ON THE IMPLEMENTATION OF DISTRIBUTED DELAY CONTROL LAWS

Dan POPESCU
Department of Automatic Control, University of Craiova,
A. I. Cuza str., no. 13, RO - 200585 Craiova, Romania
E-mail: dpopescu@automation.ucv.ro

Abstract: The stabilization by feedback control of systems with input delays may be considered in various frameworks. An approach to stabilize a such system, based on the Artstein transform, is the so-called finite spectrum assignment. In this case the control law that stabilize the system is a distributed delay control law. A difficulty in applying a control law of this form consists in the practical implementation of the integral term, which needs to be calculated on-line. Some recent papers outlined an instability mechanism when the distributed delay in the control law is approximated with a sum of point-wise delays, despite the asymptotic stability of the ideal closed-loop system. In this paper, we use a suitable discretization rule based on piecewise constant control signals. Then, we analyze and illustrate by some numerical examples the robustness/fragility of this control law.

Key words: time delay systems, stabilization, piecewise constant control, robustness.

1. INTRODUCTION

Systems with input delay are of interest to control theorists and practitioners for various reasons. They originate from the simplest model of process control, which assigns to the controlled plant a transfer function of the form $H(s)e^{-hs}$ where $H(s)$ is strictly proper rational function. To this transfer function, one may associate one of the following state representations:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t-h) \\ y(t) &= c^* x(t) \end{aligned} \tag{1}$$

or

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c^* x(t-h) \end{aligned} \tag{2}$$

where in both cases $c^*(sI-A)^{-1}b \equiv H(s)$. These representations have in common the obvious fact that the state space is finite dimensional but the input operator in the first case or the output one in the second case are defined on infinite dimensional extensions and are unbounded. Control problems have been formulated and solved for such systems since the classical (transfer function based) period (the '50ies) and the mostly known result is that based on Smith predictor. The systems thus designed could be either non-robust or unstable (some times). For this reason the more recent techniques based on state space ensuring feedback stabilization and optimality of some quadratic criterion were applied to (1). One can mention here some results on feedback stabilization (Olbrot, 1978; Manitius and Olbrot, 1979; Watanabe and Ito, 1981). The guiding line of these papers is the construction by *ad-hoc* methods of dynamic compensators that are stabilizing.

Another line of research was that dealing with linear quadratic optimization. Among many contributions, we may cite (Mee, 1973), which is "mathematically oriented" and (Ichikawa, 1982) which is more "applied" and in any case simpler.

These papers were followed by those of Pandolfi (Pandolfi, 1981, 1989, 1990, 1991) containing a strong criticism of the first ones; this criticism was motivated by the fact that the results had been obtained by *ad-hoc* methods and not in a standard way, by deduction from an abstract theory. The main idea of Pandolfi was to re-introduce in the model the *propagation effects*. Since pure delay occurs from lossless telegraph equations with matched boundary conditions, this model is re-introduced and the system with input delay becomes a boundary condition for the partial differential equations that replace the delay.

Within this framework, the basic linear feedback problems are embedded in the approach based on singular control. Moreover the approach of Pandolfi shows the rational way of including systems with input delay in the broad class of systems described by abstract evolution equations (Weiss and Curtain, 1997). This line is continued by the papers of Tadmor (Tadmor, 1995, 1998) where the abstract model leads finally to finite dimensional – like solutions.

The aim of this paper is a more applied one, closed to the idea of sub-optimality. The pioneering paper (Halanay and Răsvan, 1977) showed the suboptimality of the piecewise constant controls at the level of the performance in the standard linear quadratic optimal stabilization problem; further, this approach was developed in (Drăgan and Halanay, 1999). We are also guided by a paper mentioned sometimes in the bibliography of time delay systems (Artstein, 1982); in this paper, Artstein introduces a linear transformation in order to reduce the system to a delay-free one. This idea is closed to that of Tadmor (it precedes him) and closer to elementary approaches that we intend to follow throughout the paper.

