

# LQG/LTR CONTROLLER DESIGN FOR FLEXIBLE LINK QUANSER REAL-TIME EXPERIMENT

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**Abstract:** This experiment involves an elastic beam, with high flexibility, fixed at one of its free end onto a rotational base. The implication is as follows: can a rotary flexible link be made to respond to angular position commands with minimal amount of vibration and overshoot? The problem therefore becomes one of having set point tracking while regulating vibration. In addition to the electromechanical model to be present in subsequent sections, a digital feedback control design is synthesized. Since the plant has two degrees of freedom but only one actuator, the system is under-actuated and exhibits significant nonlinear behavior for large pendulum excursion.

Our purpose is to design a robust controller in order to realize a real-time control of the flexible link position using a Quanser PC board and power module and the appropriate WinCon real-time software. For the controller design is used a well-known robust method, called LQG/LTR (Linear Quadratic Gauss Ian/Loop Transfer Recovery) which implements an optimal state-feedback.

The real-time experiment is realized in the Automatic Control laboratory.

**Key words:** *MATLAB/Simulink, WinCon software, robust control, real-time experiment*

## HARDWARE DESCRIPTION OF THE EXPERIMENT



Figure 1: Quanser Flexible Link experiment

The main components to focus on in connecting the Flexible Link Quanser experiment are the **Power Module**, the **DS1104 Interface Board**, and the **servo rotary motor**. With these elements, plus the necessary interface cables, we are able to run most of the plants that were designed by Quanser.

This experiment is shown in the Figure 1.

The *Universal Power Module* realizes the power adaptation between the Quanser PC board and the servo rotary motor that drives the inverted pendulum.



Figure 2: Universal Power Module

From the Figure 2, we may identify the following connectors:

- From Analog Sensors (see boxed region in figure): These are for inputs from analog sensors (e.g., potentiometers, tachometer or any other transducer) that transform a physical quantity (e.g. angle or temperature) into a DC voltage. The area “From Analog Sensors” uses “6 pin Mini to Din / 6 pin Mini Din cables” to connect the UPM-2405 to the plant.
- To A/D: This connector uses a 5 Pin Din-stereo / 4 x RCA cable to connect all 4 of the analog inputs to the Quanser PCI card. This cable is going to be referred to here as the “To A/D cable,” and it is a black cable. The RCA termination (like for your stereo) is going to be connected to the data acquisition board and the 5 in Din-stereo is going to be connected to the power module.
- From D/A: This connector uses a 5 Pin Din-mono / RCA cable to connect one analog output from the Quanser PCI board to the plant. This cable will be referred to as the “D/A cable,” and it is black. The RCA termination is hooked up to the data acquisition board and the 5 Pin Din-mono is connected to the power module.
- To Load: This connector uses a 6 pin Din / 4 pin Din, and goes from the amplifier to the actuator in the plant. This cable is going to be referred to here as the “To Load cable,” and it is black. Note that one end of the cable has six pins, and the other has four so you must match appropriately. This cable then transfers power to the plant and the analog control input to the plant.
- Other: We have other inputs and outputs that are going to be used occasionally in some of the experiments.

The supply voltage (top left of the UPM-2405) is between 12 Volts; the test points S1-S2, S3, and S4 are used to measure the outputs of the analog sensors directly at these points (e.g., to see their values on a scope).

Next, we describe how to connect the DC servo motor system that is used in the “rotational series” of the Quanser plants. The “rotational series” that we have is the SRV-02ET (E-encoder, T-tachometer), and the DC servo is shown in the Figure 3. A high quality DC servomotor is mounted in a solid aluminum frame. The motor drives a built-in Swiss-made 14:1 gearbox whose output drives an external gear. The motor gear drives a gear attached to an independent output shaft that rotates in a precisely machined aluminum ball bearing block. The output shaft is equipped with an encoder. This second gear on the output shaft drives an anti-backlash gear connected to a precision potentiometer. The potentiometer is used to measure the output angle. The external gear ratio can be changed from 1:1 to 5:1 using various gears. Two inertial loads are supplied with the system in order to examine the effect of changing inertia on closed loop performance. In the high gear ratio configuration, rotary motion modules attach to the output shaft using two 8-32 thumbscrews. The square frame allows for installations resulting in rotations about a vertical or a horizontal axis.

