A DIRECT ADAPTIVE CONTROLLER FOR A CLASS OF NONLINEAR PROCESSES USING NEURAL NETWORKS

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Abstract: This paper proposes a direct adaptive control strategy for a class of nonlinear systems for which the dynamics is incompletely known and time varying. The nonlinear controller design is based on the input-output linearizing technique. The only information required about the process is the measurements of the state variables and its relative degree. Unknown controller functions are approximated using neural networks. The form of the controller and the adaptation laws for the neural controller are derived from a Lyapunov analysis of stability. Under certain conditions the state vector remains bounded and the plant output tracks with user specified dynamics the output of a linear reference model. The technique is applied to a nonlinear and time varying biotechnological process model. Computer simulations are included to demonstrate the performances of this controller by comparison to an exactly linearizing non-adaptive controller.

Keywords: Nonlinear systems, Nonlinear control, Adaptive control, Neural networks.

1. INTRODUCTION

It is well known that traditional control design involves complex mathematical analysis and has many difficulties especially in controlling highly nonlinear and time-varying plants as well. To overcome these difficulties many researchers (Chen and Liu 1994), (Mayosky and Cancelo 1999), (McLain *et al*. 1999), (Spooner and Passino 1999), (Petre 2000, 2002) have suggested neural networks as powerful building blocks for nonlinear control strategies. The basic idea behind the neural network (NN) based control is to use a neural network estimator to identify the unknown nonlinear dynamics and compensate for it.

The main advantage of using neural networks in control applications is based both on their ability to uniformly approximate arbitrary input-output mappings and on their learning capabilities that help the controller design to be rather flexible, especially when plant dynamics are highly nonlinear and time-varying. Also, the neural network based approach can deal with the control of nonlinear systems that may not be linearly parameterizable, as required in many adaptive approaches (Hovakimyan *et al*. 2002).

In this paper a neural direct adaptive strategy for the affine class of nonlinear systems having unknown or uncertain dynamics is presented. The control signals are generated based on approximate feedback linearization technique (Isidori 1995) using the neural network approximation of the functions representing the unknown dynamics. Adaptation in this direct adaptive controller requires the on-line adjustment of the parameters of neural networks. The structure of the controller and the adaptation laws are derived in a manner similar to the classical Lyapunov based model reference adaptive control design, where the stability of the system in the presence of the adaptation is ensured. So, under certain conditions, the state vector remains bounded and the plant output tracks with user specified dynamics the output of a linear reference model. The technique is applied to a nonlinear and time varying biotechnological process model.

Computer simulations are included to demonstrate the performances of this controller by comparison to an exactly linearizing controller.

2. PROPOSED CONTROL STRATEGY

2.1. Class of nonlinear systems

Consider the class of single-input, single-output (SISO) affine nonlinear systems given by

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)u \\ y(t) &= h(x) \end{aligned} \tag{1}$$

where $x \in \Re^n$ is the state vector, $u \in \Re$ and $y \in \Re$ are the input and the output, respectively. Assume that the unknown nonlinear functions $f(\cdot), g(\cdot) \in \Re^n$ and $h \in \Re$ are smooth. If the relative degree of the system (1) is $\delta \le n$, then differentiating y with respect to time δ times, the system output dynamics may be rewritten as:

$$y^{(\delta)} = L_f^{\delta} h(x) + L_g L_f^{\delta-1} h(x) u(t) = \alpha(x) + \beta(x) u(t) \tag{2}$$

where $\alpha(x) = L_f^{\delta} h(x)$ and $\beta(x) = L_g L_f^{\delta-1} h(x)$ are Lie derivatives of the system dynamics. Also, assume that the control gain $\beta(x)$ in (2) is bounded away from zero i.e. $\beta(x) > \beta_0 > 0$.

It is well known that for the system (1a), (1b) there is a state diffeomorphism $z = \Phi(x)$ defined as

$$\Phi(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \vdots \\ \phi_\delta(x) \\ \phi_{\delta+1}(x) \\ \vdots \\ \phi_n(x) \end{bmatrix} = \begin{bmatrix} L_f^{(0)}h \\ L_f^{(1)}h \\ \vdots \\ L_f^{(\delta-1)}h \\ \phi_{\delta+1}(x) \\ \vdots \\ \phi_n(x) \end{bmatrix} = \begin{bmatrix} z_{11} \\ z_{12} \\ \vdots \\ z_{1\delta} \\ z_{21} \\ \vdots \\ z_{2,n-\delta} \end{bmatrix} \qquad (3)$$