We start with an elementary form of the stabilizing feedback (Olbrot, 1978; Manitius and Olbrot, 1979; Watanabe and Ito, 1981) and discuss the dynamics of the closed loop system. Then, we approach the practical implementation of the resulting distributed delay control law. The feedback control by piecewise constant signals is considered from the point of view of stability preserving and the robustness with respect to parameter uncertainties is analyzed.

2. STABILIZING FEEDBACK

We shall consider here the following system with input delay (Kwon and Pearson, 1980):

$$\dot{x}(t) = Ax(t) + B_0 u(t) + B_1 u(t-h) \tag{3}$$

Obviously, its solution is defined for $t > 0$ if there are given the initial conditions $(x_0, u_0(\cdot))$ and the control $u(t)$ for $t > 0$; here $u_0(\theta)$ is some initial function defined for $\theta \in [-h, 0)$. The Artstein transform (Artstein, 1982) in this case is given by

$$z(t) = x(t) + \int_{-h}^{0} e^{-A(\theta+h)} B_1 u(t+\theta) d\theta \tag{4}$$

and the result of Artstein takes the form of the following equivalence.

Proposition 1. Let $(x(t), u(t); t > 0)$ be a solution (admissible pair) for (3), defined by some initial condition $(x_0, u_0(\cdot))$. Then $(z(t), u(t); t > 0)$ with $z(t)$ defined by (4) is a solution (admissible pair) for the system

$$\dot{z}(t) = Az(t) + (B_0 + e^{-Ah} B_1) u(t) \tag{5}$$

with the initial condition $z_0 = z(0)$. Conversely, let $(z(t), u(t); t > 0)$ be a solution of (5) defined by some initial condition z_0. Then, given some $u_0(\cdot)$ defined on (-h, 0) and taking

$$x_0 = z_0 - \int_{-h}^{0} e^{-A(\theta+h)} B_1 u_0(\theta) d\theta \tag{6}$$

the solution of (3) defined by these initial conditions and by $u(t)$, $t > 0$ is given by

$$x(t) = z(t) - \int_{-h}^{0} e^{-A(\theta+h)} B_1 u(t+\theta) d\theta \tag{7}$$

The proof of this result is straightforward.

Furthermore, we shall use the following result (Kwon and Pearson, 1980) that may be found implicitly in (Ichikawa, 1982; Olbrot, 1978; Manitius and Olbrot, 1979; Watanabe and Ito, 1981; Mee, 1973; Răsvan and Popescu, 2001).

Proposition 2. Let $u = Fz$ be a feedback stabilizing scheme for (5). Then the control

$$u(t) = F\left[x(t) + \int_{-h}^{0} e^{-A(\theta+h)} B_1 u(t+\theta) d\theta \right] \tag{8}$$

is stabilizing for (3).

The proof is straightforward and relies entirely on Proposition 1.

The structure defined by (8) may be used as a stabilizing compensator since the solution of the closed loop system

$$\begin{aligned} &\dot{x}(t) = Ax(t) + B_0 u(t) + B_1 u(t-h) \\ &u(t) - F\int_{-h}^{0} e^{-A(\theta+h)} B_1 u(t+\theta) d\theta = Fx(t) \end{aligned} \tag{9}$$

may be constructed by steps (Răsvan and Popescu, 2001), hence this system is well defined. Moreover, we may differentiate the second equation of (9) in order to obtain a usual time delay system:

$$\begin{aligned} &\dot{x}(t) = Ax(t) + B_0 u(t) + B_1 u(t-h) \\ &\dot{u}(t) = FAx(t) + F(B_0 + e^{-Ah} B_1) u(t) + \\ &\quad + F\int_{-h}^{0} e^{-A(\theta+h)} AB_1 u(t+\theta) d\theta \end{aligned} \tag{10}$$

The solution of this system may be constructed following the line of Halanay (Halanay, 1966) applying a contraction principle on $\Re^n \times \Re^m \times L^2((-h,0); \Re^m)$.

