A PRACTICAL CONTROL METHOD FOR MULTIVARIABLE SYSTEMS

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Abstract: The main purpose of the paper is to present a robust practical method for experimentally decoupling, compensating and control of two input-two output process. The decoupler channels are first order lag plus dead time elements, which satisfy the following requirements: the direct channels have unit gain and two channels have dead time equal to zero. The decoupler can be simplified in addition taking two appropriate channels with the lag time constant equal to zero. After decoupling, each output of the decoupled process is controlled by a special method, which consists in monotonic compensating and standard IMC control of the both direct channels of the decoupled process. The results obtained by simulation validate the proposed control procedure.

***Key words*:** decoupling controller, decoupler, monotonic compensation, standard IMC algorithm.

1. INTRODUCTION

Many industrial processes are multivariable, exhibiting input-output cross-coupling which cannot be neglected because it provides difficulties in process control. Multivariable process control using monovariable controllers cannot yield satisfactory performance, because of mutual interactions between monovariable loops. The multivariable controller use makes possible partial or total elimination (only in the theoretical case) of these self-disturbing interactions. Usually, a multivariable controller with n inputs and n outputs consists of a block with n monovariable controllers (possibly of PID type) and a process decoupling block. Using a decoupling controller, the system tuning problem is reduced to the independent tuning of the monovariable controllers of every control loop. Most of the controller synthesis methods for multivariable control systems are based on knowing the process model as precise as possible. The relation between the process, decoupler and decoupled process transfer matrixes has the form:

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}=\begin{bmatrix} (G_{11})_d & 0 \\ 0 & (G_{22})_d \end{bmatrix}. \tag{1}$$

From decoupling equations

$$\begin{cases} G_{11}D_{12}+G_{12}D_{22}=0 \\ G_{21}D_{11}+G_{22}D_{21}=0 \end{cases}, \tag{2}$$

we obtain the decoupled transfer functions:

$$\begin{cases} (G_{11})_d=\dfrac{(1-f)D_{11}}{G_{11}} \\ (G_{22})_d=\dfrac{(1-f)D_{22}}{G_{22}} \end{cases}, \tag{3}$$

where

$$f=\frac{G_{12}G_{21}}{G_{11}G_{22}} \tag{4}$$

is the *process coupling factor*. Because there are two decoupling equations and the decoupler has four transfer functions, two of the decoupler transfer functions, usually the diagonal transfer functions D_{11} and D_{22}, can be arbitrary chosen. The non-diagonal transfer functions of the decoupler are then given by the expressions:

$$\begin{cases} D_{12}=\dfrac{-G_{12}D_{22}}{G_{11}} \\ D_{21}=\dfrac{-G_{21}D_{11}}{G_{22}} \end{cases} \tag{5}$$

Usually, to have a simple decoupler structure, the diagonal transfer functions D_{11} and D_{22} are chosen equal to 1. If the transfer functions D_{12} and D_{21} are improper (not realizable), then D_{11} and D_{22} are chosen as follows

$$D_{11}=1/(\tau_1 s+1)\,,\ D_{22}=1/(\tau_2 s+1)\,, \tag{6}$$

where the time constants τ_1 and τ_2 are 5…10 times less than the process dominant time constant. In this case

$$\begin{cases} D_{12}=\dfrac{-G_{12}}{G_{11}(\tau_1 s+1)} \\ D_{21}=\dfrac{-G_{21}}{G_{22}(\tau_2 s+1)} \end{cases}. \tag{7}$$

Usually, process modeling and identification tasks are time-consuming and demand specific knowledge in control theory and an advanced practical experience. Moreover, the decoupling controller structure is dependent of the process model structure. In the case of strong cross-interaction, even if the process has a monotonous and finite step response on the direct channels, the direct channels of the decoupled process can be non-monotonic (of non-minimal phase, with large overshoot or of oscillating type) or even unstable. For such decoupled process, the monovariable controllers design is not a simple problem and the control performance may not be acceptable.

