FLUTTER SUPPRESSION BY ACTIVE CONTROL. A TYPICAL PROBLEM

Marius Stoia-Djeska
POLITEHNICA University Bucharest, Department of Aerospace Sciences, Polizu 1-6, Bucharest and STRAERO S.A., Iuliu Maniu 220, Bucharest 06 1099

Ioan Ursu
STRAERO S. A., Iuliu Maniu 220, Bucharest 06 1099
e-mail: iursu@aero.incas.ro

Felicia Ursu
INCAS “Elie Carafoli”, Iuliu Maniu 220, Bucharest 06 1099

Abstract. Flutter is an aeroelastic self-excited oscillation of aerospace vehicles structures. The motion of the aerospace structures can be exceptionally complicated due to the interactions of aerodynamic, elastic structure and servo effects. Consequently, flutter prevention is an important design task. The present paper addresses the problem of control synthesis for weakly damped aeroelastic wing structures by means of the primary flight controls servos. In fact, an active control for a typical section in unsteady incompressible flow is thought. First, we briefly review the reasons why the research activities on flutter suppression are important for the development of aerospace industry. Then, a simplified two-dimensional representation of an aeroservoelastic system, consisting of an airfoil with a trailing-edge flap, controled by a aervoactuator, is developed. The essential features of the airfoil’s aerodynamics are described by a quasi-steady model for the lift, pitching moment and hinge moment. The flap deflection is due to the active control law applied to the servoactuator. The control aims at suppression of flutter type instability by changing the wing configuration to cause the total aerodynamic lift and moment variations. The airstream speed was considered in this study as a parametric uncertainty. Thus a linear system with structured uncertainty is obtained. The optimal linear quadratic regulator with state observer is employed to get the active control of even unstable in open loop speeds. Thus, flutter speed can be maximized, no mater how much by using a succession of gains of control law. Two sufficient conditions concerning the stability robustness and performance robustness of the derived active control were introduced. Numerical examples that illustrate the design are presented, using the data of an experimental model in aerodynamic tunnel.

Keywords: aeroservoelasticity, flutter, typical section, active control, LQR, observer, uncertainties, stability robustness, performance robustness.

1. INTRODUCTION

The increasing performance requirements and the reduction of the Direct Operating Cost (DOC) of new aircrafts lead to an important increase of size of aerospace vehicles and also optimization of structural masses. Aeroelastic problems of aircraft light weight structures are the result of interactions between deformations of the elastic structure and aerodynamic forces induced by the structure deformations. In modern aircrafts with any type of active control systems and mainly in those with fly-by-wire flight control systems, additional interactions between the airframe and control systems are possible. The impedance of powered control systems and the special connected flutter problems got rise to a new control topic: aeroservoelasticity (I. Ursu and F. Ursu, 2002). Indeed, the aeroelastic phenomena may not only strongly influence the structural dynamics and dynamic flight stability, but also the overall performance and controllability of the aircraft. Since flutter, which is an aeroelastic self-excited oscillation, tends to occur in almost all of the flight regimes, flutter avoidance is essential in getting certificate for newly designed aerospace vehicles (Dowell, 1995; Ursu *et al.*, 1996; Matsushita, 2001). Moreover, the aeroelastic vibrations characterized by weak damping reduce de fatigue life of the structure and consequently increase the DOC and also may lead to catastrophic failures of the aircraft components.

The overall active control originates in aerospace systems. Nowadays, it is used both for flutter suppression and structural load alleviation. This role is accomplished through an additional functionality of the primary flight control surfaces and of their actuation system. A typical active control system for flutter suppression is designed to perform three functions: a) sensing the flutter mode, b) feeding back the signal, and c) controlling the flutter mode. Various paradigms of the applied control have been put to the proof; see e.g. Ohta *et al.*, 1989, in which LQR approach is used; and also, Carl and Gojny, `htttp://fluid.power.net/techbriefs/papers/proc_gojny.pdf`, with a parameter space design.

The present paper addresses the problem of control synthesis for weakly damped aeroelastic wing structures by means of the primary flight controls servos. In order to outline a general approach, the aeroelastic model consists of a quasi-steady formulation of the aerodynamic lift and moment of a typical section (Dowell, 1995) with flap, which is connected with a servoactuator. So, after a brief description of the obtained two-degrees of freedom aeroelastic model, we present the methodology of active control flutter suppression, by designing a LQR control law based on a standard observer. The airstream speed was considered in this study as a parametric uncertainty. This means that a linear system with structured uncertainty is defined. The obtained law is employed to get the active control of even unstable in open loop speeds. Consequently, flutter speed can be maximized, no mater how much by using a succession of gains of control law. Two sufficient conditions concerning the stability robustness and performance robustness of the derived active control were introduced. Finally, numerical results for a representative base-case problem are reported and then some conclusions are drawn for this study.

