DC Motor Identification
Based on Distributions Method

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Abstract: In this paper one presents an algorithm for a DC motor parameters identification from sample data using the distribution approach. While most of the latest methods used in identification utilize a discrete-time model, the distribution method is an alternative approach to directly identify a continuous-time model from discrete-time data. The relation between the state variables is represented by functionals using techniques from distribution theory. Based on these relations, an algorithm for off-line parameter identification is developed. The method is applied to identify the parameters of a real experimental platform.

Keywords: parameter identification, DC motor, distributions.

1. INTRODUCTION

Since the advance of digital computers and the availability of digital data furnished by the acquisition boards, most system identification algorithms usually aim at identifying the parameters of discrete-time models based on sampled input-output data. Over the last period there has been an increasing interest in continuous-time approaches for system identification from sampled data Johansson, 1994), (Sinh, 1991). Identification of continuous-time models is indeed a problem of considerable importance in various disciplines.

A simplistic way of estimating the parameters of continuous-time models by an indirect approach is to use the sampled data to first estimate a discrete-time model and then convert it into an equivalent continuous-time model (Unbehauen, 1990). The second step, i.e. obtaining an equivalent continuous-time model from the estimated discrete-time model, is not always easy. Difficulties are encountered whenever the sampling time is either too large or too small (Middleton, 1990). Whereas a large sampling interval may lead to loss of information, a small sampling period may create numerical problems because the poles are constrained to lie in a small area of the z-plane close to the unit circle. Some conversion methods use the matrix logarithm which may produce complex arithmetic when the matrix has negative eigenvalues. Moreover, the zeros of the discrete-time model are not as easily transformable to continuous-time equivalents as the poles are. In every tuning algorithm, the most difficult phase is the identification one, the whole control design depending on it. We can underline two approaches of identification algorithms: on-line identification algorithms and off-line identification algorithms. In on-line identification approach, the result is obtained in the same moment with a new observation data acquisition. The on-line identification deals with parametric methods (deterministic or stochastic), which identify the parameters of a mathematical model with a structure apriori known. The main on-line methods can be found in (Soderstrom and Stoica, 1989) or (Eykhoff, 1974).

In off-line identification approach it is possible to identify both the structure of linear time invariant systems and the parameters of the mathematical model using observations over a larger time interval, including the steady state (Bastogne et. al., 1996). The distribution method used in this paper is an off-line integral method. In this approach the set of linear differential equations describing the state evolution is mapped into a set of linear algebraic equations respect to the model parameters.

DC motors have long been widely used in many industrial applications. A dc motor can be considered as a single input, single output (SISO) system having torque-speed characteristics compatible with most mechanical loads. This makes a DC motor controllable over a wide range of speeds by proper adjustments of its terminal voltage. Mathematical modelling is one of the most important and often the most difficult step towards understanding a physical system. In modelling a dc motor, the aim is to find the governing differential equations that relate the applied voltage with the produced speed of the rotor and to determine the parameters of the model. System identification of dc motors is a topic of great practical importance, because for almost every servo control design a mathematical model is needed. This paper is structured as follows. Section 2, describes the dynamic of the separately excited dc motor. Section 3 presents the problem statement of continuous time systems identification based on distribution approach. Section 4 is dedicated to identification algorithm analysis. In section 5, the identification algorithms are applied to the parameter identification of a dc motor. Finally, conclusions of the paper are summarized in section 6.

2. DC MOTOR MODEL

A mathematical model for a physical device must often reflect a compromise. It must not attempt to mirror the real device in such great detail that the model becomes cumbersome; on the other hand it should not be so simplified that predictions and explanations based on it are either trivial or far from reality.

In this work one used the second order linear model over other models due to its simplicity. The main difficulty with the nonlinear models is the requirement of numerical solution and the use of this model in those applications of adaptive and optimal control which require a digital computer. The second-order linear model assumes the following:

1. The static friction is negligible and the frictional torque can be considered directly proportional to angular velocity.
2. The brush voltage drop is negligible.
3. Armature reaction can be neglected.
4. The resistance and the inductance of the armature can be regarded as constant.

There is a variation of the inductance of the armature with armature current, so conventional methods for dc motor parameters identification is not accurate and lead to poor controlling (Sinha, 1984).

A DC motor is designed to run on DC electric power. By far the most common DC motor types are the brushed and brushless types, which use internal and external commutation respectively to create an oscillating AC current from the DC source so they are not purely DC machines in a strict sense. The classic DC motor design generates an oscillating current in a wound rotor, or armature, with a split ring commutator, and either a wound or permanent magnet stator. A rotor consists of one or more coils of wire wound around a core on a shaft; an electrical power source is connected to the rotor coil through the commutator and its brushes, causing current to flow in it, producing electromagnetism. The commutator causes the current in the coils to be switched as the rotor turns, keeping the magnetic poles of the rotor from ever fully aligning with the magnetic poles of the stator field, so that the rotor never stops but rather keeps rotating indefinitely (as long as power is applied and is sufficient for the motor to overcome the shaft torque load and internal losses due to friction, etc.).