The motor can be used in two configurations: the low gear, and the high gear ratio. Normally, the first one is used in the position control experiments, and the high gear ratio is used when you want to place other elements, such the flexible joint, the pendulum, etc. on the DC servo. Figures 4 shows the high gear ratio that we use.



Figure 3: SRV02ET

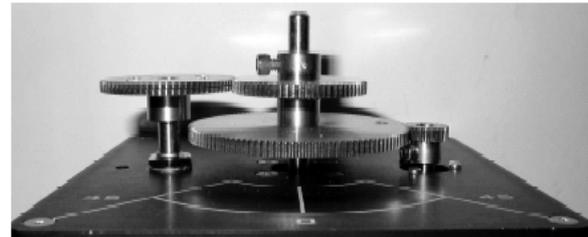


Figure 4: High gear ratio.

The PC running WinCon Server must have a compatible version of The MathWorks' MATLAB installed, in addition to Simulink, and the Real-Time Workshop toolbox. WinCon presently supports **MATLAB 5.3.x or 6.x** (i.e. Release 11.x or 12.x, respectively) with the corresponding Simulink (i.e. Version 3.0.x or 4.x, respectively) and Real-Time Workshop (i.e. Version 3.0.x or 4.x, respectively). Additionally, the Control System Toolbox can be useful for controller design.

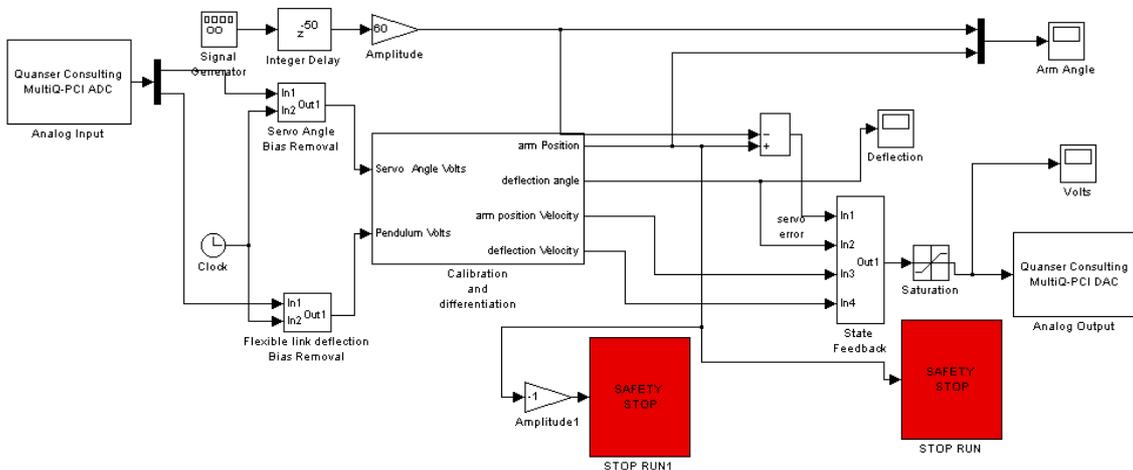


Figure 5: The Simulink model for the real-time experiment

**Simulink Real-Time Application Model**

In Figure 6 is presented the Simulink model for the real-time experiment. Using RTW (Real-Time Workshop) provided with MATLAB/Simulink, the WinCon software generate from this Simulink model the real-time code used for the flexible link control.

For real-time control purposes, the above model generates the reference for the flexible link position using a *Signal Generator*. Thus the commands are stepped.

