# Absolute Stability of a longitudinal Blended Wing Body Aircraft Model with Rate Limiting of the Actuator<sup>\*</sup>

Claudia Alice State\*

\* Doctoral School of Control Engineering and Computers, University of Craiova, A.I.Cuza, 13, Craiova, RO-200585, Romania (e-mail: cldstate@ yahoo.com).

**Abstract:** This paper is focused on P(ilot) I(n-the-Loop) O(scillations) of Category II associated to quasi-linear models, which are induced by nonlinearities determined by the saturation of position or rate limited elements. The theoretical model of the airplane is a Blended Wing Body (BWB) configuration and the human operator is expressed by the Synchronous Pilot Model, represented by a simple gain. The absolute stability for the longitudinal BWB aircraft model proposed is investigated using a frequency Popov-type criterion. The mathematical model presented in this article is a pilot-aircraft coupled system used for describing the longitudinal motion of the Blended Wing aircraft and techniques from the frequency domain are applied. The transfer function obtained from open-loop analysis has a double pole at the origin. Therefore, the pilot-aircraft system is in the critical case of a double zero root and the Popov criterion, in the case of the infinite parameter, is applied in order to investigate the absolute stability for the longitudinal BWB aircraft model in the presence of the rate saturation of the actuator.

*Keywords:* Absolute stability, Popov criterion, Nonlinear system, Blended Wing Body, Frequency domain inequality

# 1. INTRODUCTION

Known to have been the cause for several aircraft incidents and accidents, the pilot-induced oscillations (PIO) are dangerous and complicated interactions between the human pilot and the aircraft dynamics that can lead to destruction of the aircraft, and it often occurs when the pilot of an aircraft proves to be unable to adapt himself to a sudden change of the vehicle dynamics during a high demanding flight task.

A PIO event can be considered as a closed-loop instability caused by dynamic coupling between the pilot and the aircraft, which is described in the specific literature as "sustained or uncontrollable oscillations resulting from efforts of the pilot to control the aircraft" (Jeram and Prasad (2003)) or "inadvertent, sustained aircraft oscillation which is the consequence of an abnormal joint enterprise between the aircraft and the pilot" (McRuer (1992)).

According to common references (see, for example, McRuer et al. (1996)), PIOs can be separated into four categories.

*PIO Category I* mainly concerns to linear pilot-vehicle system oscillations. These PIOs result from linear phenomena such as excessive lags introduced by filters, actuators, feel system and digital system time delays. They are the simplest to model and prevent.

*PIO Category II* is characterised by quasi-linear pilotvehicle models, but with some nonlinear contribution, such as rate or position limiting. The closed loop pilotvehicle system has a nonlinear behavior, mainly characterized by the saturation of position or rate limited elements.

- *PIO Category III* is enough evasive defined as completely nonlinear. The closed loop pilot-vehicle system has a highly non-linear behaviour, with no further peculiar characteristic. These severe life-threatening PIOs are caused by nonlinearities and transitions in pilot or effective airplane dynamics.
- *PIO Category IV* which refers to coupling effects between pilot inputs and the aircraft structural modes, is characterised by highly non-linear pilot-vehicle system oscillations. These PIOs are theoretically considered and less studied.

This paper deal with the analysis of Category II PIOs, which are mainly characterised by nonlinearities determined by rate or position saturations of control surface actuators. Caused by dynamic coupling between the human pilot and the aircraft, these oscillations can occurs with motions about all or any symmetry axes of the aircraft, and this could lead to instability in the systems.

One of the goal of this paper is to provide a frequency criterion to establish whether a given aircraft is free from PIO of Category II. Pilot-in-the-loop analysis of the aircraft dynamics involves adoption of mathematical models of the human pilot, which can be a useful tool for predicting these PIOs. The theoretical model of the airplane is a Blended Wing Body (BWB) tailless configuration which is claimed to have a superior aerodynamic performance. As

 $<sup>^{\</sup>star}$  This work has been partly supported by the Research Project CNCSIS ID-95



Fig. 1. BWB concept aircraft

a mathematical model of the human pilot, Synchronous Pilot Model is considered, represented by a simple gain.

#### 1.1 BWB concept

Defined as having no definite fuselage and only a single wing, the Blended Wing aircraft generate less noise and offers a greater lift-to-drag ratio than traditional aircraft. The BWB aircraft model also offers a reduction in the number of parts required relating to reduced manufacturing costs. Figure 1 ilustrates a representative BWB tailless aircraft.