The proposed control method eliminates or reduces these disadvantages. In the proposed structure of a multivariable control system (fig. 1), F1 and F2 are serial filters for decoupled process compensating, and C1 and C2 are standard IMC controllers.

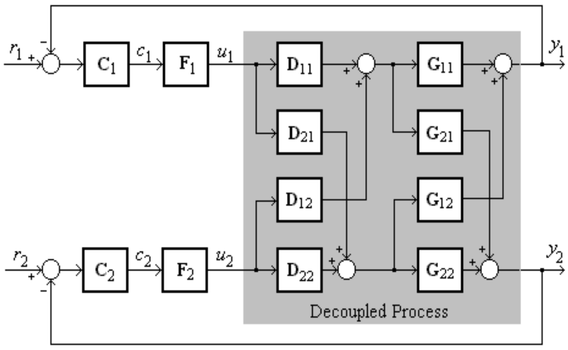

Fig. 1. Proposed structure for multivariable control system

The proposed decoupling solution is based on the following idea: two parallel-opposite channels with the same gain, the same dead time and very close transient time accomplish a satisfactory compensation for a process having all the input-output channels of proportional type (with finite step response). The transient time is the whole response time to input step change minus the dead time. Each process channel is thus described by 3 parameters, which can be easily determined by experimental way: gain $(K_p)_{ij}$, dead time $(\tau_p)_{ij}$ and transient time $(T_t)_{ij}$, i.e.

$$\text{Process}: \begin{bmatrix} (K_{p11},\tau_{p11},T_{t11}) & (K_{p12},\tau_{p12},T_{t12}) \\ (K_{p21},\tau_{p21},T_{t21}) & (K_{p22},\tau_{p22},T_{t22}) \end{bmatrix}.$$

2. FIRST TYPE DECOUPLER

The proposed decoupler fulfils the following conditions:

1) each channel is at most first order lag plus dead time element;
2) the direct input-output channels have gain equal to 1;
3) at least two channels are dead time equal to zero.

Taking into account the first two decoupling conditions, the decoupler structure is in the form

$$D=\begin{bmatrix} \dfrac{e^{-\tau_{11}s}}{T_{11}s+1} & \dfrac{k_{12}e^{-\tau_{12}s}}{T_{12}s+1} \\ \dfrac{k_{21}e^{-\tau_{21}s}}{T_{21}s+1} & \dfrac{e^{-\tau_{22}s}}{T_{22}s+1} \end{bmatrix}, \tag{8}$$

with

$$k_{12}=-K_{p12}/K_{p11},\quad k_{21}=-K_{p21}/K_{p22},$$
$$T_{11}=\frac{T_{t22}}{4},\quad T_{12}=\frac{T_{t12}}{4},\quad T_{21}=\frac{T_{t21}}{4},\quad T_{22}=\frac{T_{t11}}{4}. \tag{9}$$

From the third decoupler property and the dead time compensation relations

$$\tau_{11}+\tau_{p21}=\tau_{21}+\tau_{p22},\quad \tau_{22}+\tau_{p12}=\tau_{12}+\tau_{p11}, \tag{10}$$

we get the decoupler dead-times such as:

$$\tau_{p22}\le\tau_{p21} \Rightarrow \tau_{11}=0,\tau_{21}=\tau_{p21}-\tau_{p22};$$
$$\tau_{p22}>\tau_{p21} \Rightarrow \tau_{21}=0,\tau_{11}=\tau_{p22}-\tau_{p21};$$
$$\tau_{p11}\le\tau_{p12} \Rightarrow \tau_{22}=0,\tau_{12}=\tau_{p12}-\tau_{p11};$$
$$\tau_{p11}>\tau_{p12} \Rightarrow \tau_{12}=0,\tau_{22}=\tau_{p11}-\tau_{p12}. \tag{11}$$

From here, it follows that depending on the process dead times, the decoupler structure can be in four ways: with zero dead time at 1-1 or 2-1 channel, and at 1-2 or 2-2 channel. The decoupler has eight parameters: two gains, four lag time constants and two dead times. The decoupler designed in such a way can be experimentally refined by suitably adjusting the two lag time constants T_{12} and T_{21}. If the steady state decoupling is perfect, the refining operation yields

$$\int_0^\infty [y_1(t)-y_1(0)]dt\cong 0,\quad \int_0^\infty [y_2(t)-y_2(0)]dt\cong 0, \tag{12}$$

where y_1 and y_2 are the step input responses of decoupled process crossing channels.