2. AEROELASTIC EQUATIONS

A simplified two degrees of freedom representation of an airplane wing is used in this work as the aeroelastic system for which flutter suppression is desired. The model is the well-known typical section with trailing edge flap (Dowell, 1995; Ko, 1997; Lind and Brenner, 1999). The two degrees of freedom are the downward vertical displacement h of the elastic axis and a leading edge up angular rotation α about this line and are sketched in Figure 1. These degrees of freedom correspond to the bending and torsion displacements of a high aspect ratio wing under real loads (Dowell, 1995).

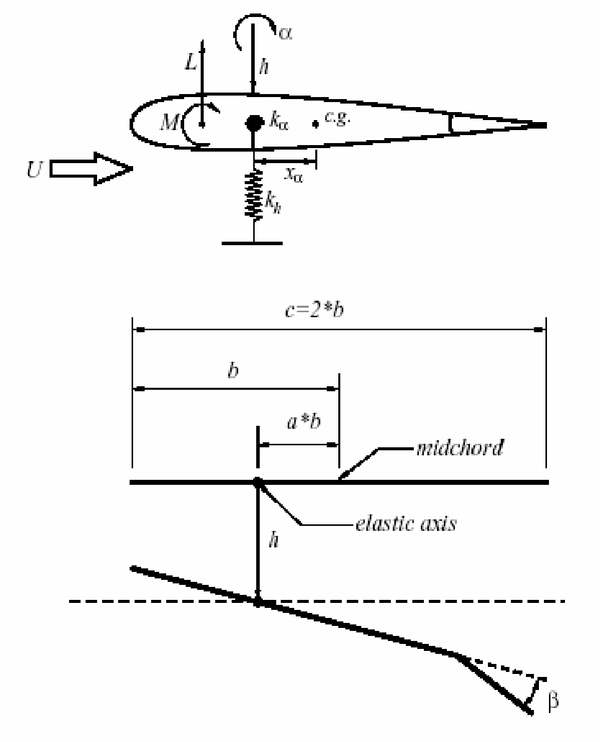

Figure 1. The aeroelastic model

In what follows, we assume that the stiffness of the flap hinge is very large compared with the constant stiffness k_h of the plunging motion and with the constant stiffness k_α associated with the pitching motion. Further, we also assume that the flap moves instantaneously so that the contribution of the flap motion to the kinetic energy of the typical section will be neglected. Due to this last assumption, the flap can still be used as a control surface but its deflection β does not represent an additional degree of freedom for the stability and remains only a control parameter of the aeroelastic system's dynamics. Finally, the external forces acting on the wing section are due only to the pressure difference distribution on the typical section.

The equations of motion describing the plunge and pitch during an aeroelastic response are derived starting from the Lagrange equations (Dowell, 1995)

$$\frac{\partial}{\partial t}\left(\frac{\partial(T-U)}{\partial \dot{\bar{q}}}\right)-\frac{\partial(T-U)}{\partial \bar{q}}=\bar{Q}_q \tag{1}$$

where the set of the generalized coordinates is $\bar{q}=\{h,\alpha\}^T$. The corresponding generalized forces are derived here as $Q_h=-L$ and $Q_\alpha=M$, where L and M are the lift and moment, respectively. The kinetic energy is given by

$$T=\frac{1}{2}m\dot{h}^2+mx_\theta\dot{h}\dot{\alpha}+\frac{1}{2}I_\theta\dot{\alpha}^2 \tag{2}$$

and the potential energy of strain is

$$U=\frac{1}{2}k_\alpha\alpha^2+\frac{1}{2}k_h h^2. \tag{3}$$

Hence, the governing equations of the aeroelastic system under considerations are

$$\begin{bmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{bmatrix}\ddot{\bar{q}}+\begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\bar{q}=\begin{bmatrix} -L \\ M \end{bmatrix} \tag{4}$$

where m is the mass of the typical section, I_α is the mass moment of inertia about the elastic axis, x_α is the dimensionless distance between the elastic center and center of mass and b is the semichord length. Following Lind and Brenner (1999), in the above system one can introduce a structural dissipation through the structural damping coefficients in pitch and plunge, c_h and c_α, so that the equations are

$$\begin{bmatrix} m & mx_\alpha b \\ mx_\alpha b & I_\alpha \end{bmatrix}\ddot{\bar{q}}+\begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\dot{\bar{q}}+\begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix}\bar{q}=\begin{bmatrix} -L \\ M \end{bmatrix} \tag{5}$$