An intuitive presentation of the BWB concept can be found in (Chambers (2005)). Also, several researches regarding this particular configuration are, for example, (Rahman and Whidborne (2008)) and (Smith and Abbasi (2004)).

The aerodynamic BWB model used here was obtained taking into consideration (Castro (2003)) and (Rahman et al. (2009)). The linear low order BWB aircraft model for the uncoupled longitudinal dynamics was considered taking into account the case of short-period approximation. In addition, only elevator control  $\delta_e$  was retained.

### 1.2 Actuator Rate Limiting

For absolute stability analysis, the nonlinear equations of motion describing the aircraft dynamics were obtained using the rate limiting of the actuator, representing the nonlinear part of the longitudinal model presented in this article.

Rate limiting of the actuator is one of main factors contributing to Category II PIO. A block diagram of a basic rate-limited actuator model is presented in Figure 2, where the pilot input command u to the actuator produces the actual actuator deflection. The output deflection  $\delta$  is fed back to the input surface command to produce an error signal. In the forward path the error signal serves as the input to the nonlinear saturation block. Actuator rate limiting occurs when pilot input command error requires a higher rate than the actuator can actually provide. The output from the saturation block is the surface rate  $\dot{\delta}$ . This



Fig. 2. Simplified actuator model with rate limiting



Fig. 3. Saturation nonlinearities

signal is then integrated to produce the surface deflection δ.

Rate limiting introduces additional phase lag, increasing the delay between the pilot input command and aircraft response. Depending upon the characteristics of the aircraft, this alone can be sufficient to lead to PIO events and tends to destabilize the closed-loop system.

# 2. THEORETICAL BACKGROUND

General stabilization of the pilot-aircraft system, using the rate limiter is shown in the Figure 5, in which the limiter is of "AIAA type", as in (Răsvan (2011)).

In Figure 5, r = 0 represents the null reference and the following elements are used:

- $\chi$  is the output of the system;
- k<sub>p</sub> represents the model of pilot;
   δ<sub>p</sub> is the control signal elaborated by the pilot;
- $\hat{G}(s)$  is the open-loop transfer function for the aircraft model, where

$$G(s) = c^T (sI - A)^{-1} b$$

•  $\psi$  designate a non-linear function (a saturation, like in Figure 3), which fulfills the following sector condition

$$0 \leq \bar{\psi} \leq \frac{\psi(y)}{y} \leq \bar{\psi} \leq \infty, \ \psi(0) = 0$$

where  $\overline{\psi}$ ,  $\underline{\psi}$  and y can be seen in Figure 4.



Fig. 4. Sector restricted nonlinearity

2.1 Nonlinear feedback analysis - The Lurie problem

Absolute stability problem (see (Răsvan (1975)) and (Răsvan and Danciu (2011))) refers to the global asymptotic of the zero equilibrium of the general nonlinear system

$$\dot{x}(t) = Ax(t) - b\psi(c^{\mathrm{T}}x(t)) \tag{1}$$

having sector of restricted nonlinearities of the form

$$0 \le \underline{\psi} \le \frac{\psi(y)}{y} \le \overline{\psi} \le +\infty, \psi(0) = 0$$
(2)

and the property of the equilibrium being valid for all the linear and nonlinear functions verifying (1).

Further details about the global asymptotic stability property of dynamical systems can be also found in (Voicu (1986)).

From Figure 5 we can observe that in the feedback structure composed of the aircraft and human pilot dynamic, a saturation nonlinearity occurs. An early nonlinear feedback system analysis problem was formulated by Lurie. The following transformation of the system who has rate saturation can be written as in Figure 6. In the example considered in this paper we can use the following equivalence between the systems (3) and (4).

$$\dot{x}_{\alpha}(t) = A_{\alpha} x_{\alpha}(t) - b_{\alpha} \psi(y(t)) \tag{3}$$

where

$$y(t) = c_{\alpha}^{\mathrm{T}} x_{\alpha}(t)$$

and

$$\begin{aligned} \dot{x}_{\beta}(t) &= A_{\beta} x_{\beta}(t) + b_{\beta} \delta_{e}(t) \\ \dot{\delta}_{e}(t) &= -\psi(\omega_{0}(c_{\beta}^{\mathrm{T}} x_{\beta}(t) + \delta_{e}(t))) \end{aligned}$$
(4)

where

- $\delta_e$  was introduced as a state;
- $A_{\beta}$  should be Hurwitz matrix to make sure of the stability of the system;
- $c_{\beta}$ ,  $A_{\beta}$ ,  $b_{\beta}$  are smaller in dimension than in (3);



