FAULT DETECTION AND ISOLATION USING STATISTICAL SIGNAL PROCESSING

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Abstract: In this work, statistical signals processing are employed in fault detection and isolation scheme. The procedure was applied to detect the failed sensor and suppose the use of mathematical model of process and the correlation (covariance) function calculated in real time between the signals, which represent the commands, and the measurements. If the values of these statistical functions are great, that means the signals are correlated. When these values are small and this situation is preserved long time that means we have a failed sensor.
The technique was tested by simulation and the results are good.

Key words: Fault detection and isolation, analytical redundancy, statistical signal processing.

INTRODUCTION

Changes (faults) can make the system unsafe and less reliable. Productivity of the automatic system can degrade because changes can impose performance limitations on the system and may also require frequent system shut downs for its maintenance. In the case of technologically challenging applications, like space or underwater technology, where a system's full automation is expected, the presence of changes can limit what engineers can accomplish in their designs. The finally effect of the changes is on environmental and human safety, cost and ability of creation of autonomous system.

Fortunately, al of the above-described situations can be managed by giving the system *self-diagnostic* capabilities, which allow it to detect any changes, analyze them and handle them appropriately. The system's ability to learn how its environment has changed makes it more self-sufficient and intelligent, and improves its behavioral decision. Self-diagnosis of the system can be accomplished by the introduction of either analytical or hardware redundancy. In the hardware redundancy approach, additional physical instrumentation is introduced, sensors for instance. In the analytical redundancy approach, additional software is introduced which usually employs model-based techniques (Kmelnitsky, 2002), (Polycarpou, Helmicki, 1995), (Polycarpou, Trunov, 2000).

Analytical redundancy is less expensive, much easier to upgrade and has more potential. It requires a lot of computational resources, because of its on-line application (Kmelnitsky, 2002).

STATISTICAL SIGNAL PROCESSING

Let consider the signal $x(t)$ and $y(t)$ like stationary random processes. The function of cross-correlation is a statistical quantity defined as:

$$R_{xy}(\tau) = E\{x(t)y(t+\tau)\} \tag{1}$$

Also, the cross-covariance is the mean-removed:

$$C_{xy}(\tau) = E\{(x(t)-\mu_x)(y(t+\tau)-\mu_y)\} \tag{2}$$

or, in terms of the cross-correlation

$$C_{xy}(\tau) = R_{xy}(\tau) - \mu_x\mu_y \tag{3}$$

where μ_x and μ_y are the mean values.

For continuous stochastic process, the cross-correlation function is:

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)y(t+\tau)dt \tag{4}$$

In practice, it is necessary to estimate this sequence, because it is possible to access only a finite segment of the infinite-random process. A common estimate based on N samples of $x(t)$ and $y(t)$ (x_n and y_n) is the deterministic cross-correlation (5) also called the time-ambiguity function.

$$R_{xy}(m) = \sum_{n=0}^{N-m-1} x_n y_{n+m}, \; m \geq 0 \tag{5}$$

where we assume for this discussion that x_n and y_n are indexed from 0 to N-1 (MathWorks, 1999). In the same conditions, the cross-corelation function (3) and the mean values (μ_x and μ_y) have the expressions (6), (7) and (8).

$$C_{xy}(m) = \sum_{n=0}^{N-m-1} x_n y_{n+m} - \mu_x \mu_y, \; m \ge 0 \qquad (6)$$

$$\mu_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_n \qquad (7)$$

$$\mu_y = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} y_n \qquad (8)$$

An important parameter who characterize the correlated process is the cross-correlation coefficient (Spataru, 1987):

$$\rho(x_n, y_n) = \frac{C_{xy}}{\sigma_x \sigma_y} \qquad (9)$$

where

$$\sigma_x = \sqrt{D_x} \qquad (10)$$

$$\sigma_y = \sqrt{D_y} \qquad (11)$$

and D_x , D_y represent the variances with the expressions:

$$D_x = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (x_n - \mu_x)^2 \qquad (12)$$

$$D_y = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (y_n - \mu_y)^2 \qquad (13)$$

The values of coefficient (9) are limited to the interval:

$$0 \le |\rho(x_n, y_n)| \le 1 \qquad (14)$$

For $|\rho|=1$ we have two stochastic processes full correlated, during for $\rho=0$, the processes are uncorrelated (it is very possible to be independents).