As an example, for the multivariable process proposed by Ho, et al. (1996)

$$G(s)=\begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}, \tag{13}$$

the unadjusted transfer matrix of the first type decoupler is

$$D(s)=\begin{bmatrix} \dfrac{1}{14.4s+1} & \dfrac{1.48e^{-2s}}{21s+1} \\ \dfrac{0.34e^{-4s}}{10.9s+1} & \dfrac{1}{16.7s+1} \end{bmatrix}. \tag{14}$$

3. SECOND TYPE DECOUPLER

The general structure of the second type decoupler has the form (8), but it fulfils in addition the condition: 4) at least two channels are lag time constant T_{ij} equal to zero. Consequently, the decoupler has the structure simpler than that of the first type decoupler. Moreover, the decoupled process is faster. Taking into account the fourth decoupler property and also the relations

$4T_{11}+T_{t21}=4T_{21}+T_{t22}$, $\qquad 4T_{22}+T_{t12}=4T_{12}+T_{t11}$, (15)

which approximately express the equality of mutual compensation channel transient times, it follows the decoupler lag time constants:

$$T_{t22}\le T_{t21} \Rightarrow T_{11}=0,\, T_{21}=(T_{t21}-T_{t22})/4\,;$$
$$T_{t22}>T_{t21} \Rightarrow T_{21}=0,\, T_{11}=(T_{t22}-T_{t21})/4\,;$$
$$T_{t11}\le T_{t12} \Rightarrow T_{22}=0,\, T_{12}=(T_{t12}-T_{t11})/4\,;$$
$$T_{t11}>T_{t12} \Rightarrow T_{12}=0,\, T_{22}=(T_{t11}-T_{t12})/4\,. \tag{16}$$

From here it follows that depending on the process transient times, the decoupler structure can be in four ways: with zero lag constant time on 1-1 or 2-1 channel, and on 1-2 or 2-2 channel. The decoupler gains and dead times follow from (9) and (12), like the first decoupler. Therefore, depending on the process dead times, the decoupler structure can be also in four ways. Each of the 16 possible structures can be experimentally refined by suitably adjusting the non-zero time constants. The decoupler has only six parameters: two gains, two lag time constants and two dead times. For example, for the multivariable process (13), the unadjusted transfer matrix of the second type decoupler is

$$D(s)=\begin{bmatrix} \dfrac{1}{3.5s+1} & \dfrac{1.48e^{-2s}}{4.3s+1} \\ 0.34e^{-4s} & 1 \end{bmatrix}. \tag{17}$$

The second decoupler form (17) is much simpler and faster that the first form (14).

4. DECOUPLED PROCESS COMPENSATION

Compensating both direct channels of the decoupled process has the aim to improve its dynamics, so that the input step response of each channel to be monotonic and as fast as possible (Cîrtoaje, 2002). The most usual method of monovariable process compensating is to connect a lead-lag filter in front of the process. The lead-lag transfer function has the form

$$G_f(s)=\frac{K_f T_f s+1}{T_f s+1}. \tag{18}$$

For robustness reason, in the case of a stable and monotonic process, we recommend a filter time constant with the value

$$T_f=T_t/10\,, \tag{19}$$

where T_t is the transient time. The filter gain K_f must be chosen as large as possible, but respecting two condition: a) $K\le 5$, for robustness reason; b) the compensated process to remain also monotonic. In the case of a stable but non-monotonic process, we recommend that K_f to be equal to zero and T_f to be sufficiently large so that the compensated process to become monotonic.