where $L_f^{(i)}h$, $i = 0, 1, \ldots, \delta-1$ are the Lie derivatives that transform the process (1) into a normal form in the new coordinate $z = [z_1^T \; z_2^T]^T = \Phi(x)$ (Isidori 1995), (Petre 2000):

$$\begin{aligned} \dot{z}_{1k} &= z_{1,k+1}, \; k = 1, 2, \ldots, \delta-1 \\ \dot{z}_{1\delta} &= \alpha(x) + \beta(x)u(t) \\ \dot{z}_2 &= \Psi(z_1, z_2) \\ y &= z_{11} \end{aligned} \qquad (4)$$

where $z_1 = [z_{11} \ldots z_{1\delta}]^T \in \Re^\delta$ contains the process output and its $(\delta-1)$ derivatives, $z_2 = [z_{21} \ldots z_{2,n-\delta}]^T \in \Re^{n-\delta}$ are the states associated with the internal dynamics, the elements of the continuous vector function $\Psi(z_1, z_2)$ are $\psi_i(z_1, z_2) = L_f\phi_{\delta+i}(z_1, z_2)$, $i = 1, \ldots, n-\delta$ and $x = \Phi^{-1}(z_1, z_2)$. If $x = 0$ is the equilibrium point of the undriven system and $h(x) = 0$, the zero dynamics of the system are defined to be

$$\dot{z}_2 = \Psi(0, z_2) \qquad (5)$$

It was demonstrated (Sastry and Bodson 1989) that if the function $f(x)$ and $g(x)$ in (1) are known, the zero-dynamics $\Psi(0, z_2)$ are globally exponentially stable and $\Psi(z_1, z_2)$ are Lipschitz in z_1, z_2. Consequently, under the process (1) the following assumptions can be made.

Assumption 1. The process can be defined by (1) and transformed to (2) with input gain bounded by $0 < \beta_0 \le \beta(x) \le \beta_1$. Much more, the input gain rate of change is bounded by $|\dot{\beta}(x)| \le B$ where $B \in \Re$ is a finite constant. The zero dynamics of the system (1) is exponentially stable.

2.2. Problem statement

Our main objective is to design an adaptive control system which will cause the plant output y to asymptotically track a desired output trajectory y_d in the presence of unknown disturbances, using only local measurements. Usually, the desired output trajectory may be defined as the output of a stable linear reference model, with relative degree greater than or equal to δ which characterizes the desired performances. This requirement is summarized in the following assumption.

Assumption 2. The desired output trajectory and its $\delta-1$ derivatives $y_d, \dot{y}_d, \ldots, y_d^{(\delta-1)}$ are measurable and bounded.

2.3. Direct adaptive control

Assuming that the state x is accessible, firstly, in an ideal case, we may define an input-output linearizing control law, which compensates for the dynamics of the system, as

$$u^*(x, v) = \frac{-\alpha(x) + v(t)}{\beta(x)} = u_u(x, v) + u_k(x) \qquad (6)$$

where the signal v is considered as a new input that will be defined below, $u_u(x, v)$ is the unknown portion of the control law that is smooth in its arguments, and $u_k(x)$ is a known part of the control law, which is assumed to be well-defined apriori (Spooner and Passino 1999).

The direct adaptive control law is defined using a radial basis neural network (RBNN) with adjustable parameters to approximate $u_u(x, v)$.

A RBNN is made up of a collection of $p > 0$ parallel processing units called nodes. The output of the i th node is defined by a Gaussian function $\gamma_i(x) = \exp\left(-|x - c_i|^2 / \sigma_i^2\right)$, where $x \in \Re^n$ is the input to the network, c_i is the center of the i th node, and σ_i is its size of influence. The output of a RBNN, $y = F(x, W)$, may be calculated either by a weighted sum as

$$F(x, W) = \sum\nolimits_{i=1}^{p} w_i \gamma_i(x) \qquad (7)$$

or by a weighted average

$$F(x, W) = \sum\nolimits_{i=1}^{p} w_i \gamma_i(x) \Big/ \sum\nolimits_{i=1}^{p} \gamma_i(x) \qquad (8)$$

where $W = [w_1 \; w_2 \ldots w_p]^T$ is a vector of network weights. The equations (7) and (8) may be rewritten as

$$F(x, W) = W^T \Gamma(x), \qquad (9)$$

where $\Gamma(x)$ is a set of radial basis functions defined by $\Gamma(x) = [\gamma_1(x) \; \gamma_2(x) \ldots \gamma_p(x)]^T$.