*Servo Angle Bias Removal* and *Flexible Link deflection Bias Removal* are grouped Simulink blocks.

*Calibration and differentiation block* converts the rotating arm position voltage and deflection angle voltage to the appropriate units. Each signal is then differentiated to obtain the angular rates in degrees/sec.

**FLEXIBLE LINK NONLINEAR AND LINEAR MODEL**

The flexible link experiment is composed of a mechanical and electrical subsystem. The modeling of the mechanical model is used to describe the tip deflection and the base rotation for the experiment. The

electrical subsystem involves modeling a DC servomotor that dynamically relates voltage to torque.

The Flexible Link module consists of a flat flexible arm at the end of which is a hinged potentiometer. The flexible arm is mounted to the hinge. Measurement of the flexible arm deflection is obtained using a strain gage. The objective of the experiment is to design a feedback control system that positions the flexible arm as well as regulating vibration.

## Mechanical System Modeling

The equations of motion involving a rotary flexible link, involves modeling the rotational base and the flexible link as rigid bodies. As a simplification to the partial differential equation describing the motion of a flexible link, a lumped single degree of freedom approximation is used. We first start the derivation of the dynamic model by computing various rotational moment of inertia terms. The rotational inertia for a flexible link and a light source attachment is given respectively by

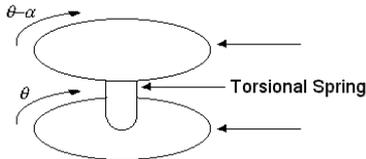
$$J_{link} = \frac{1}{3} m_{link} L^2 \quad (1)$$

where  $m_{link}$  is the total mass of the flexible link, and  $L$  is the total flexible link length.

For a single degree of freedom system, the natural frequency is related with torsional stiffness and rotational inertia in the following manner

$$\omega_n = \sqrt{\frac{K_{stiff}}{J_{link}}} \quad (2)$$

where  $\omega_n$  is found experimentally and  $K_{stiff}$  is an equivalent torsion spring constant as delineated through the following figure



In addition, any frictional damping effects between the rotary base and the flexible link are assumed negligible. Next, we derive the generalized dynamic equation for the tip and base dynamics using Lagrange's energy equations in terms of a set of generalized variables  $a$  and  $q$ , where  $a$  is the angle of tip deflection and  $q$  is the base rotation given in the following

$$\begin{aligned} \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{\theta}} \right) - \frac{\partial T}{\partial \theta} + \frac{\partial P}{\partial \theta} &= Q_p, \\ \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{\alpha}} \right) - \frac{\partial T}{\partial \alpha} + \frac{\partial P}{\partial \alpha} &= Q_a, \end{aligned} \quad (3), (4)$$

where  $T$  is the total kinetic energy of the system,  $P$  is the total potential energy of the system, and  $Q_i$  is the  $i$ th-generalized force within the  $i$ th degree of freedom. Kinetic energy of the the base and the flexible link are given respectively as

$$\begin{aligned} T_{base} &= \frac{1}{2} J_{base} \dot{\theta}^2 \\ T_{link} &= \frac{1}{2} J_{link} (\dot{\theta} - \dot{\alpha})^2 \end{aligned} \quad (5), (6)$$

The total kinetic energy of the mechanical system is computed as the sum of (5) and (6)

$$T = \frac{1}{2} J_{base} \dot{\theta}^2 + \frac{1}{2} J_{link} (\dot{\theta} - \dot{\alpha})^2 \quad (7)$$

Potential energy of the system provided by the torsional spring given as

$$P = \frac{1}{2} K_{stiff} \alpha^2. \quad (8)$$

Applying equation (7) and (8) into (3) and (4) results in the following dynamic equations

$$\begin{aligned} (J_{base} + J_{link}) \ddot{\theta} - J_{link} \ddot{\alpha} &= Q_p \\ -J_{link} \ddot{\theta} + J_{link} \ddot{\alpha} + K_{stiff} \alpha &= Q_a \end{aligned} \quad (9), (10)$$