FAULT DETECTION AND ISOLATION SCHEME

The structure used for fault detection and isolation is represented in fig. 1. In this case, the output of the process is measured by two identically sensors. If $p < 1$ is the probability to have one failed sensor, the probability to have simultaneous two failed sensors (considered like two independent process) is:

$$p^2 << p \qquad (15)$$

To install two sensors is not a difficult problem, but in this case the voting method is inapplicable. The author propose for the *Fault management decision block* (fig.1) the structure represented in fig. 2.

The idea consists to processing the signal purchased from the process, $u(t)$ and $z_i(t), i = 1,2$ and to calculate an equivalent output signal $z_e(t)$ and to generate an alarm signal if the failures arise. The steps are the next:

- On calculate the cross-correlations function and the cross-correlation coefficients for $u(t)$ and $z_i(t), i = 1,2$.
- If

$$\rho(u_n, z_{1,n+m}) \ge \rho_0 \qquad (16)$$

and

$$\rho(u_n, z_{2,n+m}) \ge \rho_0 \qquad (17)$$

we can accept that the both sensors are in good conditions; ρ_0 is a decision threshold and m is fixed in function by the time delay constant of the sensors.
- The value of equivalent output signal $z_e(t)$ is calculated with the formula:

$$z_e(t) = \begin{cases} \frac{1}{2}[r_1 z_1(t) + r_2 z_2(t)], & \text{if } r_1 = r_2 = 1 \\ r_1 z_1(t), & \text{if } r_1 = 1 \text{ and } r_2 = 0 \\ r_2 z_2(t), & \text{if } r_1 = 0 \text{ and } r_2 = 1 \end{cases} \qquad (18)$$

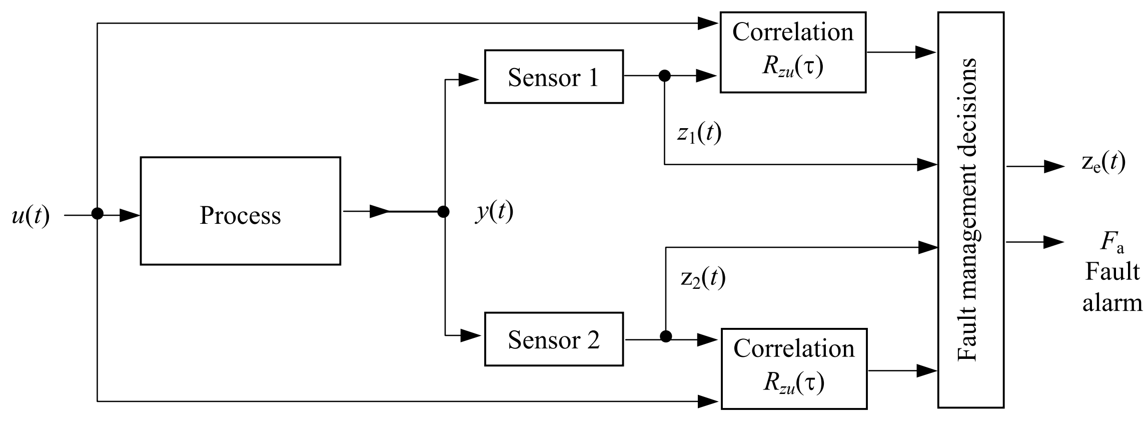
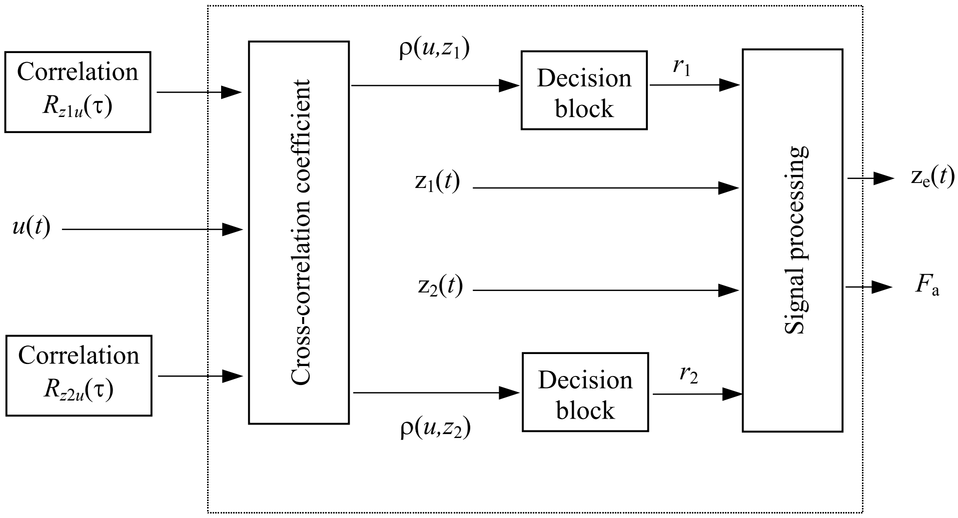