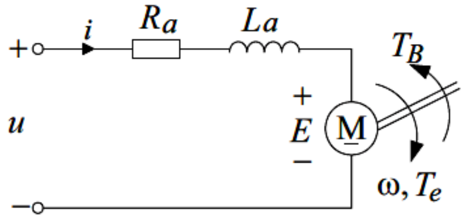


Fig. 1. DC motor equivalent circuit.

A DC motor can be considered as a single input, single output (SISO) system having torque-speed characteristics compatible with most mechanical loads. A DC motor consists of two sub-processes: electrical and mechanical. The electrical sub-process consists of armature inductance, armature resistance and the magnetic flux of the stator. A second sub-process in the motor is a mechanical one. The traditional model of DC motor is a 2-order linear one, which ignores the dead nonlinear zone of the motor. The DC motor equivalent circuit under rating excitation is shown in Fig. 1.

The motor torque, T, is related to the armature current, ia, by a constant factor K_t. The back emf, E, is related to the rotational velocity by the constant factor K_e.

The voltage balance equation of DC motor armature circuit is expressed as

$$u = R_a \cdot i + K_e \cdot \omega + L_a \frac{di}{dt} \tag{1}$$

where, i is armature current (A); L_a is equivalent inductance of armature circuit (H); R_a is equivalent resistance of armature circuit (Ω); u is terminal voltage of armature circuit (V); K_e is voltage coefficient of DC motor (V·s/rad). The torque balance equation of DC motor is expressed as

$$K_t \cdot i - B \cdot \omega = J \frac{d\omega}{dt} \tag{2}$$

where, J is the inertia moment of the rotor (Kg·m^2); K_t is the torque coefficient of DC motor (N·m/A); B is viscous friction coefficient (N·m·s/rad). In the state-space form, the equations above can be expressed by choosing the rotational speed and electric current as the state variables and the voltage as an input. The output is chosen to be the rotational speed, so by representing (1) and (2) in a model of state space form provides:

$$\begin{cases} J \dfrac{d\omega}{dt} = -B \cdot \omega + K_t \cdot i \\ L_a \dfrac{di}{dt} = -K_e \cdot \omega - R_a \cdot i + u \end{cases} \\ y = \omega \tag{3}$$

Eliminating state variable i from this system of equation one obtains the input – output differential equation:

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_0 u \tag{4}$$

where

$$\begin{aligned} a_2 &= L_a \cdot J \\ a_1 &= L_a \cdot B + R_a \cdot J \\ a_0 &= K_e \cdot K_t + R_a \cdot B \\ b_0 &= K_t \end{aligned} \tag{5}$$

3. DISTRIBUTION BASED IDENTIFICATION OF LINEAR SYSTEMS

Accurate mathematical models and their parameters are essential when designing controllers because they allow the designer to predict the closed loop behaviour of the plant (Wang, 1999). Errors in parameter values can lead to poor control and instability.

The conventional way of characterizing a dc motor is to perform a separate test for each parameter, but this is not only time consuming, but can yield misleading results if the parameters are measured under static or no load conditions (Jung,1992). Therefore, estimation techniques must be used to estimate the unknown or inaccurate parameters values with precision. Each identification session consists of a series of basic steps. Some of them may be hidden or selected without the user being aware of his choice. This can result in poor or suboptimal results. In each session the following actions should be taken:

- Collecting information about the system.
- Selecting a model structure to represent the system.
- Choosing the model parameters to fit the model as well as possible to the measurements: selection of a "goodness of fit" criterion.
- Validating the selected model.

Estimation approaches can be divided into two categories: offline estimation and online estimation. Offline techniques use specific test inputs, measure the corresponding output signals and then try to establish the relation between them. Online techniques use, for example, observers and Kalman filters to recursively estimate parameters. Distribution based technique is an off-line estimation method.

A choice should be made within all the possible mathematical models that can be used to represent the system. Again a wide variety of possibilities exist, such as:

- Parametric versus nonparametric models: In a parametric model, the system is described using a limited number of characteristic quantities called the parameters of the model, whereas in a nonparametric model the system is characterized by measurements of a system function at a large number of points (Bos,2007). Examples of parametric models are the transfer function of a filter described by its poles and zeros and the motion equations of a piston. An example of a nonparametric model is the description of a filter by its impulse response at a large number of points. Usually it is simpler to create a nonparametric model than a parametric one because the modeller needs less knowledge about the system itself in the former case. However, physical insight and concentration of information are more substantial for parametric models than for nonparametric ones.