Given a single RBNN, it is possible to approximate a wide variety of (nonlinear) functions $f(x)$ simply by making different choices for W. In particular, if there is a sufficient number of nodes within the network, then there is some W^* such as

$$\sup_{x \in S_x} \left| F(x, W^*) - f(x) \right| < \varepsilon \qquad (10)$$

where S_x is a compact set, and $\varepsilon > 0$ is a finite constant provided $f(x)$ is continuous (Spooner and Passino, 1999).

The ideal control function (6) may be represented by a RBNN, F_u such that:

$$u^* = F_u(x, v, W_u^*) + u_k(x) + \mu_u(x, v) \qquad (11)$$

where the vector of ideal control parameters is defined as

$$W_u^* = \arg \min_{W_u \in \Omega_u} \left[\sup_{x \in S_x,\, v \in S_v} \left| F_u(x, v, W_u) - u_u(x, v) \right| \right] \qquad (12)$$

and $\mu_u(x,v)$ is the representation error, which arises when $u_u(x,v)$ is represented by an RBNN of finite size. From the universal approximation property of RBNN, it is known that for a given approximator structure, there exists W_u^* such that $|\mu_u| \le M_u$ for some finite $M_u > 0$. The subspaces S_x and S_v are defined as compact sets through which the state trajectories x of the system and the signal v may travel. The subspace Ω_u is the convex compact set, which contains feasible parameters for W_u^*.

Assumption 3. Also, assume that the representation error $\mu_u(x,v)$ defined above is bounded by some $M_u > 0$, i.e. $|\mu_u(x,v)| \le M_u$. If $x \in L_\infty^n$, then $u_k \in L_\infty$.

Subsequently, an adaptive algorithm will be defined to estimate W_u^* with $\hat{W}_u$. These estimates are then used to define the control laws

$$u = F_u(x,v,\hat{W}_u) + u_k(x) \tag{13}$$

where $F_u(x,v,\hat{W}_u)$ is the RBNN output used to approximate an ideal controller for the system (1). A parameter error vector is defined as $\tilde{W}_u = W_u - W_u^*$.

Define by $e = [e_t, \dot{e}_t, \ldots, \overset{(\delta-1)}{e_t}]^T = \bar{y}_d - \bar{y}$ the output error vector, where $e_t = y_d - y$ is the tracking error of the system (1), and $\bar{y}_d = [y_d, \dot{y}_d, \ldots, y_d^{(\delta-1)}]^T$ and $\bar{y} = [y, \dot{y}, \ldots, y^{(\delta-1)}]^T$.

It is desired that the output error of the system (1) follows $e^{(\delta)} + \lambda_{\delta-1}e^{(\delta-1)} + \lambda_{\delta-2}e^{(\delta-2)} + \cdots + \lambda_0 e = 0$, where the coefficients $\lambda_i, i = 0,1,\ldots,\delta-1$ are picked so that the polynomial:

$$L(s) = s^\delta + \lambda_{\delta-1}s^{\delta-1} + \lambda_{\delta-2}s^{\delta-2} + \cdots + \lambda_0 \tag{14}$$

is Hurwitz. The system error dynamics may be expressed as

$$e^{(\delta)} = y_d^{(\delta)} - \alpha(x) - \beta(x)u \tag{15}$$

Adding and subtracting $\beta(x)u^*$ and using the definition of u^* from (6), one obtains

$$e^{(\delta)} = y_d^{(\delta)} - \beta(x)(u - u^*) - v(t) \tag{16}$$

Let $a_k(t)$ and $a_{\mu u}(t)$ be two scalar time functions and

$$v = y_d^{(\delta)} + \lambda_{\delta-1}e^{(\delta-1)} + \cdots + \lambda_0 e + a_k(t)\frac{e^T Pb}{2} + \beta_1 a_{\mu u}(t)\,\mathrm{sgn}(e^T Pb) \tag{17}$$

where $P \in \Re^{\delta\times\delta}$ is a positive definite matrix defined by a Lyapunov matrix equation and $b \in \Re^\delta$ is a vector. These will be both defined below.