We assume that the aerodynamic lift and moment are determined according to a quasi-steady model (Dowell, 1995; Lind and Brenner,1999)

$$L=\rho U^2 bc_z^\alpha\left(\alpha+\frac{\dot{h}}{U}+\left(\frac{1}{2}-a\right)b\frac{\dot{\alpha}}{U}\right)+\rho U^2 bc_z^\beta\beta \tag{6}$$

$$M = \rho U^2 b^2 c_m^\alpha \left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a \right) b \frac{\dot{\alpha}}{U} \right) + \rho U^2 b^2 c_m^\beta \beta \tag{7}$$

where ρ is the density of the air, U is the velocity of the free air stream, c_z^α, c_m^α are the lift and moment coefficients per angle of attack and c_z^β, c_m^β are the lift and moment coefficients per flap deflection and a is the dimensionless distance between the midchord and the elastic axis.

After substituting the lift and moment into the equations of motion we obtain the standard, structural type, second order multidimensional linear time invariant system

$$M\ddot{X} + C\dot{X} + KX = B_\beta \beta, \quad X := \begin{bmatrix} h \\ \alpha \end{bmatrix} \tag{8}$$

$$M := \begin{bmatrix} m & mbx_\alpha \\ mbx_\alpha & J_\alpha \end{bmatrix}, C := C_0 + C_1(U)$$
$$C := \begin{bmatrix} Ch + \rho b C_z^\alpha & \rho b^2 (1/2 - a) C_z^\alpha U \\ -\rho b^2 C_m^\alpha U & c_\alpha - \rho b^3 (1/2 - a) C_m^\alpha U \end{bmatrix}$$
$$K := \begin{bmatrix} k_h & \rho b C_z^\alpha \\ 0 & k_\alpha - \rho b^2 C_m^\alpha U \end{bmatrix} := K_0 + K_1(U) \tag{8'}$$
$$B_p := \begin{bmatrix} -\rho b C_z^\beta U^2 \\ \rho b C_z^\beta U^2 \end{bmatrix} := B_0 U^2.$$

These equations serve as starting point for the development of the complete aeroservoelastic model.

3. AGGREGATE AEROSERVOELASTIC MODEL

Introducing the state equation of the actuator

$$\tau\dot{\beta} + \beta = k_s u \tag{9}$$

where τ is the time constant and k_s is the gain (aileron deflection β - current $u := i$) and then defining the state variables

$$x_1 := h, x_2 := \alpha, x_3 := \dot{h}, x_4 := \dot{\alpha}, x_5 := \beta$$
$$x := [x_1, x_2, x_3, x_4, x_5]^{\mathrm{T}} \tag{10}$$

one obtains the state space form of the aggregate structure-actuator aeroservoelastic model

$$\dot{x} = A_U x + Bu \tag{11}$$

$$A_U := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -M^{-1}K & & -M^{-1}C & & M^1 B \\ 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_s/\tau \end{bmatrix}.$$

Let us consider the nominal system described by

$$\dot{x} = Ax + Bu$$
$$A := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -M^{-1}K_0 & & -M^{-1}C_0 & & M^1 B_0 U_{nom}^2 \\ 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix} \tag{12}$$

and the system with uncertainty (11) defined as

$$\dot{x} = Ax + Bu + \Delta A(t)x$$
$$\Delta A(t) := \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -M^{-1}K_{1\Delta} & & -M^{-1}C_{1\Delta} & & M^{-1}B_0\left(U^2 - U_{nom}^2\right) \\ 0 & 0 & 0 & 0 & -1/\tau \end{bmatrix} \tag{13}$$
$$K_{1\Delta} := K_1(U) - K_1(U_{nom}), \quad C_{1\Delta} := C_1(U) - C_1(U_{nom})$$

Thus, the speed $U = U(t)$ is thought as a time dependent parameter uncertainty. The problem is now to synthesize a control law for the system (13), to face up to the uncertainties $\Delta A(t)$.

In practice, the dynamic instability of the airspace structures described by systems (12), (13) is named *flutter*.