It is interesting to check the characteristic equation of (9) and (10). Considering the Euler type solutions $x(t) = e^{st} p$, $u(t) = e^{st} q$ with $s \in C$, $\dim(p) = \dim(x)$, $\dim(q) = \dim(u)$, we obtain:

$$\begin{aligned} &(sI - A)p - (B_0 + e^{-sh} B_1)q = 0 \\ &-Fp + \left[I - F\left(\int_{-h}^{0} e^{(sI-A)\theta} d\theta \right) e^{-Ah} B_1 \right] q = 0 \end{aligned}$$

If we take into account (4) viewed along Euler solutions, the above equalities become

$$(sI - A)\left[p + \left(\int_{-h}^{0} e^{(sI-A)\theta} d\theta \right) e^{-Ah} B_1 q \right] - \\ - (B_0 + e^{-Ah} B_1) q = 0$$

$$-F\left[p+\left(\int_{-h}^{0}e^{(sI-A)\theta}d\theta\right)e^{-Ah}B_1 q\right]+q=0$$

From here, we deduce the characteristic

$$\det\begin{pmatrix} sI-A & -(B_0+e^{-Ah}B_1) \\ -F & I \end{pmatrix}=$$

$$=\det(sI-A-(B_0+e^{-Ah}B_1)F)=0 \quad (11)$$

hence the spectrum is finite and may be assigned from the stabilization problem for (5). In a similar way the characteristic equation for (10) is obtained as

$$s^m\det(sI-A-(B_0+e^{-Ah}B_1)F)=0 \quad (12)$$

where $m=\dim(u)$ and the factor s^m is a consequence of differentiation showing that the system evolves along an invariant manifold.

3. IMPLEMENTATION OF DISTRIBUTED DELAY CONTROL LAWS

The continuous time (optimal) stabilization gives to the control engineer the structure and the main parameters of the control device, which can be determined by off-line computation.

A difficulty in applying a control law of the form (8) consists of the practical implementation of the integral term, which needs to be calculated on-line. As explained in (Manitius and Olbrot, 1979), obtaining this term as the solution to a differential equation must be discarded because it involves unstable pole-zero cancellations when A is unstable. A possibility is to approximate the distributed delay by a sum of point-wise delays by using a numerical quadrature rule (Michiels, Mondie and Roose, 2003):

$$u(t)=F\left(x(t)+e^{-Ah}\sum_{j=0}^{q}h_{j,q}e^{A\theta_{j,q}}B_1u(t-\theta_{j,q})\right) \quad (13)$$

In the past few years the effects of such a semi-discretization on the stability of the closed-loop system has been examined thoroughly. It was demonstrated with a scalar example that for some parameters values the control law (13) may not stabilize the system (3), for arbitrarily large values of q. Furthermore, the stability of the closed-loop system may be destroyed by infinitesimal relative perturbations on the delay parameters $\theta_{j,q}$ in (13), due to the neutral nature of the problem.

Now, we consider the system (3) and apply the piecewise constant control

$$u(t)=u_k,\quad k\delta\le t<(k+1)\delta,\quad k=0,1,2,\cdots \quad (14)$$

where $\delta=h/N$. As in Halanay and Răsvan (1977) and Drăgan and Halanay (1999) we associate the discrete time system

$$x_{k+1}=\mathbf{A}(\delta)x_k+\mathbf{B_0}(\delta)u_k+\mathbf{B_1}(\delta)u_{k-N} \quad (15)$$

where

$$\mathbf{A}(\delta)=e^{A\delta}\ ,\quad \mathbf{B_i}(\delta)=\left(\int_0^{\delta}e^{A\theta}d\theta\right)B_i\ ,i=0,1 \quad (16)$$

Let $(x_0,u_0(\cdot))$ be the initial condition associated with (3). Since the discretized system is satisfied by $x_k=x(k\delta)$, $x(\cdot)$ being the solution of (3) with piecewise constant control, it is only natural to choose the discretized initial condition $(x_0;u_{-i}^0=u_0(-i\delta)\ ,\ i=\overline{0,N})$. We may define

$$z_k=x_k+\sum_{-N}^{-1}\mathbf{A}(\delta)^{-(N+j+1)}\mathbf{B_1}(\delta)u_{k+j} \quad (17)$$

which is the discrete analogue of Artstein transform and find the associate system

$$z_{k+1}=\mathbf{A}(\delta)z_k+(\mathbf{B_0}(\delta)+\mathbf{A}(\delta)^{-N}\mathbf{B_1}(\delta))u_k \quad (18)$$

It is worth mentioning that (17) might be obtained by writing (4) at $t=k\delta$ and computing the integral for piecewise constant control signals.