- White box models versus black box models: In the construction of a model, physical laws whose availability and applicability depend on the insight and skills of the experimenter can be used (Kirchhoff's laws, Newton's laws, etc.). Specialized knowledge related to different scientific fields may be brought into this phase of the identification process. The modelling of a loudspeaker, for example, requires extensive understanding of mechanical, electrical, and acoustical phenomena (Diniz, 2002). The result may be a physical model, based on comprehensive knowledge of the internal functioning of the system. Such a model is called a white box model. Another approach is to extract a black box model from the data. Instead of making a detailed study and developing a model based upon physical insight and knowledge, a mathematical model is proposed that allows sufficient description of any observed input and output measurements. This reduces the modelling effort significantly. The choice between the different methods depends on the aim of the study: the white box approach is better for gaining insight into the working principles of a system, but a black box model may be sufficient if the model will be used only for prediction of the output.

- Linear models versus nonlinear models: In real life, almost every system is nonlinear. Because the theory of nonlinear systems is very involved, these are mostly approximated by linear models, assuming that in the operation region the behaviour can be linearized Pearson, 2000). This kind of approximation makes it possible to use simple models without jeopardizing properties that are of importance to the modeller. This choice depends strongly on the intended use of the model. For example, a nonlinear model is needed to describe the distortion of an amplifier, but a linear model will be sufficient to represent its transfer characteristics if the linear behaviour is dominant and is the only interest.

One important direction in continuous-time system identification is to transform the system of differential equations to an algebraic system that reveals the unknown parameters (Marin, 2002). By using some measures, the direct computation of the input-output data derivatives can be completely avoided. For linear system identification, several methods are reported on this direction: identification based on the Laplace transformation and then use the Laguerre filter or transforming the continuous-time system to the frequency domain. The idea of utilizing test functions in system identification was proposed by Pearson and Lee (Pearson, 1985) in terms of modulating functions in order to ameliorate the noise handling for deterministic least-squares identification based on time limited data. In this approach the set of linear differential equations describing the state evolution is mapped into a set of linear algebraic equations respect to the model parameters. Using techniques utilized in distribution approach, the measurable functions and their derivatives are represented by functionals on the fundamental space of testing functions (Schwartz, 1951). The main advantages of this method are that a set of algebraic equations with real coefficients results and the formulations are free from boundary conditions (Ohsumi, 2002).

Let us denote by Ω_n the fundamental space from the distribution theory of the real functions $\varphi : \Re \to \Re, t \to \varphi(t)$ with compact support *T*, having continuous derivatives at least up to the order n. Let $q : \Re \to \Re, t \to q(t)$ be a function which admits a Riemann integral on any compact interval *T* from $\Re$. Using this function, a unique distribution (or generalized function)

$$F_q : \Omega_n \to \Re, \varphi \to F_q(\varphi) \in \Re \tag{6}$$

can be built by the relation:

$$F_q(\varphi) = \int_{\Re} q(t)\varphi(t)dt, \forall \varphi \in \Omega_n \tag{7}$$

In distribution theory, the notion of *k*-order derivative is introduced. If $F_q \in \Omega_n$, then its *k*-order derivative is a new distribution $F_q^{(k)} \in \Omega_n$ uniquely defined by the relations:

$$F_q^{(k)}(\varphi) = (-1)^k F_q(\varphi^{(k)}), \forall \varphi \in \Omega_n \tag{8}$$

$$\varphi \to F_q^{(k)}(\varphi) = (-1)^k \int_{\Re} q(t)\varphi^{(k)}(t)dt \in \Re \tag{9}$$

where

$$\varphi^{(k)} : \Re \to \Re, t \to \varphi^{(k)}(t) = \frac{d^k \varphi(t)}{dt^k} \tag{10}$$

is the *k*-order time derivative of the test function.

When $q \in C^k(\Re)$, then

$$F_q^{(k)}(\varphi) = \int_{\Re} q^{(k)}(t)\varphi(t)dt = (-1)^k \int_{\Re} q(t)\varphi^{(k)}(t)dt \tag{11}$$

that means the k-order derivative of a distribution generated by a function $q \in C^k(\Re)$ equals to the distribution generated by the k-order time derivative of the function q. So, in place of the states and their time derivatives of a system one utilize the corresponding distributions and, in some particular cases, it is possible to obtain a system of equations linear in parameters. If the system is compatible the model parameters are structurally identifiable.

In our study have been utilized three types of test functions characterized by a bounded support $T = (t_a \quad t_b), t_a < t_b$, all of these accomplishing the condition:

$$\varphi(t) = 0; \forall t \in (-\infty, t_a] \cup [t_b, +\infty) \tag{12}$$

The nonzero restriction is one of the following three types:

1. Exponential:

$$\varphi(t) = \exp\left(\frac{|t_a t_b|}{(t - t_a)(t - t_b)}\right) \tag{13}$$

2. Sinusoidal:

$$\varphi(t) = \sin^p((t - t_b)\pi / (t_b - t_a)) \tag{14}$$

3. Polynomial:

$$\varphi(t) = (t - t_a)^p (t - t_b)^p \tag{15}$$

where $p \geq 2$ is an integer.