Next we compute the amount of virtual work,  $W$ , applied into the system. The amount of virtual work is given to be

$$\delta W = \tau \delta \theta + 0 \delta \alpha \quad (11)$$

where  $\tau$  is the torque applied to the rotational base. Rewriting equation (13) into a general form of virtual work given as

$$\delta W = Q_p \delta \theta + Q_a \delta \alpha. \quad (12)$$

We obtain the virtual forces applied onto the generalized coordinates  $Q_\theta$  and  $Q_\alpha$ , respectively to be

$$\begin{aligned} Q_p &= \tau, \\ Q_a &= 0. \end{aligned} \quad (13), (14)$$

After decoupling the acceleration terms of (9) and (10), the dynamic equations for the mechanical subsystem are

$$\ddot{\theta} = -\frac{K_{stiff}}{J_{base}} \alpha + \frac{1}{J_{base}} \tau \quad (15), (16)$$

$$\ddot{\alpha} = -K_{stiff} \left( \frac{1}{J_{link}} + \frac{1}{J_{base}} \right) \alpha + \frac{1}{J_{base}} \tau$$

Next, rewriting equations (15) and (16) into a state space form gives

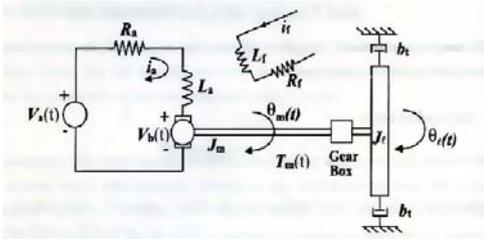
$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \ddot{\theta} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{stiff}}{J_{base}} & 0 & 0 \\ 0 & -K_{stiff} \left( \frac{1}{J_{link}} + \frac{1}{J_{base}} \right) & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_{base}} \\ \frac{1}{J_{base}} \end{bmatrix} \tau \quad (17)$$

## Electrical System Modeling

Since the control input into the mechanical model of equation (19) is a torque  $\tau$ , an electrical dynamic equation relating voltage to torque is needed. First, the torque applied to the rotational base, on the right hand side of equation (19), is converted to the torque applied to the gear train by the DC servomotor by means of a gear ratio  $K_g$  given as

$$\tau = K_g \tau_m, \quad (18)$$

where  $\tau_m$  is the torque applied by the servomotor. Next, the circuitry within the DC motor system consists of a resistor, an inductor, an external voltage source, and a back emf placed in a series circuit. The following figure describes the circuit as previously described



Applying Kirchoff's voltage law to the above figure, produces

$$V_{PC}(t) = I(t)R + L \frac{dI(t)}{dt} + V_b(t), \quad (19)$$

where  $I$  is the current in the circuit,  $R$  is the resistor,  $L$  is the inductor,  $V_b$  is the back emf, and  $V_{PC}$  is the voltage supplied by the data acquisition board. The DC servomotor is an electromechanical device that relates torque to current through a proportionality gain  $K_T$  given in the following equation

$$\tau_m = I(t)K_T. \quad (20)$$

In addition, the back emf is a voltage applied by the motor shaft to the circuit, which is directly proportional to the angular velocity of the motor

$$V_b(t) = K_b \omega_m, \quad (21)$$

where  $K_b$  is the back emf constant and  $\omega_m$  is the angular velocity of the motor. Relating the motor angular velocity with the base angular velocity gives

$$V_b(t) = K_b K_g \omega_b(t) = K_b K_g \dot{\theta}. \quad (22)$$

Substituting equations (22) and (24) into (21) gives

$$V_{PC}(t) = \tau_m(t) \frac{R}{K_T} + \left( \frac{L}{K_T} \right) \frac{d\tau_m(t)}{dt} + K_b K_g \dot{\theta}(t). \quad (23)$$