Let F be a stabilizing feedback for (18), i.e. is such that $\mathbf{A}(\delta)+(\mathbf{B_0}(\delta)+\mathbf{A}(\delta)^{-N}\mathbf{B_1}(\delta))F$ has its eigenvalues inside the unit disk. We deduce that the compensator

$$u_k=Fx_k+\sum_{-N}^{-1}F\mathbf{A}(\delta)^{-(N+j+1)}\mathbf{B_1}(\delta)u_{k+j} \quad (19)$$

is stabilizing for (15). On the other hand, if we consider the closed loop system

$$\begin{aligned} x_{k+1}&=\mathbf{A}(\delta)x_k+\mathbf{B_0}(\delta)u_k+\mathbf{B_1}(\delta)u_{k-N} \\ u_k&=Fx_k+\sum_{-N}^{-1}F\mathbf{A}(\delta)^{-(N+j+1)}\mathbf{B_1}(\delta)u_{k+j} \end{aligned} \quad (20)$$

one may see that this is a feedback system with an augmented dynamics:

$$\begin{aligned} x_{k+1}&=\mathbf{A}(\delta)x_k+\mathbf{B_1}(\delta)v_k+\mathbf{B_0}(\delta)u_k \\ v_{k+1}&=w_k^1 \\ &\cdots\cdots\cdots\cdots \\ w_{k+1}^{N-1}&=u_k \\ u_k&=F\Big[x_k+\mathbf{A}(\delta)^{-1}\mathbf{B_1}(\delta)v_k+\cdots \\ &+\mathbf{A}(\delta)^{-(N-1)}\mathbf{B_1}(\delta)w_k^{N-2}+\mathbf{A}(\delta)^{-N}\mathbf{B_1}(\delta)w_k^{N-1}\Big] \end{aligned} \quad (21)$$

Since $w_k^{N-1}=u_{k-1}$ the corresponding initial condition is $w_0^{N-1}=u_{-1}=u_0(-\delta)$; further, $w_0^{N-2}=u_{-2}=u_0(-2\delta)$, $\cdots$, $w_0^1=u_0(-(N-1)\delta)$,

$v_0 = u_0(-N\delta)$. Obviously (21) is exponentially stable. This follows from the fact that $u = Fz$ is exponentially stabilizing system (18) and making use of (17). The result may be obtained also spectrally, as in (Răsvan and Popescu, 2001a).

To end the analysis we have to show how stabilization of the associated discrete time system ensures stabilization for the initial continuous time system. This problem will be tackled on the transformed system (5) with the stabilizing feedback

$$u(t) = Fz(k\delta) = Fz_k \,, \quad k\delta \le t < (k+1)\delta \qquad (22)$$

The system is of the type considered in Halanay and Răsvan (1977); a straightforward application of the results from the above paper will ensure the exponential stability of the closed loop hybrid system. Further, if z(t) satisfies an exponential estimate then using (4) the exponential estimate for $x(t)$ is obtained what ends the proof of the following result.

Proposition 3. Consider the system (3) under the assumption that $(A, B_0 + e^{-Ah}B_1)$ is a stabilizable pair. Then $(\mathbf{A}(\delta), \mathbf{B_0}(\delta) + \mathbf{A}(\delta)^{-N}\mathbf{B_1}(\delta))$ is stabilizable and a stabilizing feedback for this couple is stabilizing for (3) provided $\delta > 0$ is small enough. Here $\mathbf{A}(\delta), \mathbf{B_0}(\delta), \mathbf{B_1}(\delta)$ are defined by (16) and $\delta = h/N$. Moreover, a stabilizing feedback for (5) is stabilizing also if the implementation is performed using samples i.e. state values measured at $k\delta,\ k = 0, 1, 2, \cdots$.