Figures 1 and 2 present the exponential type test function and its first-order derivative for T = [0.1 0.9]. One can note that these functions and their derivatives vanish on the

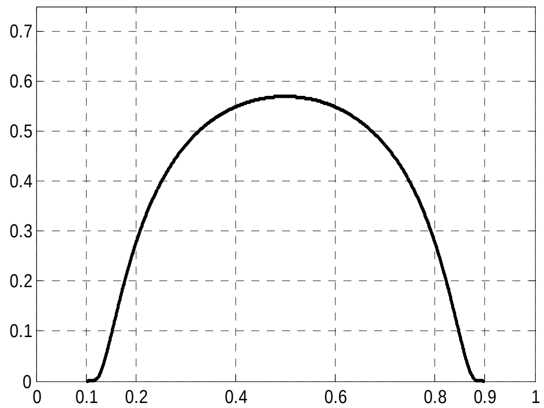


Fig. 2. Exponential type test function

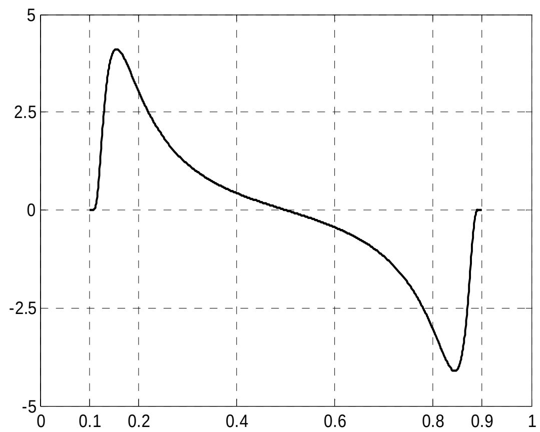


Fig. 3. First-order derivative of exponential type test function

For a linear system consider the input – output differential equation

$$\sum_{k=0}^{n} a_k y^{(k)}(t) = \sum_{k=0}^{m} b_k u^{(k)}(t),\ m \le n, a_n \ne 0 \qquad (16)$$

Multiplying both sides with a test function $\varphi(t)$ and integrating over $\Re$ one get the following algebraic equation:

$$\sum_{k=0}^{n} a_k F_y^{(k)}(t) = \sum_{k=0}^{m} b_k F_u^{(k)},\ m \le n, a_n \ne 0 \qquad (17)$$

where

$$\begin{aligned} F_y^{(k)}(\varphi) &= F_{y^{(k)}}(\varphi) = \int_R y^{(k)}(t)\varphi(t)dt = \\ &= (-1)^k \int_R y(t)\varphi^{(k)}(t)dt \end{aligned} \qquad (18)$$

$$\begin{aligned} F_u^{(k)}(\varphi) &= F_{u^{(k)}}(\varphi) = \int_R u^{(k)}(t)\varphi(t)dt = \\ &= (-1)^k \int_R u(t)\varphi^{(k)}(t)dt \end{aligned} \qquad (19)$$

The unknown parameters are grouped in a column vector:

$$\theta = [b_m, ..., b_k, ..., b_0, a_n, ..., a_k, ..., a_1]^t = [\theta_1, ..., \theta_p]^t \qquad (20)$$

Because θ has p components it is necessary to use a finite number $N \ge p$ of test functions $\varphi_i, i = 1:N$ to get a linear system of algebraic equations in the unknown parameters:

$$F_w\theta = F_v \qquad (21)$$

where F_w is a real matrix $(N \times p)$

$$F_w = [F_w^t(\varphi_1), ..., F_w^t(\varphi_i), ..., F_w^t(\varphi_N)]^t \qquad (22)$$

If the matrix rank is $r=\mathrm{rank}(F_w)=p$ then the system has a unique solution:

$$\hat{\theta} = (F_w^t F_w)^{-1} F_w^t F_v = \theta^* \qquad (23)$$

Remark 1. The consistency of estimates is influenced by the sampling period. The consistency analysis of estimates using integral filters is presented in section 4.

Remark 2. This procedure can also be applied in the case of state space equations (that are first order linear differential equations) for identification of state space matrices. Obviously, the states must be measureable. In order to illustrate this, in section 5 are presented the numerical results obtained by simulation.

4. ANALYSIS OF THE ALGORITHM PROPERTIES

In the following one presents some consistency and numerical aspects related to the presented algorithm.

4.1 Identifiability

Identifiability is a necessary prerequisite for model identification; it concerns uniqueness of the model parameters determined from the input–output data, under ideal conditions of noise-free observations and error-free model structure. A remarkable feature of distribution-based identification procedure is that it provides a linear reparameterization of the input–output relation of the nonlinear system. This reparametrization of the system involves a very simple identifiability condition to be accomplished, that is the existence of the matrix $(F_w^T \cdot F_w)^{-1}$ or, equivalently, F_w is of full rank.