Since the effect of the inductance in the circuit is relatively small in comparison with other circuit components, the derivative term of torque can be neglected eliminated to give the following approximate proportionality equation between voltage to torque and angular velocity

$$V_{PC}(t) = \tau_m(t) \frac{R}{K_T} + K_b K_g \dot{\theta}(t). \quad (24)$$

The torque applied by the motor is solved for in equation (26) to be

$$\tau_m(t) = \frac{K_T V_{PC}(t)}{R} - \frac{K_b K_T K_g \dot{\theta}(t)}{R}. \quad (25)$$

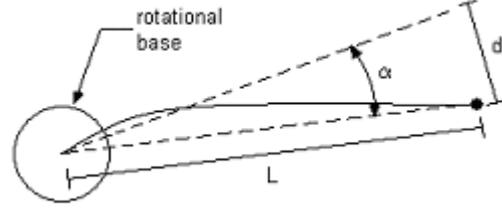
As a result, the state space model of (19) can be rewritten to utilize an electrical control voltage in (20) and (27) to give

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{stiff}}{J_{base}} & -\frac{K_T K_b K_g^2}{J_{base} R} & 0 \\ 0 & -K_{stiff} \left( \frac{1}{J_{link}} + \frac{1}{J_{base}} \right) & -\frac{K_T K_b K_g^2}{J_{base} R} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_T K_g}{J_{base} R} \\ -\frac{K_T K_g}{J_{base} R} \end{bmatrix} V_{PC} \quad (26)$$

Next, a transformation between relative angular position and relative displacement about a neutral axis is used within the state space model of (28). The relative angular position, velocity with respect to the rotating base is proportional to the relative displacement, velocity of the flexible link tip (i.e.  $\sin(\alpha) \approx \alpha$ )

$$\begin{aligned} d &= \alpha L, \\ \dot{d} &= \dot{\alpha} L. \end{aligned} \quad (27), (28)$$

The following figure shows the relationship of these three parameters.



Substituting the above equations of (29) and (30) into the state space dynamics of (28) gives the following state space equation

$$\begin{bmatrix} \dot{\theta} \\ \dot{d} \\ \ddot{\theta} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{stiff}}{J_{base} L} & -\frac{K_T K_b K_g^2}{J_{base} R} & 0 \\ 0 & -\frac{K_{stiff}}{L} \left( \frac{1}{J_{link}} + \frac{1}{J_{base}} \right) & -\frac{K_T K_b K_g^2}{J_{base} R} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ d \\ \dot{\theta} \\ \dot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_T K_g}{J_{base} R} \\ -\frac{K_T K_g}{J_{base} R} \end{bmatrix} V_{PC}. \quad (29)$$

## Control Objective

The objective for the rotary flexible link dynamic system is to achieve an asymptotically stable system response for flexible link. A Linear Quadratic Regulator (LQR) based controller achieves asymptotic stable response for a controllable state space model. For the state variable of  $d(t)$ , an LQR controller drives the flexible dynamic response to zero asymptotically. However, for the angular position tracking of a new state variable is required to allow for set point tracking. To achieve error regulation for an angular displacement error and an angular velocity error term is defined respectively as

$$\begin{aligned} e(t) &\triangleq \theta(t) - \theta_d, \\ \dot{e}(t) &\triangleq \dot{\theta}(t), \end{aligned} \quad (30), (31)$$

where  $\theta_d$  is a desired constant angular position for the flexible link. In addition, an integral controller coupled in the rigid body dynamics is defined within the state space dynamics of (31)

$$\dot{\phi}(t) = e(t), \quad (32)$$

so that the state space dynamics of (31) is augmented with (32) to give the linear model

## Controller Design

Given a 5th order system given in equation (35), the control objective involves error regulation for the absolute angular displacement of the rotary base and vibration control for the end of the flexible link. A full state feedback control law given by

$$u(k) = Kx(k), \quad (36)$$

where  $x(k) \in \mathfrak{R}^5$ ,  $u(k) \in \mathfrak{R}$ ,

such that the full state feedback control law of (36) satisfies the following criteria

