:

$$K=\begin{bmatrix}44.1008 & 3.8667 & 1.0051 & -0.0042\end{bmatrix} \tag{10}$$

The time variations of the plunge h and of the pitch α corresponding to the closed-loop system with the same initial conditions considered above are comparatively shown with the open-loop responses in Figures 6 and 7, respectively.

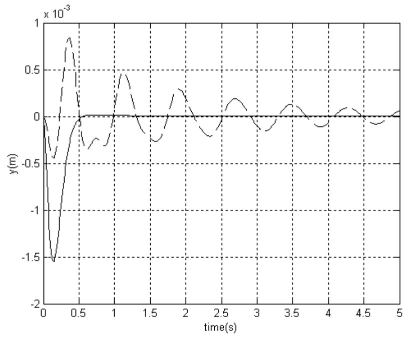

Figure 6: Plunge h time responses at $U=6\,m/s$: open-loop (dashed), closed-loop (solid)

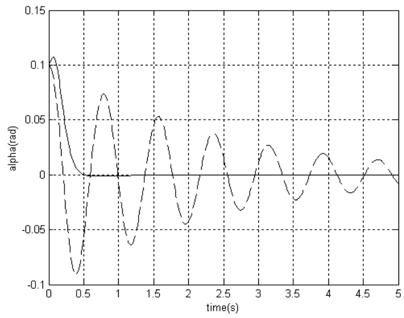

Figure 7: Pitch angle α time responses at $U=6\,m/s$: open-loop (dashed), closed-loop (solid)

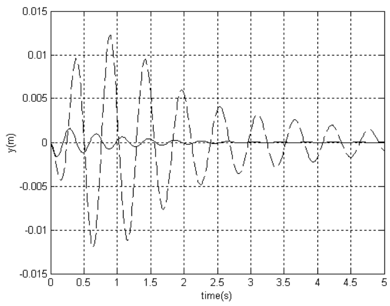

Figure 8: Plunge h time responses at $U=11\,m/s$: open-loop (dashed), closed-loop (solid)

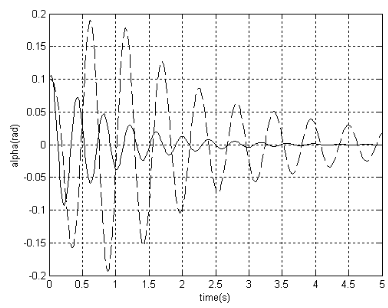

Figure 9: Pitch angle α time responses at $U=11\,m/s$: open-loop (dashed), closed-loop (solid)

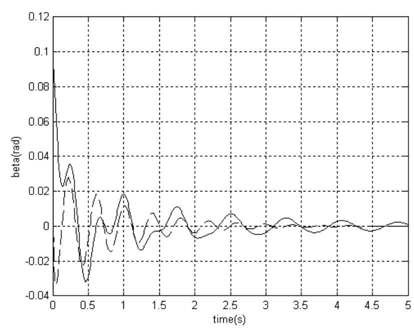

Figure 10: The control β time response at $U=6\,m/s$ (solid) and $U=11\,m/s$ (dashed)

In Figures 8 and 9 are plotted the time responses of the same states at an airspeed $U=11\,m/s$, close to its critical value $U_{crt}=11.2\,m/s$.

The time responses presented above indicate a better damping of the flexible modes both at the nominal airspeed and in the neighborhood of its critical value. The control time responses corresponding to $U = 6\,m/s$ and to $U = 11m/s$ are given in Figure 10.

The numerical results show that in both situations examined the system is stabilized with acceptable magnitude of the control effort.

CONCLUSIONS

The paper deals with the robust control of a simple aeroelastic system, in which pole placement objectives are formulated for a wide range of the airspeed. The proposed method approximates the aeroelastic plant subjected to uncertain parameters with a polytopic one. The case study presented in third section shows that a robust design methodology based on the polytopic representation of the aeroelastic uncertainty can be effectively used in order to augment the modes damping on a wide range of variation to the airspeed.

REFERENCES

Buschek H., Calise, A.J., 1993, Robust Control of Hypersonic Vehicles Considering Propulsive and Aeroelastic Effects, Proceedings of AIAA Congress.

Boyd, S., El Ghaoui, L., Feron, E., Balkrishan, V., 1994, Linear Matrix Inequalities in Systems and Control Theory. SIAM, Philadelphia, Pennsylvania.

Carl, U.B., Gojny, M.H., 2000, Damping Augmentation of Flexible Structures- A Robust State Space Approach.

Dowell, E.H., 1995, A Modern Course In Aeroelasticity, 3rd edition, Dordrecht, Kluwer.

Ko, J., Kurdilla, A.J., Strganac, T,W., 1997, Nonlinear Dynamics and Control for a Structurally nonlinear Aeroelastic System, Department of Aerospace Engineering, Texas A&M University, TX 77843-3141.

Lind, R., Brenner, M., 1999, Robust Aeroservoelastic Stability Analysis, London, Springer.

Petersen, I.R., Hollot, C.V., 1986, A Riccati equation approach to the stabilization of uncertain linear systems, Automatica, 22, p. 397-411.