4.2 Consistency

Obviously, consistency of the estimates is directly influenced by the precision of numerical integration used to compute the value of the distributions. There are available numerous integration methods (often called numerical quadrature) with various degree of precision. One presents shortly the techniques used in the simulations to the approximate calculation of a definite integral $I_f = \int_a^b f(x)dx$ where f(x) is a given function and [a, b] a finite interval. Interpolatory quadrature formulas, where the nodes are constrained to be equally spaced, are called Newton–Cotes formulas. These are especially suited for integrating a tabulated function (such is our case). Newton-Cotes numerical integration rule If it is a weighted sum of function values: $I_f = \sum_{i=0}^{N-1} c_i \cdot f(x_i)$ where coefficients $\{c_i\}$ are derived from interpolating polynomial fitted for points $\{x_i, f_i\}$. A Newton–Cotes formula of any degree n can be constructed. One of the simplest integration methods is the trapezoidal rule. The trapezoidal rule is based on linear interpolation of *f(x)* at $x_1=a$ and $x_2=b$, i.e. *f(x)* is approximated by

$$p(x) = f(a) + (x-a)\frac{f(b)-f(a)}{b-a} \qquad (24)$$

The integral of p(x) equals the of trapezoid with base (b-a) times the average height

$$\frac{1}{2}(f(a)+f(b))$$

hence

$$\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b)) \qquad (25)$$

To increase the accuracy one subdivides the interval [a, b] and assume that $f_i = f(x_i)$ is known on a grid of equidistant points $x_0 = a$, $x_i = x_0 + ih$, $x_n = b$, where $h=(b-a)/n$ is the step length. The trapezoidal approximation for the ith subinterval is

$$\int_{xi}^{x_{i+1}} f(x)dx = \frac{h}{2}(f_i + f_{i+1}) + R_i \tag{26}$$

where

$$R_i = -\frac{h^3}{12} f''(\varsigma_i), \varsigma_i \in [x_i, x_{i+1}] \tag{27}$$

Summing the contributions for each subinterval $[x_i, x_{i+1}]$, i = 0 : n, gives

$$\int_a^b f(x)dx = \frac{h}{2}(f_0 + f_n) + h\sum_{i=2}^{i=n-1} f_i + R_T \tag{28}$$

The global truncation error is

$$R_T = -\frac{h^3}{12}\sum_{i=0}^{i=n-1} f''(\varsigma_i) = -\frac{h^2}{12}(b-a)f''(\varsigma), \varsigma \in [a,b] \tag{29}$$

This shows that by choosing h small enough we can make the truncation error arbitrarily small. In other words, we have asymptotic convergence when $h \to 0$.

In the composite Simpson's rule one divides the interval [a, b] into an even number $n = 2m$ steps of length h and uses the formula

$$\int_a^b f(x)dx = \frac{h}{3}(f_0 + 4S_1 + 2S_2 + f_n) + R_T \tag{30}$$

where

$$S_1 = f_1 + f_3 + \cdots + f_{n-1},\ S_2 = f_2 + f_4 + \cdots + f_{n-2} \tag{31}$$

are sums over odd and even indices, respectively. The remainder is

$$R_T = \frac{h^4}{180}(b-a)f^{IV}(\varsigma), \varsigma \in [a,b] \tag{32}$$

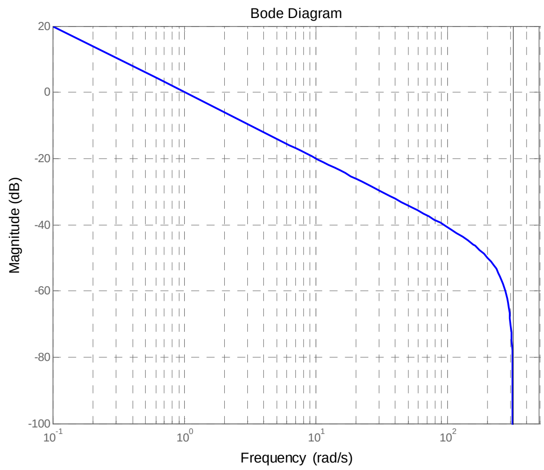
This shows that one have gained two orders of accuracy compared to the trapezoidal rule, without using more function evaluations.

Let's study properties of Newton-Cotes formulas in frequency domain (or spectral properties). Newton-Cotes rules are symmetric (hence linear phase) digital filters with finite impulse response. One of the most important characteristic of digital filter is magnitude/frequency response – function which shows how much filter damps or amplifies magnitude of particular frequency contained in input data. So, for trapezoidal rule and Simpson's rule one obtains

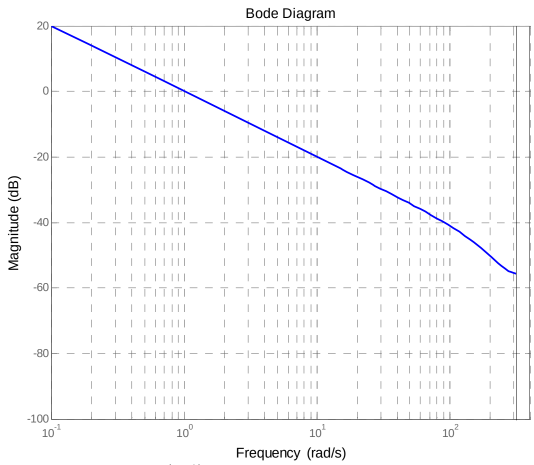
$$H(z) = \frac{h}{2}\cdot\frac{1+z^{-1}}{1-z^{-1}} \quad \text{and} \quad H(z) = \frac{h}{3}\cdot\frac{1+4z^{-1}+z^{-2}}{1-z^{-1}}$$

respectively.