Using the definition of v from (17), the system error dynamics may be expressed as

$$\dot{e} = \Lambda e + b\Big(-\beta(x)(u - u^*) - a_k(t)(e^T Pb)/2 - \beta_1 a_{\mu u}(t)\,\mathrm{sgn}\,(e^T Pb)\Big) \tag{18}$$

where

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\lambda_0 & -\lambda_1 & -\lambda_2 & \cdots & -\lambda_{\delta-1} \end{bmatrix} \tag{19}$$

and

$$b = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}^T \in \Re^\delta. \tag{20}$$

In the following analysis it will be used the fact that:

$$u - u^* = \tilde{W}_u^T \zeta_u - \mu_u, \tag{21}$$

where $\zeta_u^T = \dfrac{\partial F_u(x,v,W_u)}{\partial W_u}$ with $|\mu_u| \le M_u$.

Consider the following update laws:

$$\dot{\hat{W}}_u = \eta_u \zeta_u e^T Pb \tag{22}$$

$$\dot{a}_k = \eta_k (e^T Pb)^2 \tag{23}$$

$$\dot{a}_{\mu u} = \eta_\mu \, | e^T Pb | \tag{24}$$

where $\eta_u > 0$, $\eta_k > 0$ and $\eta_\mu > 0$ are adaptation gains. The update law (22) is used to estimate the dynamics of the system under control, while the update law (23) is used to stabilize the system. It can be seen that a_k in (23) monotonically increases if it is required that $a_k(0) \ge 0$. So, a projection algorithm may be required to ensure that this function does not become unnecessarily large. A result is given by the following theorem like as (Spooner and Passino 1999).

Theorem 1. Given the system (1) satisfying Assumption 1 with reference model satisfying Assumption 2, and the controller satisfying Assumption 3, then the control law (13) with adaptation laws (22), (23) and (24) will ensure that if the constant $B < \beta_0 \lambda_{\min}(R)/\lambda_{\max}(P)$ where R is a symmetric positive definite matrix and $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalues of R and P, respectively, then:

1) the system output and those derivatives, $y, \dot{y}, \ldots, y^{(\delta-1)}$, are bounded;

2) the control signal is bounded, i.e. $u_u + u_k \in L_\infty$;

3) the magnitude of output error, $|e|$, decreases asymptotically to zero, i.e. $\lim_{t\to\infty} |e| = 0$;

4) $\lim_{t\to\infty} |\dot{\hat{W}}_u| = 0$, $\lim_{t\to\infty} |\dot{a}_k| = 0$.

Proof: Consider the following Lyapunov-type function:

$$V = \frac{e^T Pe}{\beta(x)} + \frac{1}{\eta_u}\tilde{W}_u^T \tilde{W}_u + \frac{1}{2\beta_1\eta_k}\tilde{a}_k^2 + \frac{1}{\eta_\mu}\tilde{a}_{\mu u}^2 \tag{25}$$

where $\tilde{a}_k = a_k - k^*$ (k^* will be defined below), $\tilde{a}_{\mu u} = a_{\mu u} - M_u$, with $M_u > 0$, and $P \in \Re^{\delta\times\delta}$ is a symmetric positive definite matrix. Taking the time derivative of V yields:

$$\dot{V} = \frac{\dot{e}^T Pe + e^T P\dot{e}}{\beta(x)} - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} + \frac{2}{\eta_u}\tilde{W}_u^T\dot{\tilde{W}}_u + \frac{1}{\beta_1\eta_k}\tilde{a}_k\dot{\tilde{a}}_k + \frac{2}{\eta_\mu}\tilde{a}_{\mu u}\dot{\tilde{a}}_{\mu u}$$

$$= \frac{1}{\beta(x)}\Big[e^T\left(\Lambda^T P + P\Lambda\right)e - 2\beta(x)\left(u - u^*\right)e^T Pb - a_k\left(e^T Pb\right)^2 - 2\beta_1 a_{\mu u}e^T Pb\,\mathrm{sgn}(e^T Pb)\Big] - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} + \frac{2}{\eta_u}\tilde{W}_u^T\dot{\tilde{W}}_u + \frac{1}{\beta_1\eta_k}\tilde{a}_k\dot{\tilde{a}}_k + \frac{2}{\eta_\mu}\tilde{a}_{\mu u}\dot{\tilde{a}}_{\mu u} \quad (26)$$