5. DECOUPLED PROCESS CONTROL

Each direct channel of the compensated process is controlled by the standard IMC method (fig. 2). The controller R is a serial connection between a proportional element with the gain K and a positive feedback loop, which is designed by means of the compensated process model $(G_m)_c(s)$. In the forward path of the controller loop there is a proportional element having the gain equal to the inverse of the model gain.

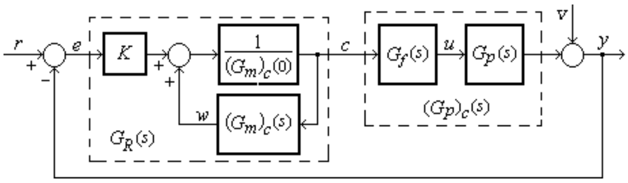

Fig. 2. Proposed IMC variant

Since the input step response of the compensated process is monotonic, we may consider its model in the form

$$(G_m)_c(s)=\frac{K_m e^{-\tau_m s}}{(T_m s+1)^2}\,, \tag{20}$$

with

$$T_m=(T_t)_c/6\,, \tag{21}$$

where $(T_t)_c$ is the transient time of the compensated process. Hence, the controller R has the continuous transfer function

$$H_R(s)=\frac{K}{K_m}\cdot\frac{1}{1-e^{-\tau_m s}/(T_m s+1)^2}\,, \tag{22}$$

or the discrete transfer function

$$H_R(z)\cong\frac{K}{K_m}\cdot\frac{1-2pz^{-1}+p^2z^{-2}}{1-2pz^{-1}+p^2z^{-2}-(1-p)^2z^{-l_m-1}}\,, \tag{23}$$

where $p=e^{-T/T_m}$ and $l_m=T_m/T$ (T - sample time). The gain K has the standard value 1. Increasing/decreasing K, the control output becomes stronger/weaker. According to (23), the controller equation in the time domain is as follows

$$c_k=2pc_{k-1}+p^2c_{k-2}-(1-p)^2c_{k-l_m-1}+$$
$$+\frac{K}{K_m}[(e_k-e_0)-2p(e_{k-1}-e_0)+p^2(e_{k-2}-e_0)]. \tag{24}$$

6. APPLICATION

Consider the multivariable process without dead time proposed by Menani and Koivo (1996), but completed here with dead times on all channels:

$$G(s)=\frac{1}{(0.1s+1)(0.2s+1)^2}\begin{bmatrix}\frac{0.5e^{-0.5s}}{0.1s+1} & -e^{-0.8s} \\ e^{-0.3s} & \frac{2.4e^{-0.6s}}{0.5s+1}\end{bmatrix}. \qquad (25)$$

From the process response to step input change (fig. 3 and fig. 4), we get the process parameters:

$$K_{p11}=0.5\,,\ K_{p12}=-1\,,\ K_{p21}=1\,,\ K_{p22}=2.4\,;$$
$$\tau_{p11}=0.6\,,\ \tau_{p12}=0.8\,,\ \tau_{p21}=0.3\,,\ \tau_{p22}=0.7\,; \qquad (26)$$
$$T_{t11}=1.4\,,\ T_{t21}=1.36\,,\ T_{t12}=1.36\,,\ T_{t22}=3\,.$$