These transfer functions have the amplitude-frequency Bode characteristics from Fig. 4.



a)



(54)

b)

Fig. 4. Amplitude-frequency bode characteristics:
a) trapezoidal rule b) Simpson's rule

As one can see classical Newton-Cotes formulas suppress high frequencies (noise) in the input data. In that sense they can be considered low-pass filters. From Fig. 4 one observes that trapezoidal rule offers a better suppression of noise than the Simpson's rule, so in the algorithm implementation one used the trapezoidal rule for numerical integration.

5. EXPERIMENTAL RESULTS

The performance of the proposed identification algorithm was tested by numerical simulations for the state space model and on a real plant (using an experimental platform) for the input – output case.

A. Identification of the state space model

If both current and speed of the load gear (state variables) are available for measurements one can identify all the motor parameters. The system described by state space equations in section 2 was simulated using the following parameter values:

Ra=2.6;
Kt=0.00767;
Ke=0.00767;
J=3.87e-7;
B=1.5e-3;
La=180e-6;

The system was simulated on a time interval of 40 seconds using a fourth order Runge-Kutta integration method in three cases:

Case 1: Ts = 40 ms, noise free.
Case 2: Ts = 40 ms, noisy measurements (SNR=40dB).
Case 3: Ts = 100 ms, noise free.

As input signal a sum of sinusoids of different amplitudes and frequencies was used that constitute a persistently exciting signal for the identification. Part of the input signal and measured signals in the noise free case are presented in figures 5 and 6.

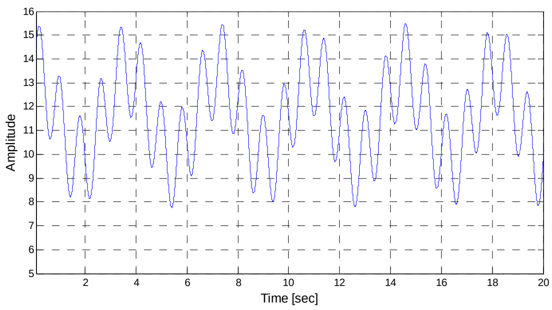


Fig. 5. Input signal (voltage)

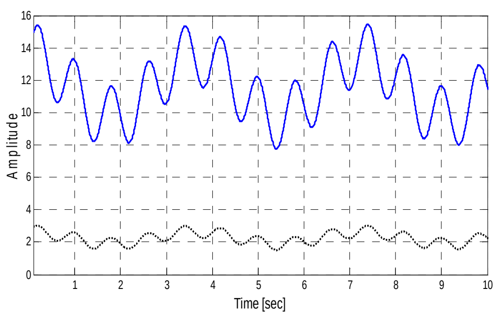


Fig. 6. System response (time evolution of state variables: speed [rad/sec] – continuous line and current [A] – dashed line)

TABLE I. CASE 1: REAL AND ESTIMATED DC MOTOR PARAMETERS (TS=40MS, NOISE FREE)

Parameter	Ra	Kt	Ke	J	B	La
real	2.6	0.00767	0.00767	3.87e-7	1.5e-3	180e-6
estimated Ts=40 ms	2.6067	0.00687	0.00687	3.56e-7	0.00148	0.000178

TABLE II. CASE 2: REAL AND ESTIMATED DC MOTOR PARAMETERS (TS=40 MS, NOISY MEASUREMENTS, SNR=40DB)

Parameter	Ra	Kt	Ke	J	B	La
real	2.6	0.00767	0.00767	3.87e-7	1.5e-3	180e-6
estimated SNR=40dB	2.5344	0.00502	0.00502	2.29e-8	0.00348	0.000195

TABLE III. CASE 3: REAL AND ESTIMATED DC MOTOR PARAMETERS (TS=100 MS, NOISE FREE)

Parameter	Ra	Kt	Ke	J	B	La
real	2.6	0.00767	0.00767	3.87e-7	1.5e-3	180e-6
estimated Ts=100ms	1.8432	0.00232	0.00232	1.47e-7	0.000511	0.000049

As test functions for signals processing three functions of exponential type (and their derivatives) were used. The corresponding results for the analyzed cases are presented in Tables I – III. The simulation results reveal good noise rejection properties of the estimation algorithm. This fact is due to the filtering properties of the integration operation. The estimates are more sensitive to the sampling period that influences the truncation error in the integration stage.