Fig 1. The structure with two sensors.



where r_i, i=1,2, are coefficients calculated in interaction by the two correlation coefficients: $\rho(u,z_1)$ and $\rho(u,z_2)$. If the both sensors are fault free, r_i=1. If on decide that one of sensors is failed, r_i=0, for eliminate the influence of erroneous signal. So, we can say $r_i \in \{0,1\}$.

The decision block from fig. 2 has the internal law:

$$r_i = \begin{cases} 1, \text{ if } \rho(u, z_i) \geq \rho_0 \\ 0, \text{ if } \rho(u, z_i) < \rho_0 \end{cases}, i = 1,2 \qquad (19)$$

The structure of the signal processing block (fig. 2) is a complex one and is represented in fig. 3.

The intermediary signal $|z_1r_1 - z_2r_2|$ can be used like fault alarm signal F_a. If the both sensors are in good conditions (fault free), than

$$\left| z_1(t)r_1 - z_2(t)r_2 \right| \approx 0 \qquad (20)$$

If a failures is arising, than

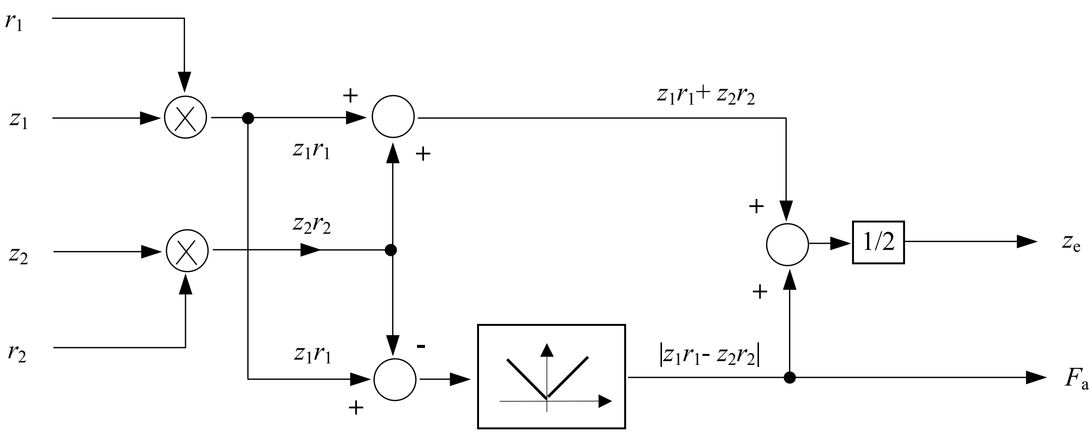
$$\left| z_1(t)r_1 - z_2(t)r_2 \right| \neq 0 \qquad (21)$$

Another application is represented by the case when we use a single sensor. A possibility to detect a failure, at the sensor level, consist in the utilization of a similarly scheme (fig. 4). This time, the role of the second sensor is assumed by the mathematical model, which represents the entire process (input-output). The structures of the blocks for fault management and for signal processing are the same.