Fig. 5. The block diagram of the generic coupled pilotaircraft system with rate limiter

•  $x_{\alpha}(t) = [x_{\beta}(t) \ \delta_e(t)]$  and  $c_{\alpha}^{\mathrm{T}}(t) = [c_{\beta}^{\mathrm{T}}\omega_0 \ \omega_0].$ 

Noting that

$$\begin{cases} y(t) = \omega_0 (c_\beta^{\mathrm{T}} x_\beta(t) + \delta_e(t)) \\ u(t) = -\psi(y(t)) \end{cases}$$
(5)

and substituting (5) into the system (4), the simplified system below is obtained:

$$\begin{cases} \dot{x}_{\beta}(t) = A_{\beta}x_{\beta}(t) + b_{\beta}\delta_{e}(t) \\ \dot{\delta}_{e}(t) = u(t) \end{cases}$$
(6)

Applying the Laplace transformation we obtain:

$$\begin{cases} s\tilde{x}_{\beta}(s) = A_{\beta}\tilde{x}_{\beta}(s) + b_{\beta}\tilde{\delta}_{e}(s) \\ s\tilde{\delta}_{e}(s) = \tilde{u}(s) \end{cases}$$
(7)

and

$$\tilde{y}(s) = \omega_0(c_\beta^{\mathrm{T}}\tilde{x}_\beta(s) + \tilde{\delta}_e(s)) \tag{8}$$

From (7) results:

$$\begin{cases} \tilde{x}_{\beta}(s) = (sI - A_{\beta})^{-1} b_{\beta} \tilde{\delta}_{e}(s) \\ \tilde{\delta}_{e}(s) = \frac{1}{s} \tilde{u}(s) \end{cases}$$
(9)

From the above system is obtained:

$$\tilde{x}_{\beta}(s) = \frac{1}{s}(sI - A_{\beta})^{-1}b_{\beta}\tilde{u}(s)$$
(10)

From (8), (9) and (10) results

$$\tilde{y}(s) = c_{\beta}^{\mathrm{T}} (sI - A_{\beta})^{-1} b_{\beta} \frac{\omega_0}{s} \tilde{u}(s) + \frac{\omega_0}{s} \tilde{u}(s)$$
(11)

Using the notation

$$\mathbf{G}(s) = c_{\beta}^{\mathrm{T}} \left( sI - A_{\beta} \right)^{-1} b_{\beta}$$
(12)

from (11) and (12) the following relation is determined

$$\tilde{y}(s) = \omega_0 \frac{\mathbf{G}(s) + 1}{s} \tilde{u}(s) \tag{13}$$

which is equivalent to

$$\tilde{y}(s) = \mathcal{T}(s)\tilde{u}(s) \tag{14}$$

The transfer function T(s) (in the case of rate limiter) is

$$T(s) = \frac{\tilde{y}(s)}{\tilde{u}(s)} = \omega_0 \left(\frac{1}{s} + \frac{G(s)}{s}\right) \tag{15}$$

This shows clearly that the system is in the critical case of a single zero root - H(s) has a pole at s=0.



# Fig. 6. Absolute stability feedback structure

We can observe that the control system from Figure 5 is transformed into the terms of Lurie feedback connection, as in Figure 6, where the feedback structure of the absolute stability contains the saturation nonlinearity  $\psi$ . In the case under study, the block **L** from the forward path represents the linear differential equations of the model and the nonlinear block **N** represents the rate saturation of the actuator.

#### 2.2 The Popov criterion

The Popov criterion used in this paper in order to provide the absolute stability of the longitudinal BWB model with rate limited actuator, considers the stability of the Lurie system. For practical considerations the following expression of Popov criterion is used (from (Sastry (1999))).

Theorem 1. Consider a Lurie system with a nonlinearity  $\psi$  in the sector. The equilibrium in the origin is globally asymptotically (exponentially) stable, provided that there exists  $\xi > 0$  such that the following inequality is true:

$$\frac{1}{k} + Re\left[\left(1 + j\omega\xi\right) \mathrm{T}\left(j\omega\right)\right] > 0, \forall \omega \in \mathbf{R}$$
(16)

The Popov condition express a frequency condition for the global asymptotically stability property of a dynamically system in the condition of the Lurie problem (Khalil (2002)).