B. Identification of input – output model

If we want to build a model for a system, we should get information about it. This can be done by just watching the natural fluctuations, but most often it is more efficient to set up dedicated experiments that actively excite the system. In the latter case, the user has to select an excitation that optimizes his own goal (for example, minimum cost, minimum time, or minimum power consumption for a given measurement accuracy) within the operator constraints. The quality of the final result can depend heavily on the choices that are made.

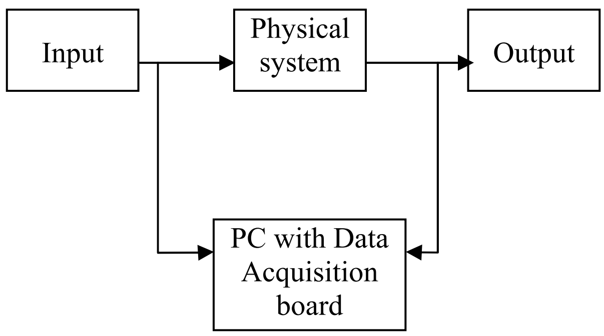


Fig. 7. System identification experimental setup

Fig. 7 shows the experimental setup requirement prior to the parameter identification. This is the recording phase.

1. Deploy a data acquisition system, which can record the input and output at the required sampling frequency (according to the system dynamics).
2. Feed the system with rich inputs. (Inputs must change with time).
3. Record the inputs and corresponding outputs simultaneously using this data acquisition system.

To illustrate the performance of the proposed identification algorithm, one identifies a real Quanser experiment using a DC servomotor with built in gearbox. The "rotational series" that we have is the SRV-02ET (E-encoder, T-tachometer), and the DC servo is shown in the Fig. 8 (Quanser, 1998). A high quality DC servo motor is mounted in a solid aluminium frame. The motor drives a built-in 14:1 gearbox whose output drives an external gear. The motor gear drives a gear attached to an independent output shaft that rotates in a precisely machined aluminium ball bearing block.

The output shaft is equipped with an encoder. This second gear on the output shaft drives an anti-backlash gear connected to a precision potentiometer. The potentiometer is used to measure the output angle. The external gear ratio can be changed from 1:1 to 5:1 using various gears. Two inertial loads are supplied with the system in order to examine the effect of changing inertia on closed loop performance. In the high gear ratio configuration, rotary motion modules attach to the output shaft using two 8-32 thumbscrews. The square frame allows for installations resulting in rotations about a vertical or a horizontal axis. The system is interfaced by means of a data acquisition card and driven by Wincon 5.0 based real time software.

WinCon™ is a real-time Windows application. It allows you to run code generated from a Matlab/Simulink diagram in real-time on the same PC (also known as local PC) or on a remote PC. Data from the real-time running code may be plotted on-line in WinCon Scopes and model parameters may be changed on the fly through WinCon Control Panels as well as Simulink. The automatically generated real-time code constitutes a stand-alone controller (i.e. independent from Simulink) and can be saved in WinCon Projects together with its corresponding user-configured scopes and control panels.

WinCon software actually consists of two distinct parts: WinCon Client and WinCon Server. They communicate using the TCP/IP protocol. WinCon Client runs in hard real-time while WinCon Server is a separate graphical interface, running in user mode. The measured input–output data are transferred to the computer by a data acquisition card (Quanser Q4, 33 MHz PCI bus interface, 12 bit high speed A/D converter (Quanser, 1998)). The data acquisition card permits use of user defined programs interfaced with Matlab. The output speed is obtained from the tacho-generator.

Fig. 8. Quanser SRV02ET DC Motor experiment

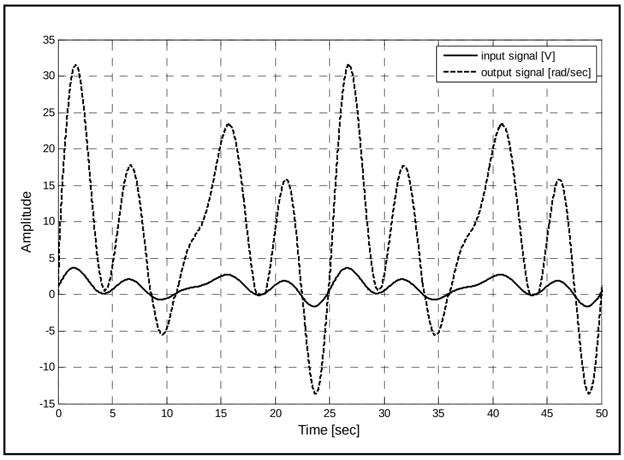


Fig. 9. The input signal and system response (dotted line)

One obtains the follwing transfer function:

$$H_{cc}(s) = \frac{b_0}{a_2 s^2 + a_1 s + a_0}$$

where

$$\begin{aligned} b_0 &= 0.00753 \\ a_2 &= 2.852e-005 \\ a_1 &= 0.00011116 \\ a_0 &= 0.000901 \end{aligned}$$

The input signal and system response are presented in Fig. 9. The validation of the model is realized by the comparison between the output of the identified model and the real plant at the same input (that was a sum of sinusoids). The result is presented in Fig. 10.