4. SIMULATION RESULTS

4.1. Example 1

Let consider a system with input delay described by the following differential equation

$$\ddot{y}(t) + \omega_0^2 y(t) = u(t) + u(t-h) \qquad (23)$$

The standard state-space form can be obtained as follows

$$\dot{x}(t) = Ax(t) + B_0u(t) + B_1u(t-h) \qquad (24)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix},\ B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (25)$$

By using the Artstein transform we obtain the system

$$\dot{z}(t) = Az(t) + Bu(t) \qquad (26)$$

where

$$B = B_0 + e^{-Ah}B_1 = \begin{bmatrix} -\dfrac{\sin(\omega_0 h)}{\omega_0} \\ 1 + \cos(\omega_0 h) \end{bmatrix}$$

The (A, B) pair is controllable for all values of the system parameters (ω_0 and h) except those for which the following equality is satisfied

$$\omega_0 h = (2k+1)\pi \qquad (27)$$

So, the (A, B) pair is *generic* controllable.

Let F be such that $A + BF$ has it eigenvalues with strictly negative real parts. Then, the following piecewise constant control is applied to the system (24)

$$u(t) = F\left[x_k + \sum_{-N}^{-1} \mathbf{A}(\delta)^{-(N+j+1)} \mathbf{B_1}(\delta) u_{k+j} \right], \qquad (28)$$

$$\text{for } \ k\delta \le t < (k+1)\delta ,$$

where

$$\mathbf{A}(\delta) = e^{A\delta} = \begin{bmatrix} \cos(\omega_0\delta) & \dfrac{\sin(\omega_0\delta)}{\omega_0} \\ -\omega_0 \sin(\omega_0\delta) & \cos(\omega_0\delta) \end{bmatrix}$$

$$\mathbf{B_1}(\delta) = \left(\int_0^{\delta} e^{A\delta} d\delta \right) B_1 = \begin{bmatrix} \dfrac{1 - \cos(\omega_0\delta)}{\omega_0^2} \\ \dfrac{\sin(\omega\ \ \delta)}{\omega_0} \end{bmatrix},\ \delta = \frac{h}{N}$$

For simulation purposes we choose $\omega_0 = \pi/2$ (rad/s) and $h = 1$ (s). Imposing the pole allocation for the system (26) in $\{-1, -2\}$ we obtain

$$F = \begin{bmatrix} \dfrac{\pi^2}{8} - \dfrac{3\pi}{4} - 1 & \dfrac{\pi}{4} - \dfrac{3}{2} - \dfrac{2}{\pi} \end{bmatrix}.$$

Choosing the initial condition associated with (24) as $\left(x_0 = [1 \quad 2]^T ; u(\tau) = 0, -h \le \tau < 0) \right)$ and $N = 10$ the state evolution (for the closed loop system) and the control input are presented in Fig.1 and, respectively, Fig.2.

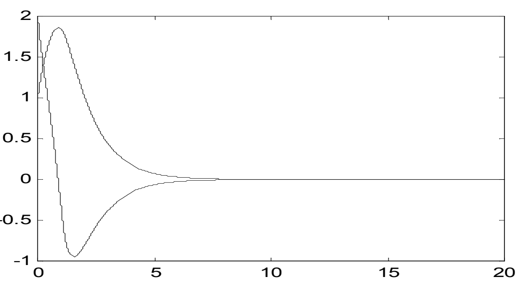

Fig.1. State Evolution (N =10, h =1)

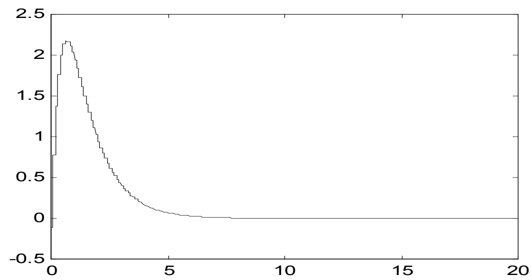

Fig.2. Input Control (N =10, h =1)

To analyse the robustness of designed controller with respect to the time delay of the plant, we have represented the state evolution for various values of h

(Fig.3 – Fig.6). Also, we repeated the above simulation for a different value of δ (Fig. 7). From the simulation results we can conclude that the proposed controller is robust both with respect h and δ .

