# A Faster Method for Robot Kinematic Modelling

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**Abstract:** This paper presents a faster method for robot modelling by direct kinematics using „*Complete Kinematical Structure* – CKS". It is a structure with 3 revolute joints and 3 prismatic joints which can assure a complete positioning and orientation in space for an object attached to its terminal. Its homogeneous transformation operator between fix frame ($R_F$) and mobile frame ($R_M$) is determined. An algorithm for robot modelling by CKS is enunciated. This structure is a useful tool for a faster establishing of the kinematical model (direct problem) for the robot arms and helps to resolve the inverse kinematical problems in some situations. By classical methods we obtain the same operator if we have the same position and orientation for the frames $R_0$ (fix) and $R_M$ (mobile). Using this procedure, a great part of tedious and time-consumer operations of matrix multiplication (present in classical method) is eliminated. Finally, two applications of some robotic structures are presented.

*Keywords:* kinematical model, Complete Kinematical Structure, modelling algorithm, DH method.

## 1. INTRODUCTION

Usually, in order to deal with the complex geometry of a manipulator and to obtain the kinematical equations of the robot models we will attach frames to the various parts of the mechanisms and then describe the relationship between these frames. Since the links of a robot arm may rotate and/or translate with respect to a reference coordinate frame, the total spatial displacement of the end-effector is due to the angular rotations and linear translations of the links (Schilling 1990). The central topic is a method to compute the position and orientation of the manipulator's end-effector relative to the base of the manipulator as a function of the join variables (Paul 1981a, 1981b, Ivanescu 1994, 2003, Groover 1986, Fu 1987, Klafter 1989). Denavit and Hartemberg (1964) proposed a systematic and generalized approach of using matrix algebra to describe and represent the spatial geometry of the links of a robot arm with respect to a fixed reference frame. The advantage of using the Denavit-Hartemberg representation of linkages is its algorithmic universality in deriving the kinematical equation of the robot arm (Asada 1986), (Bishop 2002), (Coiffet 1983), (Craig 1989). By Denavit-Hartemberg method results a homogeneous transformation matrix for every couple of two adjacent rigid mechanical links and all those matrices have to be multiplied. This transformation is a function of the four parameters: only one variable, the ather three parameters beind fixed by mechanical design (Drimer 1985, Murray 1994, Ispas 1990, Ranky 1985). By defining a frame for each link we have broken the kinematics problem in n subproblems. In order to solve each of these subproblems we have break the problem into four sub-subproblems (Renaud 1980, Davidoviciu 1986, Lamineur 1984, Pelecudi 1985, Pieper 1968, Samson 1981, Stadler 1992). Present method is based on DH approach but decreases drastically the number of the matrices and simplifies a lot the calculations of matrix product (Stoian 1994, 2003, 2006, Tzai 1984).

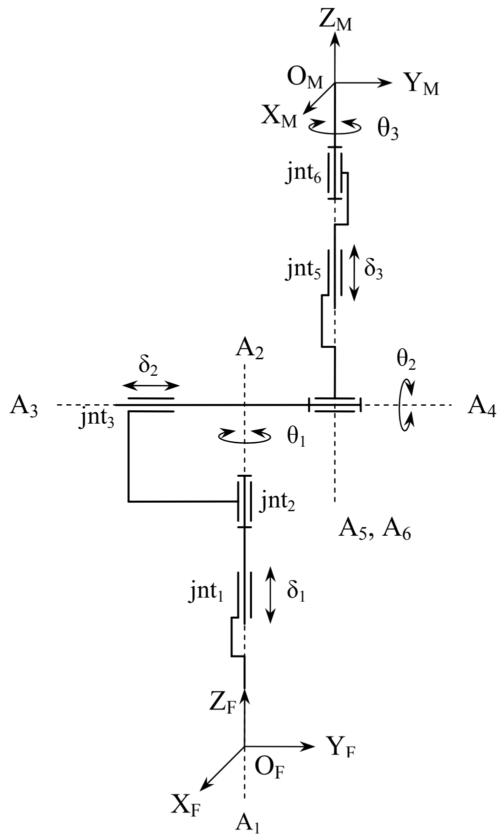


Fig. 1. Complete Kinematical Structure (CKS)

## 2. COMPLETE KINEMATIC STRUCTURE (CKS)

We name the kinematical structure from Figure 1 a "*complete kinematical structure*" (CKS) because, with 3 rotation joints and 3 translation joints, it can assure a complete positioning and orientation in space for an object attached to its terminal. The above mentioned two operations are not uncoupled. This structure is a useful tool for a faster establishing of the kinematical model (direct problem) for the robot arms and helps to resolve the inverse kinematical problems in some situations. By classical methods we obtain the same operator if we have the same position and orientation for the frames $\mathbf{R_0}$ (fix) and $\mathbf{R_N}$ (mobile). Using this procedure, a great part of tedious and time-consumer operations of matrix multiplication (present in classical method) is eliminated.

## 3. THE HOMOGENEOUS TRANSFORMATION OPERATOR $T_{CKS}$

For determining of the transfer homogeneous operator $T_{CKS}$ we will use the DH approach allocated to the structure from Figure 1. In Figure 2 we present the mapping of the body-attached coordinate frame over the rigid mechanical links by first steps of the DH algorithm.