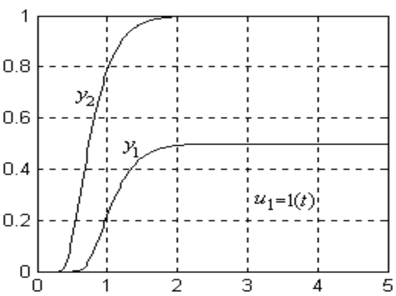

Fig. 3. Process response to $u_1=1(t)$

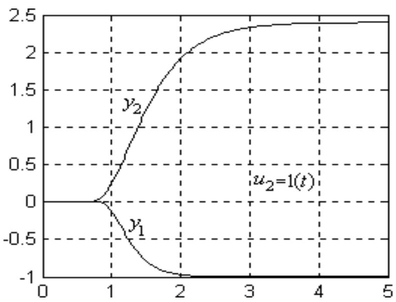

Fig. 4. Process response to $u_2=1(t)$

A. *First type decoupler*

From (8)… (11) and (26), we obtain the decoupler

$$D_1=\begin{bmatrix}\frac{e^{-0.4s}}{0.75s+1} & \frac{2e^{-0.2s}}{0.35s+1} \\ \frac{-0.417}{0.35s+1} & \frac{1}{0.35s+1}\end{bmatrix}. \qquad (27)$$

The decoupling performance is shown in fig. 5 and fig. 6. Notice that it is not necessary to adjust the time constant T_{12} or T_{21}.

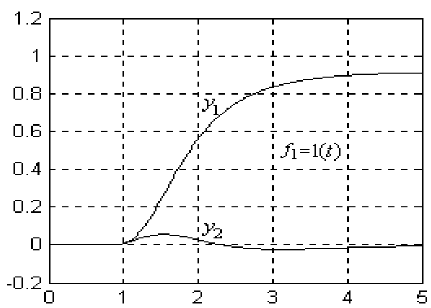

Fig. 5. Decoupled process response to $u_1=1(t)$

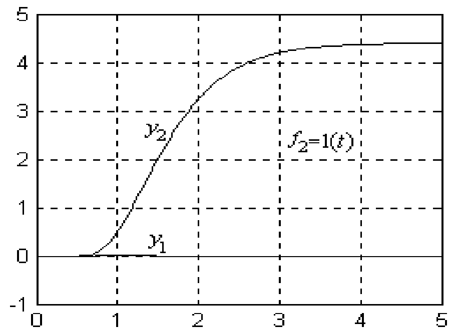

Fig. 6. Decoupled process response to $u_2=1(t)$

In order to compensate the direct channels of the decoupled process, we chose the filter time constants $T_{f1}=T_{t1}/10\cong 0.4$, $T_{f2}=T_{t2}/10\cong 0.4$. By using the filter gains $K_{f1}=2$ and $K_{f2}=1.5$, the unit step responses of the both direct input-output channels of the decoupled and compensated process remain monotonous, but faster (fig. 7 and fig. 8). From these responses, we get the parameters of the direct channels of compensated process:

$$(K_{p1})_c=0.92\,,\quad (T_{t1})_c=1.9\,,\quad (\tau_{p1})_c=1\,,$$
$$(K_{p2})_c=4.4\,,\quad (T_{t2})_c=2.4\,,\quad (\tau_{p2})_c=0.7\,.$$

According to (20) and (21), we build the suitable models

$$(H_{m1})_c(s)=\frac{0.92e^{-s}}{(0.317s+1)^2}\ ,\quad (H_{m2})_c(s)=\frac{4.4e^{-0.7s}}{(0.4s+1)^2}\,.$$

The control result for $K=1$ (obtained in Matlab-Simulink) is presented in fig. 9 and fig. 10.