Remark 2. From the general theory of functions of a complex variable with real coefficients (for example rational meromorphic functions) it is known that the real part of a transfer function is even (in  $\omega$ ) and the imaginary part is odd (but when multiplied with  $j\omega$  it is also even), so results that the above relation is even and then the condition  $\omega \geq 0$  is not restrictive:

$$\frac{1}{k} + Re\left[(1+j\omega\xi) \operatorname{T}(j\omega)\right] > 0, \forall \omega \ge 0$$
(17)

Remark 3. k is the lenght of the sector defined by:

$$k = \bar{\psi} - \underline{\psi} = V_L - 0 = V_L$$

where  $V_L$  is the rate limit value.

One should note that in relation (17), by multiplying with  $\frac{1}{\xi}$  (if  $\xi > 0$ ) which is a positive quantity, the following inequality is equivalent.

$$\frac{1}{k\xi} + \frac{1}{\xi} Re\left[(1+j\omega\xi) \operatorname{T}(j\omega)\right] =$$

$$\frac{1}{k\xi} + \frac{1}{\xi} Re\left(\operatorname{T}(j\omega)\right) - \omega Im\left(\operatorname{T}(j\omega)\right) > 0$$
(19)

For the above formally obtained relation, applying the limit  $\xi \to \infty$ , yields to:

$$-\omega Im\left[\mathrm{T}\left(j\omega\right)\right] > 0, \forall \omega \ge 0 \tag{20}$$

## 3. THE ABSOLUTE STABILITY OF THE LONGITUDINAL BWB MODEL

In this section symbolic and numeric computations for the longitudinal BWB system were performed. The longitudinal motion of the BWB aircraft model is a dynamical simplified system with one input (the elevator deflection  $\delta_e$ ) and two state variables (the angle of attack  $\alpha$  and the pitch rate q). The longitudinal aerodynamic parameters of the aircraft equations of motion which are time-invariant are substituted by their numerical values.

#### 3.1 The linear system

Taking into account (Castro (2003)) and (Rahman et al. (2009)), we consider the following differential equations of longitudinal motion describing the mathematical BWB short-period aircraft model:

$$\begin{cases} \dot{\alpha} = q\\ \dot{q} = M_q q - M_{\delta_e} \delta_e \end{cases}$$
(21)

The linear model for the mentioned uncoupled longitudinal dynamic can be expressed as

$$\dot{x} = Ax + Bu \tag{22}$$

where • A

A and B matrices are given by  

$$B = \begin{bmatrix} 0 \\ M_{\delta_e} \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & 1 \\ 0 & M_q \end{bmatrix}$$

Introducing the elevator equation into the above system, the following short-period model with actuator unsaturated is obtained:

$$\begin{cases} \alpha = q\\ \dot{q} = M_q q - M_{\delta_e} \delta_e\\ \dot{\delta}_e = (-\omega_0)[(k_\alpha + k_p)\alpha + k_q q + \delta_e] \end{cases}$$
(23)

where we have:

- $\alpha$  is the incidence angle [rad]
- q represents the pitch rate [rad/s]
- $\delta_e$  is the elevator deflection [rad]

#### 3.2 The Routh-Hurwitz criterion

At this point, by using the Routh-Hurwitz criterion, can be verified the stability of the linear system (23). The characteristic equation is

$$P(s) = s^3 + a_2 s^2 + a_1 s + a_0 = 0 (24)$$

where the following notations were made:

$$\begin{cases} a_2 = \omega_0 - M_q \\ a_1 = \omega_0 (k_q M_{\delta_e} - M_q) \\ a_0 = \omega_0 (k_\alpha + k_p) M_{\delta_e} \end{cases}$$
(25)

Remark 4. The value of the pilot gain  $k_p$  is set to unity and the following global gains of the system and their

(18)

numerical values are also used:

- $k_{\alpha} = -0.526$
- $k_q = -1.27$

The aerodynamic constants of the system can be numerically substituted and are expressed by(from (Rahman et al. (2009))):

- $\omega_0 = 20 \frac{rad}{sec}$   $M_q = -0.1556$   $M_{\delta_e} = -1.3495$

Further, the Hurwitz criterion gives the following conditions:

$$\omega_0 - M_q > 0 \tag{26}$$

$$\omega_0(k_\alpha + k_p)M_{\delta_e} > 0 \tag{27}$$

$$(\omega_0 - M_q)(k_q M_{\delta_e} - M_q) - (k_\alpha + k_p)M_{\delta_e} > 0$$
 (28)

It is easy to verify that these conditions are fulfilled.