APPLICATION FOR FLIGHT CONTROL SYSTEM

We consider an aircraft during the landing phase (fig. 5). The mathematical model is (Shin, Krishna, Yann-Hang, 1985):

$$\begin{cases} \dot{\omega}_y(t) = -0.6\omega_y(t) - 0.76\Theta(t) + 0.003w(t) + 2.34u_p(t) \\ \dot{\Theta}(t) = \omega_y(t) \\ \dot{w}(t) = 102.4\Theta(t) - 0.4w(t) \end{cases} \qquad (22)$$



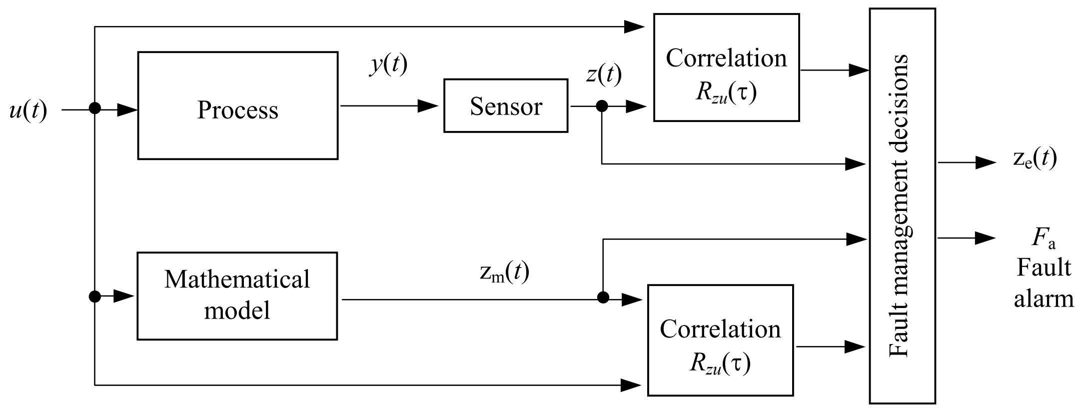


Fig. 5. The structure with one sensor and mathematical model.

where:

$\omega_y(t)$ - pitch angle rate.

$\Theta(t)$ - pitch angle.

$w(t)$ - altitude rate.

$u_p(t)$ - elevon deflection.

The aircraft is unstable, the eingevalues being:

λ_1=-0.5016+j0.8670
λ_2=-0.5016-j0.8670
λ_3= 0.0032

After LQR compensation:

$$J = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Vu(t)\right]dt \tag{23}$$

the matrix K has the expression:

$$K = V^{-1}B^T P \tag{24}$$

where P is the solution of Ricatti algebraic equation:

$$PA + A^T P - PBV^{-1}B^T P + Q = 0 \tag{25}$$

In this condition K matrix is:

$$K = [4.9415 \;\; 30.7630 \;\; 0.8799] \tag{26}$$

and the eingevalues for the compensate system are:

λ_1=-3.1839+j5.2977
λ_2=-3.1839+j5.2977
λ_3=-6.3634

In fig. 7 are represented the stabilized structure of the aircraft. Very important in this phase of the landing is the vertically speed $w(t)$. So, the algorithm described above was tested by simulation.

First, on considers the case when the signal is measured by two sensors (fig. 1). For simplify the study, let be the transfer function of the sensor as the form:

$$H_T(s) = \frac{1}{0.1s + 1} \tag{27}$$

or equivalent the input-output equation is:

$$\dot{y}_T(t) = -10 y_T(t) + 10u(t) \tag{28}$$

The simulated failure consist in a disconnection, which appear at time t_0 , of the second sensor. The real signal is substitute, from this moment by a noise with zero mean.

The experimental results are represented in figures 7 (vertically speed reference for control system), 8 (the amplitude of command for elevon actuator), 9 (vertically speed w), 10 (pitch angle rate) and 11 (pitch angle).