Since Λ is negative definite, given some positive definite matrix R, there exists a unique symmetric positive definite matrix P, satisfying the Lyapunov matrix equation $\Lambda^T P + P\Lambda = R$ so that the relation (26) may be written as:

$$\dot{V} = \frac{e^T Re}{\beta(x)} - 2\left(\tilde{W}_u^T\zeta_u - \mu_u\right)e^T Pb - \frac{a_k\left(e^T Pb\right)^2}{\beta(x)} - \frac{2\beta_1 a_{\mu u}}{\beta(x)}e^T Pb\,\mathrm{sgn}(e^T Pb) - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} + \frac{2}{\eta_u}\tilde{W}_u^T\dot{\tilde{W}}_u + \frac{1}{\beta_1\eta_k}\tilde{a}_k\dot{\tilde{a}}_k + \frac{2}{\eta_\mu}\tilde{a}_{\mu u}\dot{\tilde{a}}_{\mu u}$$

$$= \frac{e^T Re}{\beta(x)} - 2\tilde{W}_u^T\left(\zeta_u e^T Pb - \frac{1}{\eta_u}\dot{\tilde{W}}_u\right) + 2\mu_u e^T Pb - \frac{a_k\left(e^T Pb\right)^2}{\beta(x)} - \frac{2\beta_1 a_{\mu u}}{\beta(x)}e^T Pb\,\mathrm{sgn}(e^T Pb) - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} + \frac{1}{\beta_1\eta_k}\left(a_k - k^*\right)\dot{\tilde{a}}_k + \frac{2}{\eta_\mu}(a_{\mu u} - M_u)\dot{\tilde{a}}_{\mu u} \quad (27)$$

Since $1/\beta_1 \le 1/\beta(x)$ and $a_k \ge 0$, $a_{\mu u} \ge 0$, from (27), one obtains:

$$\dot{V} \le \frac{e^T Re}{\beta(x)} - 2\tilde{W}_u^T\left(\zeta_u e^T Pb - \frac{1}{\eta_u}\dot{\tilde{W}}_u\right) + 2\mu_u e^T Pb - \frac{a_k\left(e^T Pb\right)^2}{\beta_1} - \frac{2\beta_1 a_{\mu u}\left|e^T Pb\right|}{\beta_1} - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} + \frac{1}{\beta_1\eta_k}\left(a_k - k^*\right)\dot{\tilde{a}}_k + \frac{2}{\eta_\mu}(a_{\mu u} - M_u)\dot{\tilde{a}}_{\mu u} \quad (28)$$

Since $\dot{\tilde{W}}_u = \dot{\hat{W}}_u$, $\dot{\tilde{a}}_k = \dot{a}_k$, and $\dot{\tilde{a}}_{\mu u} = \dot{a}_{\mu u}$, using the definition of the adaptive laws, relation (28) may be rewritten as:

$$\dot{V} \le \frac{e^T Re}{\beta(x)} - 2\tilde{W}_u^T\left(\zeta_u e^T Pb - \frac{1}{\eta_u}\dot{\hat{W}}_u\right) + 2M_u\,|\,e^T Pb\,| - \frac{a_k}{\beta_1}\left(\left(e^T Pb\right)^2 - \frac{\dot{a}_k}{\eta_k}\right) - 2a_{\mu u}\left(|\,e^T Pb\,| - \frac{\dot{a}_{\mu u}}{\eta_\mu}\right) - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} - \frac{k^*\dot{a}_k}{\beta_1\eta_k} - \frac{2M_u\dot{a}_{\mu u}}{\eta_\mu} \quad (29)$$

By choosing $\dot{\hat{W}}_u = \eta_u\zeta_u e^T Pb$, $\dot{a}_k = \eta_k(e^T Pb)^2$, $\dot{a}_{\mu u} = \eta_\mu\,|\,e^T Pb\,|$, that is the laws (22), (23) and (24), from (29) it follows that:

$$\dot{V} \le \frac{e^T Re}{\beta(x)} - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} - \frac{k^*\left(e^T Pb\right)^2}{\beta_1} \quad (30)$$

Now, we can chose $k^* = 0$ and obtain

$$\dot{V} \le -\frac{e^T Re}{\beta(x)} - \frac{e^T Pe\dot{\beta}(x)}{\beta^2(x)} \le -\frac{e^T Re}{\beta(x)} + \frac{\lambda_{\max}(P)e^T eB}{\beta^2(x)} \le -\frac{\lambda_{\min}(R)e^T e}{\beta(x)} + \frac{\lambda_{\max}(P)e^T eB}{\beta_0\beta(x)} \le \left(-\lambda_{\min}(R) + \frac{\lambda_{\max}(P)B}{\beta_0}\right)\frac{e^T e}{\beta(x)} \quad (31)$$