Therefore, the framework of the control synthesis is flutter suppression, in other words, active control of flexible structures vibrations.

4. CONTROL SYNTHESIS FOR LINEAR SYSTEMS WITH STRUCTURED STATE SPACE UNCERTAINTY

Control synthesis for linear systems with time varying state space uncertainty will be considered. The approach represents a development of the results given in (Sobel *et al*., 1989; Yu and Sobel, 1991). These results can be obtained in a very simple manner: a) a similarity transformation involving the modal matrix of the nominal closed loop system is applied to the uncertain closed loop system; b) the Gronwall lemma is used for the solution of this system. Other standard results of the problem has been derived by using a Lyapunov approach (see references Corless and Leitmann, 1981; Barmish *et al*., 1983) where frequently used *matching conditions* were introduced), or a Popov approach (see, for example, Sparks and Bernstein, 1993). The results of Sobel and his coworkers are still especially well suited in a problem of vibration control, because involve eigenstructure assignment.

Consider a nominal linear time invariant system described by

$$\dot{x} = Ax + Bu, \; y_o = C_o x, \; y_p = C_p x \tag{14}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input (control) vector, $y_o \in \Re^r$ is the output measurement vector, $y_p \in \Re^l$ is the performance vector, and A, B, C_o, C_p are constant matrices. Suppose the nominal system is subject to linear time-varying parametric uncertainties in the entries of A, described as $\Delta A(t)$. Thus a system with uncertainty is obtained

$$\dot{x} = Ax + Bu + \Delta A(t)x, \quad y_o = C_o x. \tag{15}$$

Further, suppose that bounds are available on the absolute value of the maximum variations in the elements of $\Delta A(t)$

$$\left|\Delta a_{ij}(t)\right| \le \left(a_{ij}\right)_{max}, \quad i,j = \overline{1,n}. \tag{16}$$

Define $\Delta A^{+}(t)$ as the matrix obtained by replacing the entries of $\Delta A(t)$ by their absolute values. Also, define A_m as the matrix with entries $\left(a_{ij}\right)_{max}$.

Consider now the observer that produces the estimate of the state, $\hat{x}$, is of the form

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + L\left(y_o - C_o\hat{x}\right) \\ u &= -K\hat{x}. \end{aligned} \tag{17}$$

Thus, a separation property is assumed, that is the control is generated from linear combinations of an estimate $\hat{x}$ to the true state x. The feedback gain K is given uniquely as a solution of a LQR problem

$$\begin{aligned} \dot{x} &= Ax + Bu \\ J &= \int_0^\infty \left(x^T C_p^T Q C_p x + u^T R u\right)dt \end{aligned} \tag{18}$$

that is

$$\begin{aligned} K &= R^{-1}B^T P \\ PA + A^T P - PBR^{-1}B^T P + C_p^T Q C_p &= 0. \end{aligned} \tag{18'}$$

Q, R must be symmetric and positive, rank $B = m$, and $(A,B), (A,C_p)$ stabilizable and detectable, respectively (I. Ursu and F. Ursu, 2002); then

$$Re\,\lambda(A - BK) < 0. \tag{19}$$

The observer gain L will be select so that $(A - LC_0)$ is asymptotically stable. One may proceed as follows, by virtue of the duality property (Kwakernaak and Sivan, 1972): a) rename $\left(A^T, C_0^T\right)$ to (A, B); b) use LQR design technique to determine the stabilizing gain K; c) rename K^T to L.

The closed-loop dynamics of the overall – extended – feedback system (15), (17) are given by

$$\begin{aligned} \dot{x}_e &= A_{cl}x_e + \Delta A_{cl}x_e, \quad x_e := \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \\ A_{cl} &:= \begin{bmatrix} A & -BK \\ LC_0 & A - BK - LC_0 \end{bmatrix} \\ \Delta A_{cl} &:= \begin{bmatrix} \Delta A & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{20}$$

Consider also the nominal closed loop system

$$\dot{x}_e = A_{cl}x_e. \tag{21}$$

Stability robustness problem. Given feedback gain matrices $K \in \Re^{m \times r}, L \in \Re^{r \times m}$ such that all of the eigenvalues of the nominal system (21) exhibits desirable dynamic performance, determine if the system (20) is asymptotically stable for all $\Delta A(t)$ described by (16).

Performance robustness problem. Feedback gain matrices $K \in \Re^{m \times r}, L \in \Re^{r \times m}$ are chosen such that all of the eigenvalues of the system (21) are inside the region R. Determine if all of the eigenvalues of the system (20) are inside of the region R for all $\Delta A(t)$ described by (16).