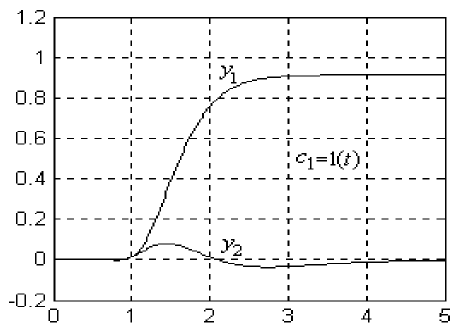

Fig. 7. Compensated process response to $c_1=1(t)$

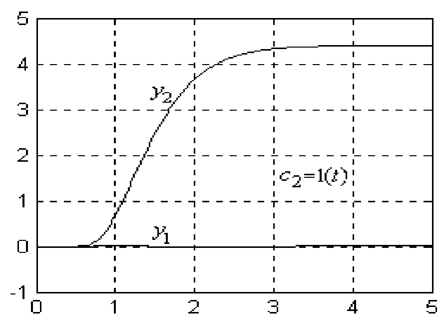

Fig. 8. Compensated process response to $c_2=1(t)$

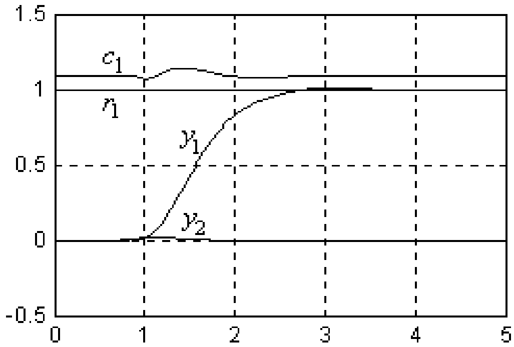

Fig. 9. Controlled process response to $r_1 = 1(t)$

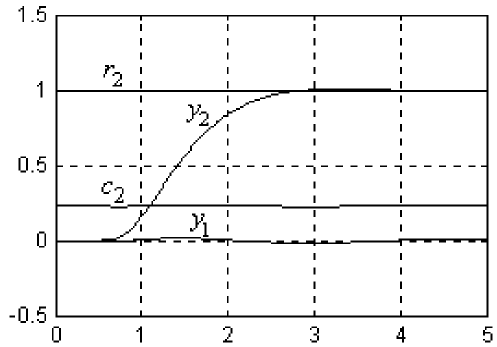

Fig. 10. Controlled process response to $r_2 = 1(t)$

B. *Second type decoupler*

From (8), (9), (11), (16) and (26), we obtain the decoupler

$$D_2 = \begin{bmatrix} \dfrac{e^{-0.4s}}{0.4s+1} & 2e^{-0.2s} \\ -0.417 & \dfrac{1}{0.01s+1} \end{bmatrix}. \qquad (28)$$

The decoupling performance does not require adjusting the time constants $T_{11} = 0.4$ or $T_{22} = 0.01$ (fig. 11 and fig. 12).

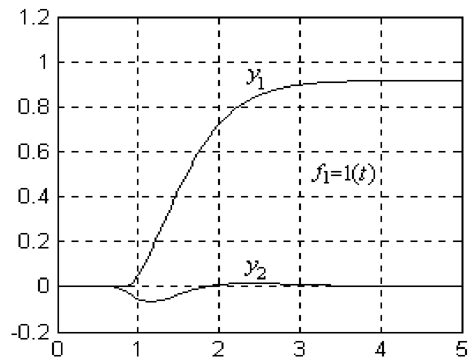

Fig. 11. Decoupled process response to $u_1 = 1(t)$

By using

$$T_{f1} = T_{t1}/10 \cong 0.3, \quad T_{f2} = T_{t2}/10 \cong 0.3,$$
$$K_{f1} = 1.8, \quad K_{f2} = 2,$$

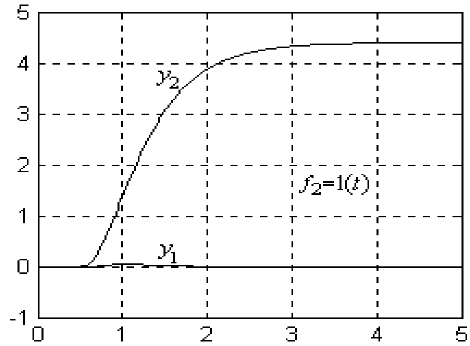

Fig. 12. Decoupled process response to $u_2 = 1(t)$

we get the compensated process responses from fig. 13 and fig. 14, which are also monotonous. These responses yield

$$(K_{p1})_c = 0.92, \quad (T_{t1})_c = 1.64, \quad (\tau_{p1})_c = 0.9,$$
$$(K_{p2})_c = 4.4, \quad (T_{t2})_c = 1.6, \quad (\tau_{p2})_c = 0.54,$$

and

$$(H_{m1})_c(s) = \frac{0.92e^{-0.9s}}{(0.273s+1)^2}, \quad (H_{m2})_c(s) = \frac{4.4e^{-0.54s}}{(0.267s+1)^2}.$$

The control performance (for $K = 1$) is little better than the performance obtained by means of first type decoupler (fig. 15 and fig. 16).