If B is chosen $B < \beta_0\lambda_{\min}(R)/\lambda_{\max}(P)$, then $\dot{V} < 0$. Particularly, if $\beta(x)$ is a constant, then:

$$B = 0 < \beta_0\lambda_{\min}(R)/\lambda_{\max}(P). \quad (32)$$

Required that $-\lambda_{\min}(R) + \lambda_{\max}(P)B/\beta_0 < -c$, where c is a finite constant, then:

$$\dot{V} < -\frac{c\,|\,e\,|^2}{\beta(x)} \le -\frac{c\,|\,e\,|^2}{\beta_0} \quad (33)$$

This implies that $V \in L_\infty$, and thus $|\,e\,|_2 \in L_\infty$. Given bounded reference signal, Part 1 is established. With exponential stability of zero dynamics, the system state x is bounded. Boundedness of the Lyapunov function thus ensures that $u_u + u_k \in L_\infty$, so Part 2 holds. By integrating of relation (33) from 0 to t, one obtains

$$V(0) - V(t) \ge \frac{c}{\beta_0}\int_0^t |\,e\,|^2\,d\tau \quad (34)$$

Since $V(t) > 0, \forall t > 0$, we have

$$\frac{c}{\beta_0}\int_0^t |\,e\,|^2\,d\tau \le V(0), \quad (35)$$

or, equivalently,

$$\int_0^t |\,e\,|^2\,d\tau \le \frac{\beta_0}{c}V(0), \quad (36)$$

so that $|\,e\,|_2 \in L_\infty$. Using Barbalat's Lemma, we thus establish that $\lim_{t\to\infty}|\,e\,|_2 = 0$, thus we are guaranteed asymptotically stable tracking of the system (1) so Part 3 holds. Much more, since the output y and the input of the system y_d is bounded and $\lim_{t\to\infty}|\,e\,| = 0$, convergence of the update law derivatives to zero is established by their definitions.

3. SIMULATION EXAMPLE AND REMARKS

The direct adaptive control strategy was applied to a nonlinear continuous biotechnological process for which dynamical kinetics and yield coefficients are not exactly known, described by the following differential equation system (McLain, *et al.* 1999), (Petre 2000):

$$\dot{X} = \mu(S)X - XD \tag{37}$$

$$\dot{S} = -1/Y_{x/s}\mu(S)X + (S_{in} - S)D \tag{38}$$

with X, S and S_{in}, the biomass, the substrate and the influent substrate concentrations, D the dilution rate, μ the specific growth rate and $Y_{x/s}$ the yield coefficient. The specific growth rate $\mu(\cdot)$ is described by the following nonlinear inhibited Haldane model:

$$\mu(S) = \mu_m \left(\frac{S}{K_m + S + S^2 / K_i} \right) \tag{39}$$

where μ_m is the maximum specific growth rate, K_m is the Monod constant, and K_i is the substrate inhibition constant.

The nominal values of the process parameters are:

$\mu_m^N = 0.48\ \text{h}^{-1}$, $K_m^N = 1.2$ g/l, $K_i^N = 15$ g/l,

$Y_{x/s}^N = 0.4$ g/l, $D = 0.2\ \text{h}^{-1}$, $S_{in} = 20$ g/l.

The manipulated input and controlled output are chosen as the dilution rate D and the biomass concentration X, respectively. From equation (37), it can be seen that the system has relative degree $\delta = 1$. It is straightforward to show that the associated zero-dynamics is locally stable via Jacobian linearization.

3.1. Exactly feedback controller

If we consider that the kinetics and yield coefficients in the fermentation model are known, then the exactly linearization feedback control law

$$D^* = \frac{-\mu(S)X + \lambda_1(X_d - X)}{-X} \tag{40}$$

where the reference X_d is the desired reference, leads to the following error model:

$$\dot{e} = -\lambda_1 e \tag{41}$$

with

$$e = X_d - X \tag{42}$$

It is clear that for any $\lambda_1 > 0$, the closed-loop system is uniformly asymptotic stable.

3.2. Neural network (NN) adaptive controller

Since the control law (40) contains the specific reaction rate $\mu(\cdot)$ considered unknown, this must be estimated by using a radial base neural network (RBNN). The control law (40) takes the form:

$$D = \frac{-\alpha(x) + \lambda_1(X_d - X)}{\beta(x)} = D_u(x) + D_k(x, X_d) \tag{43}$$

where $x = [X \;\; S]^T$ is the process state vector and $D_u(x) = -\alpha(x)/\beta(x)$. In (43) the function $\alpha(x) = \mu(\cdot)X$ is assumed to be unknown, and the function $\beta(x) = -X$ is assumed to be known and, much more, $|\beta(x)| \geq \beta_0 > 0$. Then a RBNN is used to construct an on-line estimate of $D_u(x)$ respectively of $\alpha(x)$ using the algorithm presented in subsection 2.3.