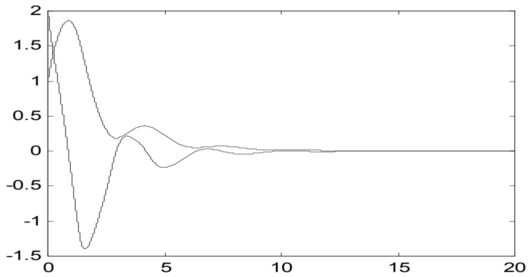

Fig.3. State Evolution (N =10, h =1.3)

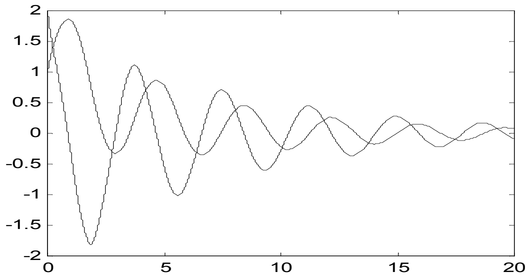

Fig.4. State Evolution (N =10, h =1.7)

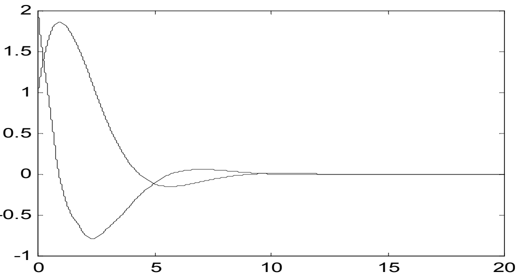

Fig.5. State Evolution (N =10, h =0.7)

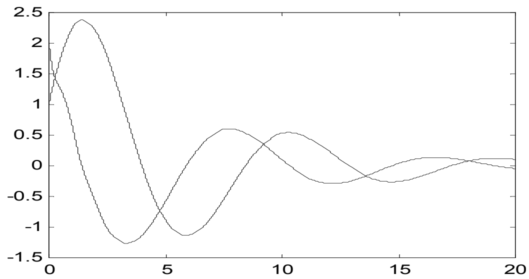

Fig.6. State Evolution (N =10, h =0.05)

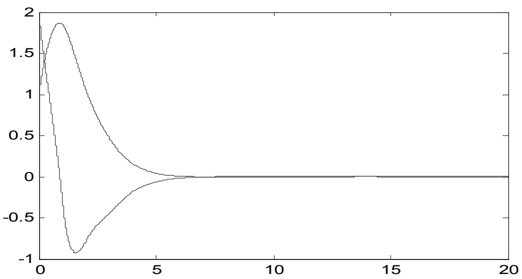

Fig.7. State Evolution (N =10, h =1, $\delta = 0.11$)

4.1. Example 2 (unstable system)

Consider the system

$$\dot{x} = x + u(t-1) \tag{29}$$

The corresponding coefficients for the form (24) are

$$A = 1,\ B_0 = 0,\ B_1 = 1,\ h = 1 \tag{30}$$

and then we obtain (see (26))

$$B = B_0 + e^{-Ah} B_1 = e^{-1} \tag{31}$$

Obvious, the (A, B) pair is controllable. Imposing the pole allocation for the system (26) in $\{-1\}$ we obtain

$$F = -2e \tag{32}$$

In the piecewise constant control law (28) the following values of coefficients are obtained:

$$\mathbf{A}(\delta) = e^{\delta},\ \mathbf{B_1}(\delta) = e^{\delta} - 1,\ \delta = \frac{h}{N}.$$

Choosing the initial condition associated with (29) as $\left(x_0 = 1; u(\tau) = 0, -h \le \tau < 0\right)$ and $N = 10$ the state evolution and the control input are presented in Fig.8 and Fig.9. Then, we repeated the simulations for different values of h and δ (see Fig. 10 – Fig. 13).