# 3.3 Nonlinear model of the BWB aircraft

In Figure 5 from section 2, the general representation of the pilot-aircraft system is shown.

By adding the rate limited actuator and the SAS (stability augmentation system) to the linear aircraft equations, the following pilot-aircraft nonlinear system is obtained:

$$\begin{cases} \dot{\alpha} = q\\ \dot{q} = M_q q - M_{\delta_e} \delta_e\\ \dot{\delta}_e = -\psi(\sigma) \end{cases}$$
(29)

where we have the output of the linear system

$$\sigma = \omega_0 [(k_\alpha + k_p)\alpha + k_q q + \delta_e]$$
(30)

and the nonlinearity

$$\psi(\sigma) = \begin{cases} \sigma, if \mid \sigma \mid \le e_L \\ e_L sgn\sigma, if \mid \sigma \mid > e_L \end{cases}$$
(31)

Remark 5.  $\sigma$  is the additive input value and  $e_L$  is the limit for saturation  $\psi$ . This saturation is crucial for oscillations, often complicating the effect of PIOs.

Using the notation  $u = -\psi(\sigma)$ , like in subsection 2.1, it is equivalent to rewrite the system (29) as follows

$$\begin{cases} \dot{\alpha} = q\\ \dot{q} = M_q q - M_{\delta_e} \delta_e\\ \dot{\delta}_e = u \end{cases}$$
(32)

The system below is obtained by applying the Laplace transform to differential equations system (32)

$$\begin{cases} \widetilde{\alpha}(s) = \frac{1}{s}\widetilde{q}(s) \\ \widetilde{q}(s) = \frac{M_{\delta_e}}{s - M_q}\widetilde{\delta_e}(s) \\ \widetilde{\delta_e}(s) = \frac{1}{s}\widetilde{u}(s) \end{cases}$$
(33)

 $\delta_e$  is substituted into the second equation of the system (33), and then  $\tilde{q}$  is also substituted into the first equation of the same system.

So, results

$$\begin{cases} \widetilde{\alpha}(s) = \frac{M_{\delta_e}}{s^2(s - M_q)} \widetilde{u}(s) \\ \widetilde{q}(s) = \frac{M_{\delta_e}}{s(s - M_q)} \widetilde{u}(s) \\ \widetilde{\delta_e}(s) = \frac{1}{s} \widetilde{u}(s) \end{cases}$$
(34)

and from (30) was obtained

$$\widetilde{\sigma}(s) = \omega_0 \left[ \frac{(k_\alpha + k_p) M_{\delta_e}}{s^2 (s - M_q)} + \frac{k_q M_{\delta_e}}{s (s - M_q)} + \frac{1}{s} \right] \widetilde{u}(s) \quad (35)$$

where

$$T(s) = \frac{\tilde{\sigma}(s)}{\tilde{u}(s)} \tag{36}$$

is the transfer function of the linear part of the system (29).

## 3.4 Stability analysis using the Popov criterion

From (35) and (36) the loop transfer function of the longitudinal BWB nonlinear system (with rate limiter) is given by

$$T(s) = \omega_0 \frac{s^2 + (k_q M_{\delta_e} - M_q)s + (k_\alpha + k_p)M_{\delta_e}}{s^2(s - M_q)}$$
(37)

From (37) it is clear that the characteristic equation has a double zero root and the criterion mentioned is used in this critical case of the absolute stability.

Making the substitution  $s = j\omega$ , we obtain the frequencydomain transfer function

$$T(j\omega) = \omega_0 \frac{-\omega^2 + (k_q M_{\delta_e} - M_q)j\omega + (k_\alpha + k_p)M_{\delta_e}}{\omega^2(M_q) - j\omega}$$
(38)

The transfer function can be written as follows:

$$T(j\omega) = P(\omega) + jQ(\omega)$$
(39)

where 
$$P(\omega)$$
 și  $Q(\omega)$  are defined by  
 $P(\omega) = Re(T(j\omega))$ 
(40)

and

$$Q(\omega) = Im(T(j\omega) \tag{41}$$

For the Popov condition (20) from subsection 2.2, we have

$$Q(\omega) = \omega_0 \frac{-\omega^2 + k_q M_{\delta_e} M_q - M_q^2 + (k_\alpha + k_p) M_{\delta_e}}{\omega(\omega^2 + M_q^2)} \quad (42)$$