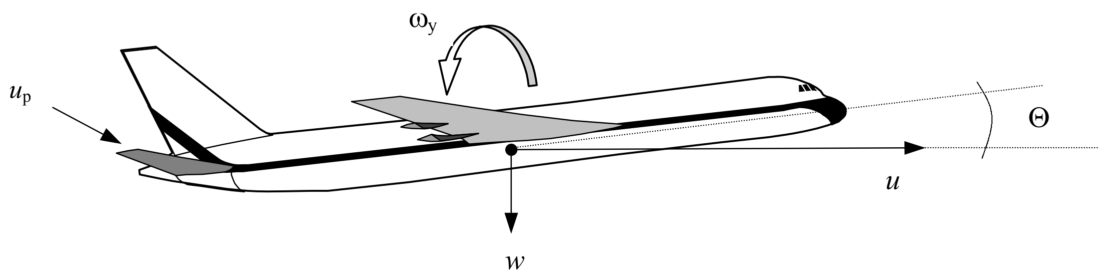


Fig. 6. The aircraft during the landing phase.

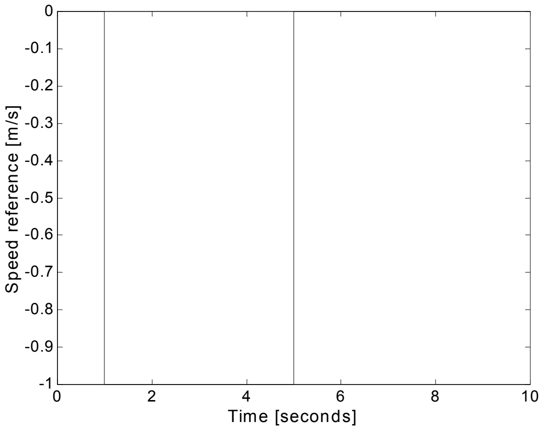


Fig. 7. Vertically speed reference

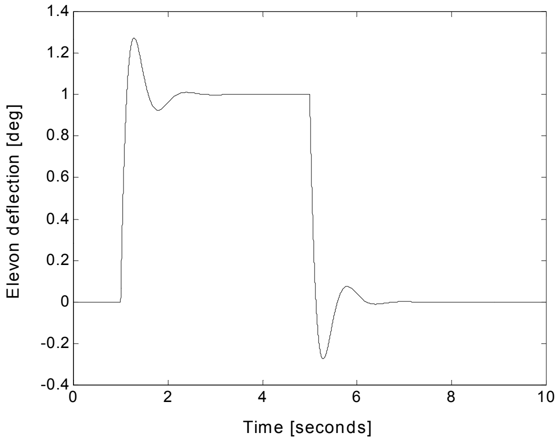


Fig. 8. The amplitude of command for elevon actuator.

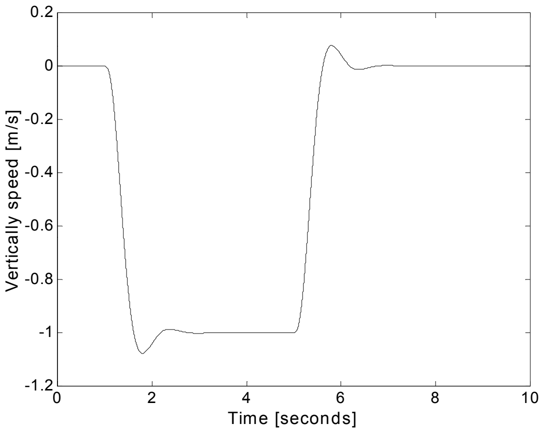


Fig. 9. Vertically speed w (both sensors are fault free).