I The closed loop state space system is asymptotically stable

II The performance functional given by

$$J(K) = \sum_{k=0}^{\infty} x^T(k)Qx(k) + u^T(k)Ru(k) \quad (37)$$

where  $x(k)$  are the state vectors and  $u(k)$  are the control inputs, respectively. The performance functional of equation (38) regulates the state trajectories of  $x(k)$  close to the origin without excessive control demand through the design of the penalty weights of  $Q$  and  $R$ . The nonnegative definite matrix  $Q \in \mathcal{R}^{5 \times 5}$ , determines the weight placed on each component of state. The nonnegative definite matrix  $R \in \mathcal{R}^1$  determines the weight placed on the control input. The state feedback control law given in (36) is computed through the following matrix equation

$$K = -R_{2a}^{-1}P_a, \quad (38)$$

where  $R_{2a}$  and  $P_a$  are defined as

$$\begin{aligned} R_{2a} &\stackrel{\Delta}{=} R + B^T P B, \\ P_a &\stackrel{\Delta}{=} B^T P A, \end{aligned} \quad (39)$$

such that the nonnegative definite matrix  $P$  solves the following Riccati equation

$$P = A^T P A + Q - P_a^T R_{2a}^{-1} P_a. \quad (40)$$

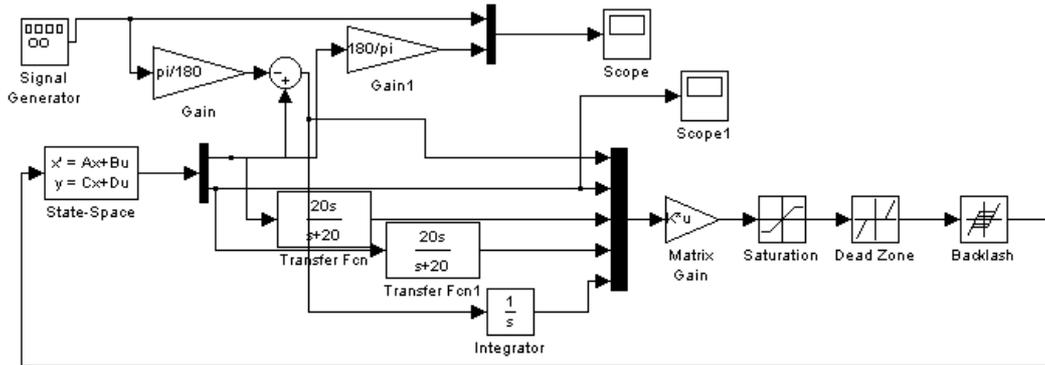


Figure 8. Simulink model of the flexible link control architecture

Using the above described controller design method, we are able to obtain the state-feedback vector.

For example, if we use

$$k = [.25, -.6, .005, .01, .02]$$

we obtain the following real-system behavior (which is compared with the simulated one).

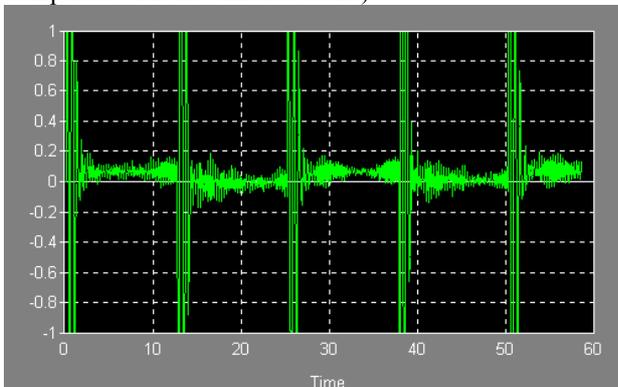


Figure 9. Flexible link angle deflection (in degrees)

The computation of equation (40) is performed numerically using MATLAB.