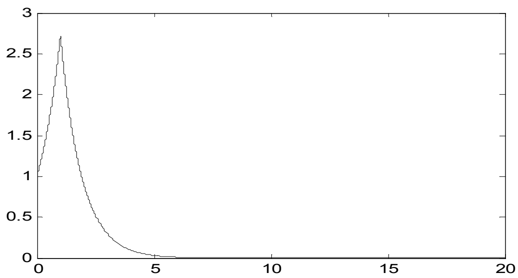

Fig.8. State Evolution (N =10, h =1)

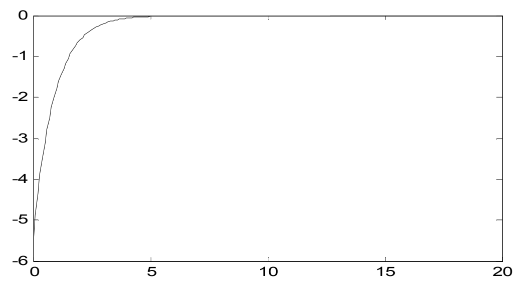

Fig.9. Input Control (N =10, h =1)

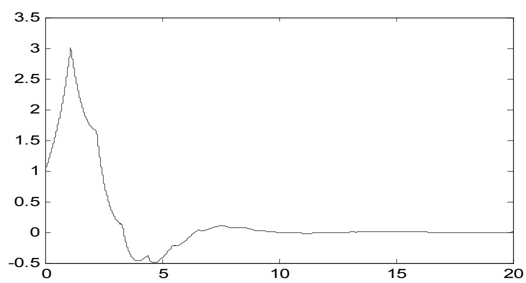

Fig.10. State Evolution (N =10, h =1.1)

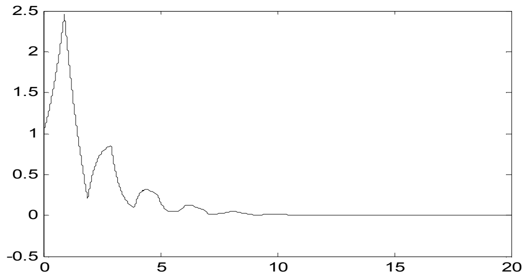

Fig.11. State Evolution (N=10, h=0.9)

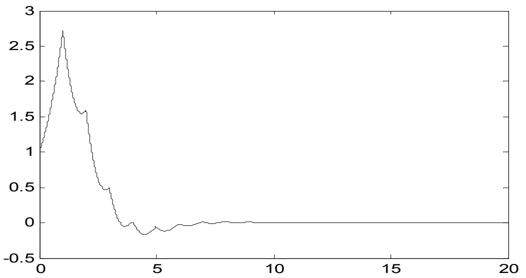

Fig.12. State Evolution (N=10, h=1, $\delta = 0.09$)

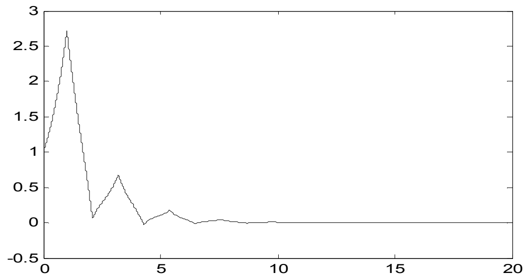

Fig.13. State Evolution (N=10, h=1, $\delta = 0.11$)

5. CONCLUSIONS

The stabilization of the systems with delayed control signal may be analyzed either in a general abstract framework or starting from the implementation of simple ideas based on Smith predictor. In this paper, we combined the second approach with the piecewise constant implementation. Such implementation associates not only a discrete-time system but also a finite dimensional one. Based on Artstein transform we have deduced a feedback stabilizing law. Then, we analyzed the robustness of the closed-loop system stability when the piece-wise constant control it is used. Further researches must be made in this direction.