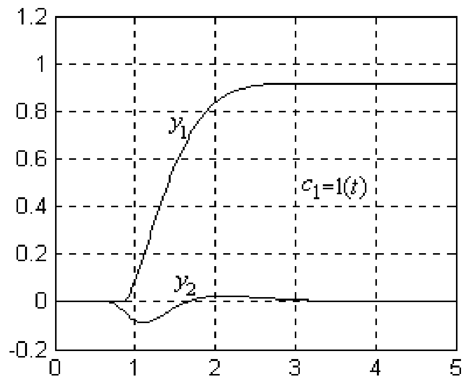

Fig. 13. Compensated process response to $c_1 = 1(t)$

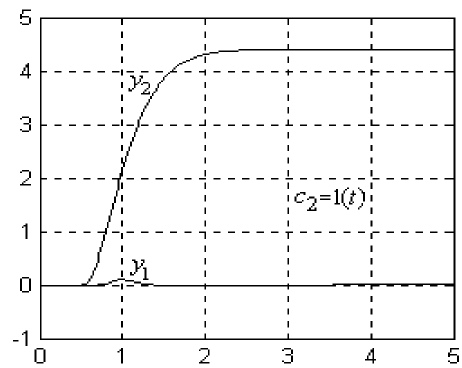

Fig. 14. Compensated process response to $c_2 = 1(t)$

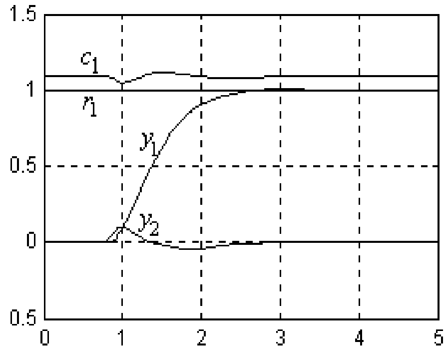

Fig. 15. Controlled process response to $r_1 = 1(t)$

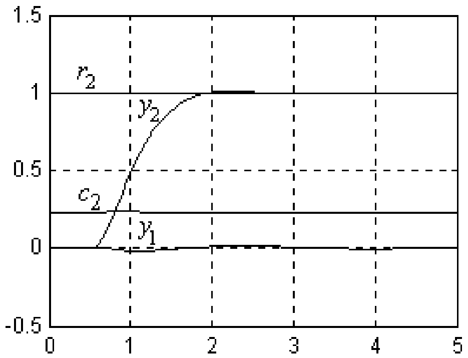

Fig. 16. Controlled process response to $r_2 = 1(t)$

7. CONCLUSIONS

This paper presents a practical method of experimentally decoupling, compensating and control for two input-two output process. The proposed control method can be implemented in four steps: 1) multivariable process decoupling; 2) decoupled process compensating, in order to obtain an overdamped (nonoscillatory) compensated process; 3) use of a second-order plus dead time model, with the same lag time constants for each direct channel of the compensated process; 4) controller design for each compensated process channel by standard IMC method.

We propose two practical decoupler type. The first decoupler has eight parameters: two gains, four lag time constants and two dead times, while the second decoupler has six parameters: two gains, two time constants and two dead times. Moreover, the second decoupler is simpler and faster than the first.

To accomplish decoupled process control, we used Cirtoaje-IMC method, based on decoupled process compensation and use of the standard variant of internal model control for the decoupled process. The simulation results have proven that the proposed control procedure is simple, practical and robust.

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