For our laboratory model, the flexible arm angle  $\theta$  and the flexible link deflection  $\alpha$  are measured by a potentiometer and a strain gage. The flexible arm angular velocity  $\dot{\theta}$  and flexible link deflection velocity  $\dot{\alpha}$  are not measured by any physical sensor, instead, we numerically compute  $\dot{\theta}$  and  $\dot{\alpha}$  by implementing a low-pass differentiator, e.g.  $\frac{20s}{s+20}$ , as a part of the overall control scheme.

In order to design an LQR controller, we need the plant dynamic parameters  $A$  and input matrix  $B$ .

## EXPERIMENTAL RESULTS VERSUS SIMULATION RESULTS

The LQG/LTR design method uses a linear model for the real plant. Due to nonlinearities the real experimental behavior is not identical with the simulated one.

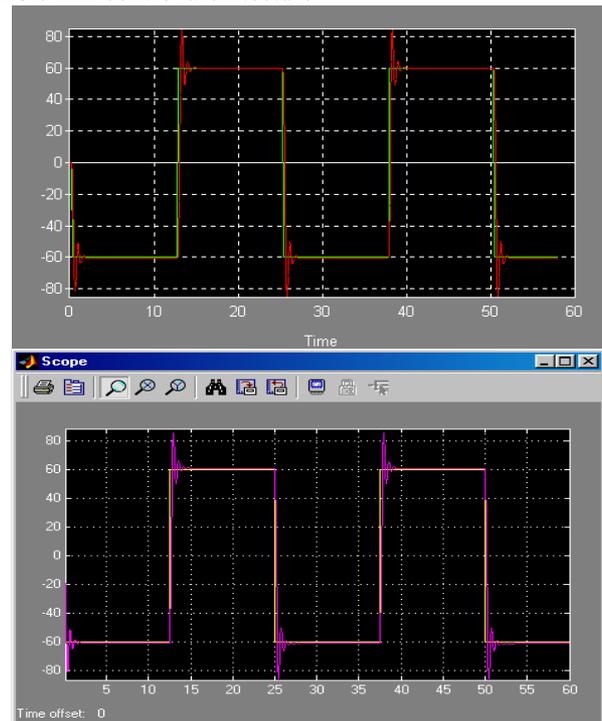


Figure 10. Real and simulated flexible link position

Using another state feedback vector, we obtain:

$$k=[.05,-.4,.005,.01,.02]$$

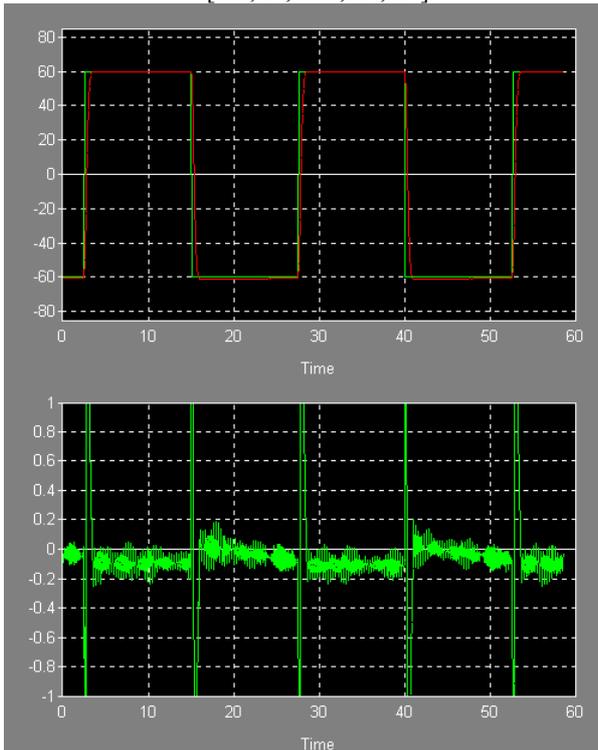


Figure 11. Flexible link position and deflection angle

Another real-time experiment tests the perturbation rejection capability of the control system.