The reference input X_d is obtained by filtering a desired piecewise constant setpoint by the reference model

$$\dot{X}_d = -\lambda_1 X_d + \lambda_1 X_{sp} \tag{42}$$

with $\lambda_1 = 0.5$, where X_{sp} is the desired setpoint.

The centers c_i of the radial basis functions are placed in the nodes of a mesh obtained by discretization of the states $X \in [7.1, 7.7]$ and $S \in [0.65, 2.45]$ with $dX = 0.05$ g/l and $dS = 0.05$ g/l, respectively.

The responses of the closed loop system with the RBNN adaptive controller by comparison to the responses of the closed loop system with the nonlinear inverse dynamic controller are given in Fig. 1 and Fig. 2. So, in Fig. 1 it is shown the behavior of closed loop system (the controlled output X and the control inputs D_u and D respectively) in the case (1) when the cinetic coefficient μ_m is time varying as:

$$\mu_m(t) = \mu_m^N (1 - 0.05\cos(\pi t / 10)) \tag{43}$$

and in Fig. 2 it is shown the behavior of closed loop system in the case (2) when the yield coefficient $Y_{x/s}$ is time varying as:

$$Y_{x/s}(t) = Y_{s/x}^N (1 + 0.025\sin(\pi t / 12)) \tag{44}$$

It must be noted that in both cases for the adaptive controller (13), (22), (23), (24) a full radial basis neural network with 481 Gaussian units with deviation $\sigma_i = 0.05$ was used. The controller parameter vector $\hat{W}_u$ is initialized to zero, and initial values of X, S and D are $X(0) = 7.64$ g/l, $S(0) = 0.895$ g/l and $D(0) = 0.2\ \text{h}^{-1}$, respectively. The parameter update law is (22) where $\eta_u = 1.5$ and $P = I_2$ (two order unit matrix).

From Fig. 1 and Fig. 2 it can be seen that the behavior of adaptive system with NN controller is very good although the process dynamics is incompletely known and time varying.

But it is clear that the number of radial basis functions that must be activated at every time step is very big and consequently the calculus time of control D increases. To reduce the RBNN dimension and implicitly to decrease the response time of RBNN controller, a procedure likes in (Petre 2000) and (Fabri and Kadirkamanathan 1996) may be used. This reduced neural network is obtained by activating only a minimum number of radial basis functions around the reference trajectory.

It must be noted that a preliminary tuning for the NN controller is not necessary.

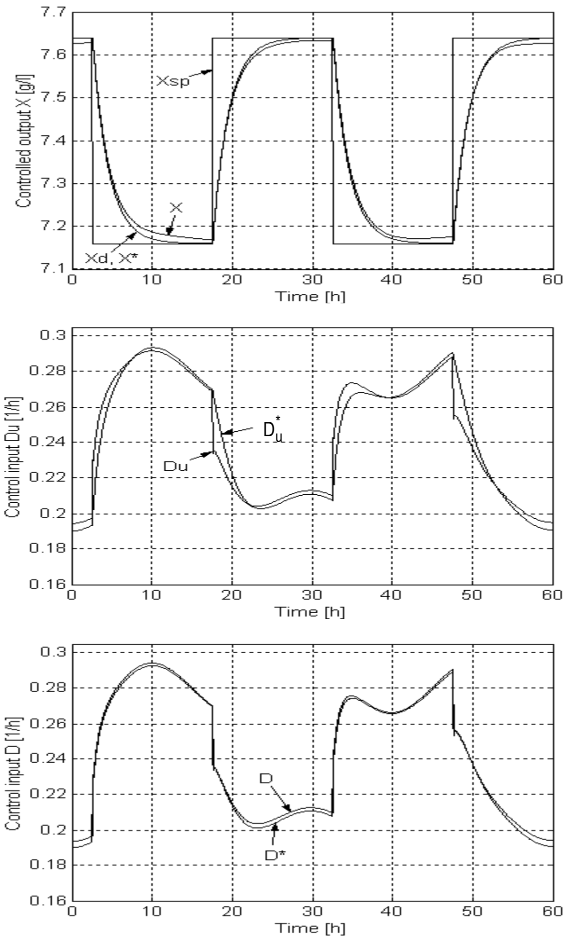

Fig. 1. Behavior of NN adaptive controller by comparison to an exactly feedback controller: *case 1*