REFERENCES

Artstein, Z. (1982). Linear Systems with Delayed Controls: a Reduction, *IEEE Trans. Aut. Control*, vol. 27, no. 4, pp. 869-879.

Drăgan, V. and A. Halanay (1999). *Stabilization of Linear Systems*, Birkhauser Verlag, Boston.

Halanay, A. (1966). *Differential equations. Stability. Oscillations. Time Lags*, Academic Press.

Halanay, A. and Vl. Răsvan (1977). General theory of linear hybrid control, *Int. J. Control*, vol. 20, no. 4, pp. 621-634.

Ichikawa, A. (1982). Quadratic control of evolution equations with delays in control, *SIAM J. Control,* vol. 20, no. 5, pp. 645-668.

Kwon, W.H. and A.E. Pearson (1980). Feedback stabilization of linear systems with delayed control, *IEEE Trans. Aut. Control,* vol. 25, pp. 266-269.

Manitius, A. and A.W. Olbrot (1979). Finite spectrum assignment for systems with delays, *IEEE Trans. Aut. Control,* AC-24, pp. 541-553.

Mee, D.H. (1973). An extension of prediction control for systems with control time delays, *Int. J. Control*, vol. 18, no. 6, pp. 1151-1173.

Michiels, W., S. Mondie and D. Roose (2003). Necessary and sufficient conditions for a safe implementation of distributed delay control laws, *Workshop: Advances in Time-delay Systems*, Paris, pp. 85-92.

Olbrot, A.W. (1978). Stabilizability, detectability and spectrum assignment for linear systems with general time delays, *IEEE Trans. Aut. Control,* AC-23, pp. 887-890.

Pandolfi, L. (1981). A state space approach to control systems with delayed controls. In: *Proceedings of Conference on Functional Differential Equations and Related Topics* (M. Kisielewicz, Ed.), pp. 256-266, Zielona Gora, Poland.

Pandolfi, L. (1989). Dynamic stabilization of systems with input delays. In: *System Structure and Control: State Space and Polynomial Methods*, Prague.

Pandolfi, L. (1990). Generalized Control Systems, Boundary Control Systems and Delayed Control Systems, *MCSS,* vol. 3, pp. 165-181.

Pandolfi, L. (1991). Dynamic Stabilization of Systems with Input Delays, *Automatica*, vol. 27, no. 6, pp. 1047-1050.

Popescu D. and Vl. Răsvan (2003). Improved Dynamic Properties by Feedback for Systems with Delay in Control, *Analysis and Optimization of Differential Systems* (V. Barbu, I. Lasiecka, D. Tiba and C. Varsan editors), Kluwer Academic Publishers, Boston/Dordrecht/London, pp. 303-313.

Răsvan, Vl. and D. Popescu (2001a). Feedback Stabilization of Systems with Delays in Control, *Control Engineering and Applied Informatics*, vol.3, no. 2, pp. 62-66.

Răsvan, Vl. and D. Popescu (2001b). Control of Systems with Input Delay by Piecewise Constant Signals, *9th Medit. Conf. on Control and Aut.*, paper WM1-B/122, Dubrovnik, Croatia.

Tadmor, G. (1995). The Nehari problem in systems with distributed input delays is inherently finite dimensional, *Syst. & Contr. Let*. vol. 26, no. 1, pp. 11-16.

Tadmor, G. (1998). Robust Control of Systems with a Single Input Lag, In: *Stability and Control of Time Lag Systems-LNCIS*, no. 228, pp. 259-282, Springer Verlag.

Watanabe, K. and M. Ito (1981). An observer for linear feedback control laws of multivariable systems with multiple delays in controls and output, *Syst. & Contr. Let.*, vol.1, no.1, pp. 54-59.

Weiss, G. and R.F. Curtain (1997). Dynamic stabilization of regular linear systems, *IEEE Trans. Aut. Control,* vol. 42, no. 1, pp. 4-21.