As we can see from Figure 11 and Figure 12, we obtain very good reference track and vibration suppression performances. Also, if the servo angle or flexible link deflection angle is perturbed, the control system acts fast and precise.

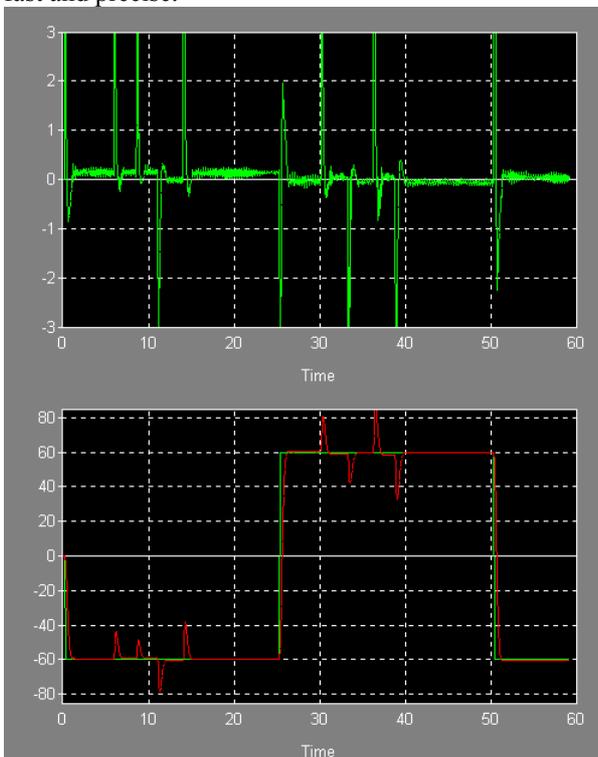


Figure 12. Perturbations rejection

## CONCLUSIONS

In this article is presented a Quanser Consulting Inc. laboratory experiment: the Flexible Link reference track and vibration suppression.

Using a single command, the DC motor voltage, we are able to control two outputs: the rotary flexible link angular position and deflection angle..

The controller design method is LQG/LTR, a robust state feedback.

Using a set of experiments is developed a nonlinear model of the rotary inverted pendulum. Using MATLAB/Simulink design environment we are able to obtain the state feedback with an integral over the flexible link angular error. The experimental results are presented using Simulink and WinCon plots.

## REFERENCES

- [1] \*\*\* Quanser Consulting Inc., *MultiQ, Programming Manual*
- [2] \*\*\* Quanser Consulting Inc., *WinCon 3.2, Real-Time Digital Signal Processing and Control under Windows 95/98 and Windows NT using Simulink and TCP/IP Technology*
- [3] \*\*\* Quanser Consulting Inc., *Flexible Link Experiment ROTFLEX*, 1998
- [4] Stein, G., M., Athans (1987) *The LQG/LTR procedure for multivariable feedback control design*, IEEE Trans. Automat. Control, AC-32, pp. 105-114
- [5] R.Y. Chiang, M.G. Safonov: *"Robust Control Toolbox"*, The MathWorks, Inc., 1992.
- [6] S. Skogestad, I. Postlethwaite: *"Multivariable Feedback Control, Analysis and design"*, John Wiley & Sons, 1995
- [7] Ivanescu M. (1994), *Industrial robots*, Ed. Universitaria, Craiova, pp. 149-186
- [8] Ortega R., Spong M.W. (1989), *Adaptive motion control of rigid robots: a tutorial*, Automatica, 25, pp. 877-888.
- [9] M. Gopal: *"Modern Control System Theory"*, Second Edition, John Wiley & Sons, 1993.
- [10] J.C. Doyle, G. Stein: *"Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis"*, IEEE Trans. on Automat. Control, AC-26, pp. 4-16, 1981.