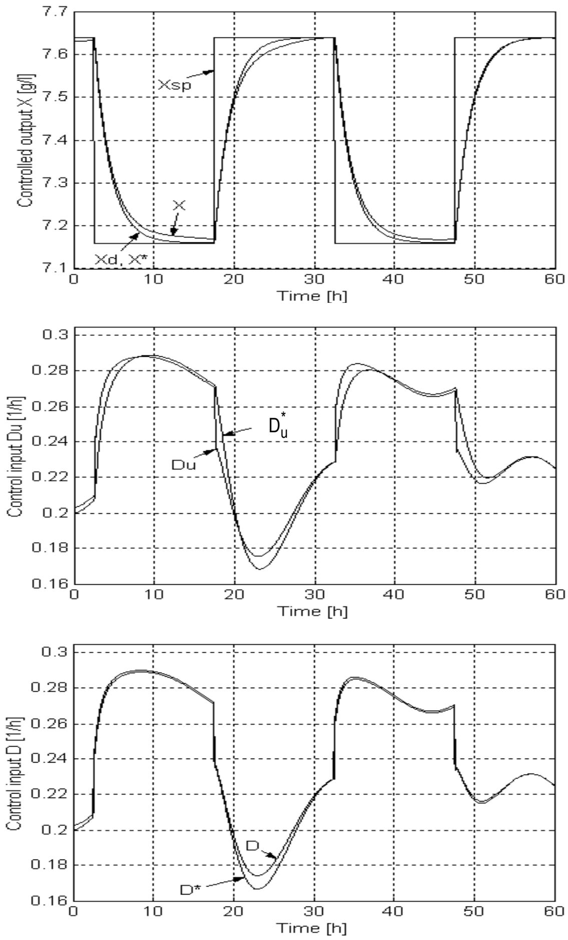

Fig. 2. Behavior of NN adaptive controller by comparison to an exactly feedback controller: *case 2*

4. CONCLUSIONS

A direct adaptive control strategy for a class of nonlinear systems for which the dynamics is incompletely known and time varying was presented. The controller design is based on the input-output linearizing technique. The unknown controller functions are approximated using radial basis neural networks. The form of the controller and the neural controller adaptation laws were derived from a Lyapunov analysis of stability. It was demonstrated that under certain conditions, the process state vector remains bounded and the plant output tracks with user specified dynamics the output of a linear reference model. The proposed algorithm was applied to a nonlinear and time varying biotechnological process. The obtained results demonstrate the effectiveness of this controller by comparison to an exactly linearizing non-adaptive controller.

REFERENCES

Chen, F.C. and C.C. Liu (1994). Adaptively controlling nonlinear continuous-time systems using multilayer neural networks. *IEEE Trans. Automat. Contr.*, **39**, **6**, pp. 1306-1310.

Fabri, S. and V. Kadirkamanathan (1996). Dynamic structure neural networks for stable adaptive control of nonlinear system. *IEEE Trans. Neural Networks*, **7**, **5**, pp. 1151-1167.

Hovakimyan, N., F. Nardi, A. Calise and N. Kim (2002). Adaptive output feedback control of uncertain nonlinear systems using single-hidden-layer neural networks. *IEEE Trans. Neural Networks*, **13**, **6**, pp. 1420-1431.

Isidori, A. (1995). *Nonlinear Control Systems*. Springer-Verlag, Berlin, Germany.

Mayosky, M.A. and G.I.E. Cancelo (1999). Direct adaptive control of wind energy conversion systems using Gaussian networks. *IEEE Trans. Neural Networks*, **10**, **4**, pp. 898-906.

McLain, R.B., M.A. Henson and M. Pottmann (1999). Direct adaptive control of partially known nonlinear systems. *IEEE Trans. Neural Networks*, **10**, **3**, pp. 714-721.

Petre, E. (2000). An adaptive controller for a class of nonlinear dynamic systems. *10-th International Symposium on Systems Theory - SINTES 10*, May 24-27, 2000, Craiova, Romania, pp. 70-74.

Petre, E. (2002). An adaptive tracking controller of a mobile robot using neural networks. *CEAI – Control Engineerig and Applied Informatics*, vol. 4, no. 3, pp. 25-32, Sept. 2002.

Sastry, S. and M. Bodson (1989). *Adaptive Control: Stability, Convergence and Robustness*, Englewood Cliffs, NJ: Prentice-Hall.

Spooner, J.T. and K.M. Passino (1999). Decentralized adaptive control of nonlinear systems using radial basis neural networks. *IEEE Trans. Automat. Contr.*, **44**, **11**, pp. 2050-2057.