In order to apply the Popov frequency-domain inequality, a straightforward computation will give

$$-\omega Im[T(j\omega)] = \omega_0 \frac{\omega^2 + P_k + M_q^2}{\omega^2 + M_q^2}$$
(43)

where, to simplify the writing,  $P_k$  is denoted by

$$P_k = -(k_\alpha + k_p + k_q M_q) M_{\delta e} \tag{44}$$

In the presented case, the frequency-domain condition can be written as:

$$\omega_0 \left[ 1 + \frac{P_k}{\omega^2 + M_q^2} \right] > 0, \forall \omega \ge 0 \tag{45}$$

which holds if  $P_k > 0$ .

But taking into account that

$$P_k = 0.9063 > 0 \tag{46}$$

it follows that the frequency domain inequality (45) is true. The limits from below are computed:

$$\lim_{\omega \to \infty} -\omega Im[T(j\omega)] = \omega_0 > 0 \tag{47}$$

and

$$\lim_{\omega \to 0} -\omega Im[T(j\omega)] = \omega_0 \left[1 + P_k M_q^2\right]$$
(48)

Numerically, taking into consideration that  $P_k > 0$ , it is easy to verify that

$$\lim_{\omega \to 0} -\omega Im[T(j\omega)] > 0 \tag{49}$$

Therefore, from (45), (47) and (48) it results that the Popov frequency-domain inequality is satisfied, in the case of the infinite Popov parameter. It follows that the absolute stability of the longitudinal BWB model with rate limiter actuator was proved, in the specified conditions, using the Popov criterion.

# 4. CONCLUSION

For the BWB configuration, the low order pilot-aircraft system is absolutely stable with the influence of actuator nonlinearity, in the mentioned conditions. As a future work it can be considered the analysis of the models that are more complex in representation, in the presence of more non-linearities of the systems.

#### REFERENCES

- Castro, H.V. (2003). Flying and Handling Qualities of a Fly-By-Wire Blended Wing Body Civil Transport Aircraft, PhD. Thesis. Cranfield University, England.
- Chambers, J.R. (2005). Innovation in flight. NASA SP-2005-4539, Boston.
- Jeram, G.J. and Prasad, J.V.R. (2003). Tactile avoidance cueing for pilot induced oscillation. Proc. AIAA Atmospheric Flight Mechanics Conference and Exhibitt, 1–9.
- Khalil, H.K. (2002). Nonlinear systems. 3rd edition. ISBN 0-13-067389-7.
- McRuer, D. (1992). Human dynamics and pilot-induced oscillations. Massachusetts Institute of Technology, Cambridge, MA.
- McRuer, D., Klyde, D.H., and Myers, T.T. (1996). Development of a comprehensive pio theory. *Proc. AIAA Atmospheric Flight Mechanics Conference*, 581–597.
- Rahman, N.U. and Whidborne, J.F. (2008). A lateral directional flight control system for the mob blended wing body platform. UKACC Int. Conf. CONTROL.
- Rahman, N.U., Whidborne, J.F., and Cooke, A.K. (2009). Longitudinal control system design and handling qualities assessment of a blended wing body aircraft. The 6th International Bhurban Conference on Applied Sciences and Technology (IBCAST). Dept. of Aerosp. Sci. Cranfield Univ.
- Răsvan, V. (1975). Absolute stability of time lag control systems. 1st ed. Bucharest: Editura Academiei, (in Romanian).

- Răsvan, V. (2011). Systems with monotone and slope restricted nonlinearities. *Tatra Mountains Mathematical Publications. Volume 48, No.1*, 165–187.
- Răsvan, V. and Danciu, D. (2011). Theoretical background for pio ii analysis. Scientific Bulletin of Politehnica University of Timisoara, Romania, Transactions on Automatic Control and Computer Science. Vol. 56(70), No. 1, March 2011, pp. 5-10, Politehnica Publ. House, ISSN 1224-600X.
- Sastry, S. (1999). Nonlinear systems; analysis, stability and control. ISBN 0-387-98513-1.
- Smith, H. and Abbasi, Y. (2004). The mob blended wing body reference aircraft. *Technical report. Cranfield* University, 71–92.
- Voicu, M. (1986). Stability Analysis Techniques of the Automatic Control Systems (in Romanian). Editura Tehnică, București.