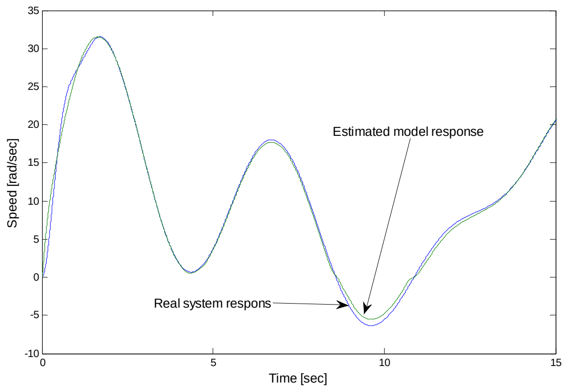


Fig. 10. Real system and estimated model responses to the same input signal

6. CONCLUSIONS

In this paper, a novel parameter estimation method for linear system identification based on the distribution algorithm was developed. The distribution algorithm has been shown to be versatile when applied to parameter estimation, without requiring a detailed mathematical representation of the identification problem. This procedure is a functional type method, which transforms a differential system of equations to an algebraic system in unknown parameters. The relation between the state variables of the system is represented by functionals using techniques from distribution theory based on testing function from a finite dimensional fundamental space. The identification algorithm has a hierarchical structure, which allows obtaining a linear algebraic system of equations in the unknown parameters. The coefficients of this algebraic system are functionals depending on the input and state variables and are evaluated through some testing functions from distribution theory. The effectiveness of system identification using the distribution algorithm was researched and a satisfactory performance was obtained. The simulation results show that the proposed method achieved a minimum tracking error and estimated the parameter values with high accuracy. The method was also applied to estimate the parameters of a DC motor commonly used in industry. The influence of the sampling period, initial conditions, test functions type, input type and noise on the parameters estimates was empirically analyzed. The algorithm provides very good results even the measurements are noise contaminated because the evaluation of states derivatives is completely avoided.

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REFERENCES

Bastogne, T., Garnier, H., Sibille, P., & Mensler, M. (1996) PMF-based subspace method for continuous-time model identification deterministic study. *Proceedings of the 11th IFAC symposium on system identification (SYSID'97)*, Fukuoka, July, pp. 1665 – 1670

Bos, A.V. (2007). *Parameter Estimation for Scientists and Engineers*. First Edition, Wiley, Inc.

Diniz, P. (2002). *Adaptive Filtering: Algorithms and Practical Implementation.* Springer Publishers, Second edition.

Eykhoff, P. (1974). *System Identification*, J. Wiley, London.

Johansson, R. (1994). Identification of continuous-time models, *IEEE Transactions on Signal Processing*, 42(4), pp. 887-896.

Jung, Y. G., Cho, K., Lim, Y., Park, J. and Change, Y. H. (1992). Time domain identification of brushless DC motor parameters, *IEEE Trans. on Industrial, Electronics*, vol. 2, 593-597.

Marin, C. (2002). System identification based on distribution theory, *Proc. of IASTED Int. Conf. Applied Simulation and Modelling (ASM)*, Crete, pp. 456-462.

Middleton, R.H. & Goodwin, G.C. (1990). *Digital Control and Estimation, a Unified Approach*, Prentice-Hall, Englewood Cliffs.

Ohsumi, A., Kameiama, K. (2002). Subspace identification for continuous-time systems via distribution-based approach. *Automatica*, 38: 63-79.

Pearson, A., Lee, F. (1985). On the identification of Polynomial input-output differential systems. *IEEE Trans. on Automatic Control*, 30(8): 778-782.

Pearson, R. K. and Pottmann, M. (2000). Grey-box identification of block-oriented nonlinear models. *Journal of Process Control*, 10(4), 301–315.

Schawarz, L. (1951). *Théorie des distribution*, Paris.

Sinh, N. K., Rao, G. P. (1991). *Identification of continuous–time systems*. Dordreht: Kluwer Academic Press.

Sinha, N., Dicenzo, C. and Szabados, B. (1984). Modelling of DC motors for control applications, *IEEE Trans. Industrial Electronics And Control Instrumentation*, vol. IECI-21, pp. 84-88.

Soderstrom, T. and Stoica, P. (1989). *System identification*. Cambridge, UK: Prentice Hall.

Unbehauen, H., Rao, G.P. (1990). Continuous-time approaches to system identification – a survey. *Automatica*, 26(1), pp. 23-35.

Wang, J., Pu, J., Moore, P. (1999). A practical control strategy for servo-pneumatic actuator systems. *Control Eng Pract* 7:1483–1488

*** Quanser Consulting Inc. (1998). *Quanser SRV02-ET Experiment. User manual.*