Proposition 1. *Suppose that K and L are such that the system* (21) *is asymptotically stable with distinct eigenvalues. Then, the system* (20) *is asymptotically stable for all* $\Delta A(t)$ *described by* (16) *if*

$$\alpha > \left\| \left(M^{-1}\right)^{+} A_m M^{+1} \right\|_2 \tag{22}$$

where

$$\alpha := -\max_i Re\left[\lambda_i\left(A_{cl}\right)\right] \tag{22'}$$

and M is the modal matrix of A_{cl}.

The value of $\|\cdot\|_2$ is usual given as Perron eigenvalue; in other words, the real non-negative eigenvalue $\lambda_{max} \ge 0$ of a non-negative matrix, such that $\lambda_{max} \ge 0$ for all eigenvalues of this matrix.

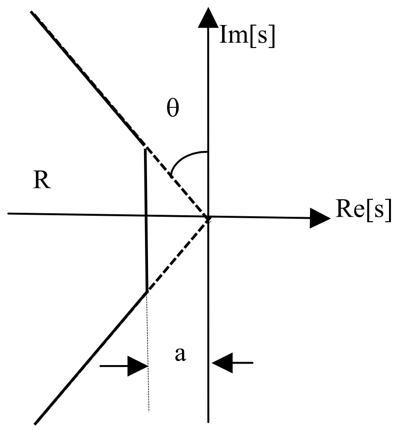

Fig. 2. Performance robustness region

Proof. Proposition is proved after some processing of the proof in (Yu and Sobel, 1991).

Proposition 2. *Suppose that K and L are such that the system* (*21*) *has only distinct eigenvalues, all of which lie inside the region* R (*see Fig.* 2). *Let D be a diagonal matrix with positive real entries, and let Q be a nonsingular matrix. Then the eigenvalues of system* (*20*) *will be in region* R *for all* ΔA *described by* (*16*) *if*

$$\min[\alpha_1, \alpha_2] > k_2(Q)\left\| \left[(MDQ)^{-1}\right]^{+} A_m [MDQ]^{+} \right\|_2 \tag{24}$$

where

$$\alpha_1 = -\max_i\left[Re\,\lambda_i\right] + a, \quad \alpha_2 = -\max_i Re\left[e^{-i\theta}\lambda_i\right] \tag{24'}$$

$k_2(Q) = \|Q\|_2 \, \|Q^{-1}\|_2$ *(condition number) and* λ_i *is an eigenvalue of* A_{cl} .

Proof. Proposition is proved after some processing of the results given in (Yu and Sobel, 1991).

5. NUMERICAL EXAMPLE AND CONCLUSIONS

Worthy noting, given the controllable and observable system (A, B, C_0) and the assumption that matrices B and C_0 are full rank, then max (m, r) closed loop eigenvalues can be assigned.

An experimental wing model for an aerodynamic tunnel will be used to illustrate the proposed active control design (Lind and Brenner, 1999). The system data were as follows: $\tau = 0.03$ s; $m = 12.387$ kg; $x_\alpha = 0.2466$; $b = 0.135$ m; $J_\alpha = 0.065$ kgm^2; $C_z^\beta = 3.358$; $c_h = 27.43$ kg/s; $c_\alpha = 0.036$ kgm^2/s; $C_z^\alpha = 6.28$; $a = -0.6$; $k_h = 2844.4$ N/m; $k_\alpha = 3.525$ Nm/rad; $C_m^\beta = 12.387$; $C_m^\alpha = -0.635$; $\rho = 1.225$ kg/m^3; $k_s = 10\pi/180/20$ rad/mA; $C_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$.