We have considered two sensors for vertically speed w. Also, the speed reference (u) and the altitude rate measurement (z_i) are supposed stochastic process. For 10 seconds data acquisition, the correlation coefficients have the values:

$$\rho(u_i, w_i) = 0.8494, i = 1, 2. \quad (29)$$

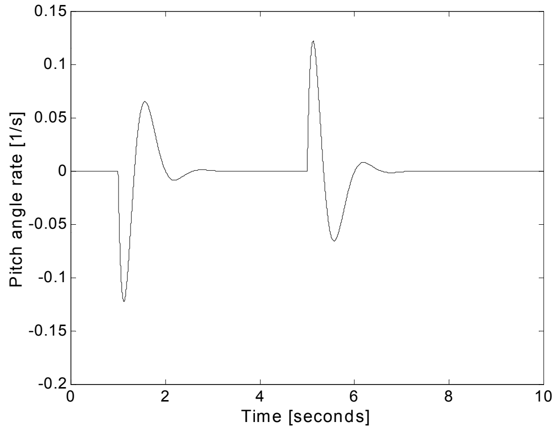


Fig. 10. Pitch angle rate.

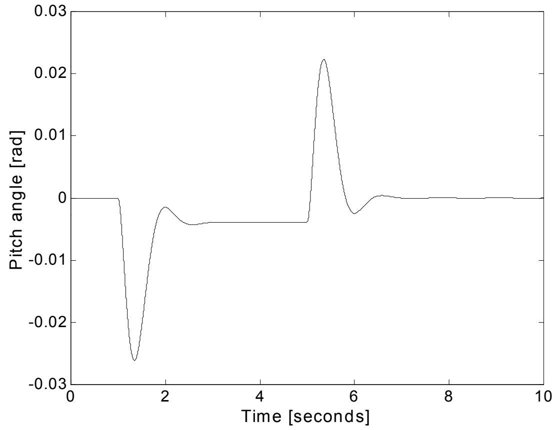


Fig. 11. Pitch angle.

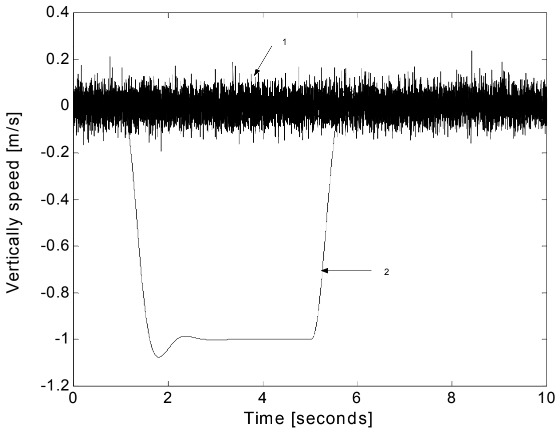


Fig. 12. Vertically speed w.
1 - signal from failed sensor (disconnect); 2 - signal from the sensor that is in good condition.

If we suppose that one of sensors is disconnected we have the situation revealed in fig. 12. The coefficients are in this case:

$$\rho(u, z_1) = 38 \cdot 10^{-6};\ \rho(u, z_2) = 0.8494 \quad (30)$$

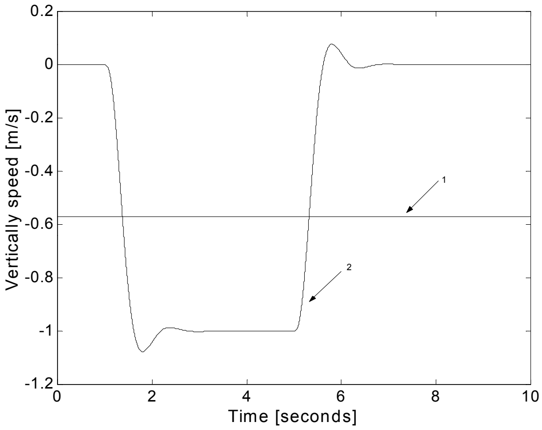


Fig. 13. Vertically speed *w*.
1 - signal from failed sensor (blocked); 2 - signal from the sensor that is in good condition.

If we have a decision threshold ρ_0 (e.g. $\rho_0 = 0.5$) it is possible to select the correct signals. In correspondence with (19) we have for (29):

$$r_1 = r_2 = 1 \qquad (31)$$

and for (30):

$$\begin{aligned} r_1 &= 0 \\ r_2 &= 1 \end{aligned} \qquad (32)$$

We have the same situation when the output y_T of the failed sensor is constant in time (fig. 13).

With the block represented in fig. 3, is selected always the correct signal. In the second case, it is generate an alarm signal.

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