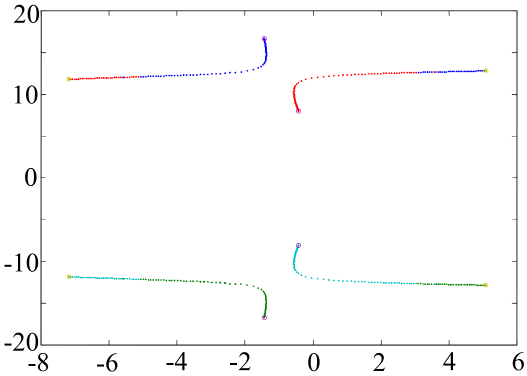

Fig. 3. Open loop root loci

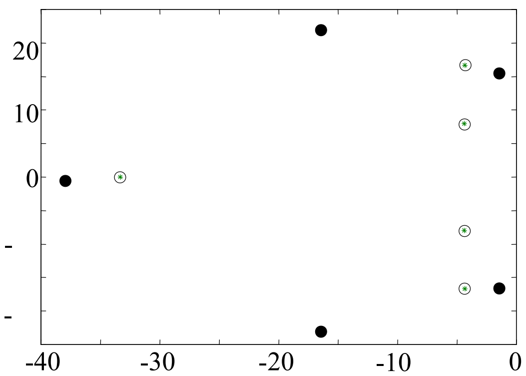

Fig. 4. Closed loop eigenvalues

Open-loop root loci of the nominal system (12) are shown in Figure 3. The system is stable until a critical speed 11.423 m/s occurs. An active control low is now built, considering $U_{nom} = 11.423$ m/s. The closed loop eigenvalues are shown in Figure 4; the chosen weghting matrices were $Q = \mathrm{diag}\left[10^7 \;\; 1500 \;\; 1 \;\; 1 \;\; 1\right]$ and $R = 1$. Very narrow bounds $U \in (11.396, 11.503)$ for stability robustness, predicted by condition (22), are not surprisingly. Indeed, it is well known that inequalities such as (22) are very conservative (Yedavalli, 1986). An improving of the bounds for robust stability can be obtained by using a direct calculus and is very spectaculous: $U \in (0, 16.42)$

Similar bounds for performance robustness provide the inequality (24), but again a direct calculus can be utilized.

The main contribution of the paper is the development of a control law, employed to get the active control of even unstable in open loop speeds. Consequently, critical flutter speed can be maximized no mater how much by using a succession of gains of control law.

6. REFERENCES

Barmish, B. R., M. Corless, G. Leitmann (1983). A new class of stabilizing controllers for uncertain dynamical systems. *SIAM Journal on Control and Optimization*, **21**, 246-255.

Carl, U. B., M. H. Gojny. Damping augmentation of flexible structures – A robust state approach. `http://fluid.power.net/techbriefs/papers/proc_gojny.pdf`

Corless, M., G. Leitmann (1981). Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems. *IEEE Transactions on Automatic Control*, **26**, 1139-1144.

Dowell, E.H. (1995). *A modern course in aeroelasticity*, 3rd edition. Kluwer, Dordrecht.

Ko, J., A. J. Kurdilla, T. W. Strganac (1997). Nonlinear dynamics and control for a structurally nonlinear aeroelastic system. Department of Aerospace Engineering, Texas A&M University, TX 77843-3141.

Lind, R., M. Brenner (1999). *Robust aeroservoelastic stability analysis*. Springer, London.

Matsushita, H., P. Granasy, E. Mosekilde (2001). Present status of the research on transonic limit cicle flutter. *Proceedings of International Conference on Unconventional Flight*, UNCONF 2001, September 12-14, Budapest.

Kwakernaak, H., R. Sivan (1972). Linear optimal control systems. Wiley Interscience, New York.

Ohta, H., A. Fujimori, P. N. Nikiforuk, M. M. Gupta (1989). Active flutter suppresion for two-dimensional airfoils. *Journal of Guidance, Control and Dynamics*, **12**, no. 2, 183-193.

Sobel, K. M., S. S. Banda, H.-H. Yeh (1989). Robust control for linear systems with structured state space uncertainty. *International Journal of Control*, **50**, no. 5, 1991-2004.

Sparks, A. G., D. S. Bernstein (1993). Real structured singular value synthesis using the scaled Popov criterion. *Journal of Guidance*, *Control and Dynamics*, **18**, no. 6, 1244-1252.

Ursu, I., F. Ursu, M. Vladimirescu (1996). About aeroservoelasticity criteria for electrohydraulic servomechanisms synthesis. *ICAS* 96*, Proceedings of* 20*th* *Congress of the International Council of the Aeronautical Sciences*, **2**, pp. 2335–2344.

Ursu, I., F. Ursu (2002). *Active and semiactive control* (in Romanian). Romanian Academy Publishing House, Bucharest.

Yedavalli, R. K., Z. Liang (1986). Aircraft control design using improved time-domain stability robustness bounds, *Journal of Guidance*, *Control and Dynamics*, **9**, no. 6, 710-715.

Yu, W., K. M. Sobel (1991). Robust eigenstructure assignment with structured state space uncertainty. *Journal of Guidance*, *Control and Dynamics*, **14**, no. 3, 621-628.