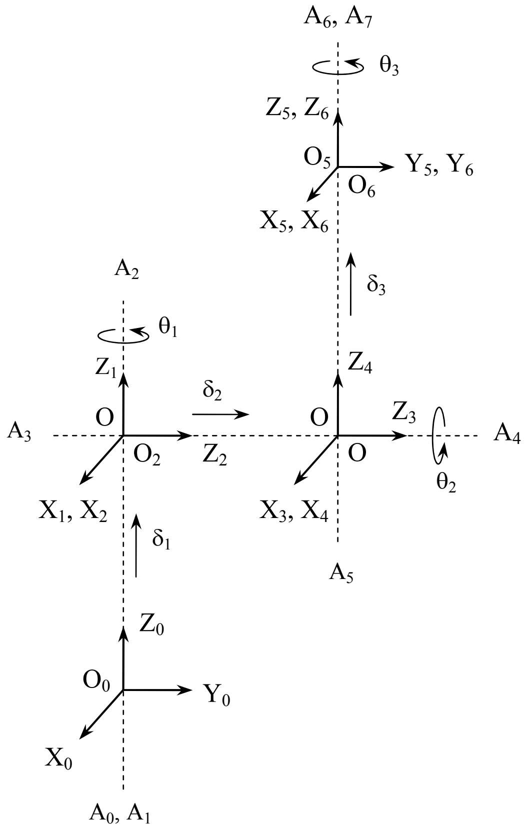


Fig. 2. Maping of the body-attached coordinate frame

The next steps establish the sets of the DH parameters for each transformation between two adjacent frames. These sets are presented in Table 1. The joint coordinate vector has the following structure:

$$\mathbf{q} = [\delta_1, \theta_1, \delta_2, \theta_2, \delta_3, \theta_3]^T \quad (1)$$

**Table 1. DH Parameters**

| | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| $T_1^0$ | 0 | 0 | $\delta_1$ | 0 |
| $T_2^1$ | 0 | $-\pi/2$ | 0 | $\theta_1$ |
| $T_3^2$ | 0 | 0 | $\delta_2$ | 0 |
| $T_4^3$ | 0 | $\pi/2$ | 0 | $\theta_2$ |
| $T_5^4$ | 0 | 0 | $\delta_3$ | 0 |
| $T_6^5$ | 0 | 0 | 0 | $\theta_3$ |

We obtain the general transfer operator $\mathbf{T_{CKS}}$ by the product between the six intermediate homogeneous transformation operators obtained from Table 1 and by DH operator (Groover et al. 1986), (Klafter 1989):

$$\mathbf{T_{CKS}} = \mathbf{T}_6^0 = \prod_{i=1}^{6} \mathbf{T}_i^{i-1} = \mathbf{T}_1^0(\delta_1)\mathbf{T}_2^1(\theta_1)\cdot$$

$$\mathbf{T}_3^2(\delta_2)\,\mathbf{T}_4^3(\theta_2)\,\mathbf{T}_5^4(\delta_3)\,\mathbf{T}_6^5(\theta_3) =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \delta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} C_1C_2C_3 - S_1S_3 & -C_1C_2S_3 - S_1C_3 & C_1S_2 & -\delta_2S_1 + \delta_3C_1S_2 \\ S_1C_2C_3 + C_1S_3 & -S_1C_2S_3 + C_1C_3 & S_1S_2 & \delta_2C_1 + \delta_3S_1S_2 \\ -S_2C_3 & S_2S_3 & C_2 & \delta_1 + \delta_3C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

## 4. ALGORITHM FOR KINEMATICAL MODEL

The next algorithm is established as follows:

- The kinematical structure of the robot is determined.

- A suitable representation with symbols of revolute/prismatic joints is made (kinematical schema).

- On this structure, a minimum number of CKSs is identified in order to cover it.

- For each identified CKS, a set of geometrical and movement parameters is appropriately established to the associated area from the kinematical schema.

- For each CKS, a homogeneous transformation operator (1) is calculated with the set of the parameters established to the previous step: $\mathbf{T^1}_{\mathbf{CKS}}$, $\mathbf{T^2}_{\mathbf{CKS}}$ …

- If multiple operators result, we make their product to obtain the general transformation operator T.

The homogeneous transformation operator, which makes the transformation between the coordinates of a point **P**, related to the mobile coordinate frame $\mathbf{R_M}$ and the point **P** coordinates related to the reference frame $\mathbf{R_F}$ considered fix, is presented bellow:

$$\mathbf{T} = \mathbf{T^1}_{\mathbf{CKS}} \cdot \mathbf{T^2}_{\mathbf{CKS}} \cdot \cdots \mathbf{T^N}_{\mathbf{CKS}} \tag{3}$$

## 5. APPLICATIONS

### *5.1. Application 1*

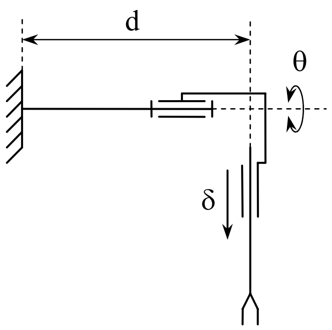


Fig. 3. Robotic kinematical structure for the Application 1

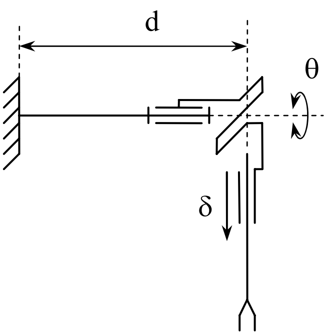


Fig. 4. Considered robotic kinematical structure

We want to obtain the kinematical model of the robotic structure from Figure 3.

This structure is redrawing in Figure 4 for an easier comparing with CKS.

In Figure 5 we represented the mapping of the necessary frames. After comparing of the structure from Figure 4 with a CKS and determining of the geometrical and moving parameters from Figure 5, we obtain the parameters presented in relations (4):

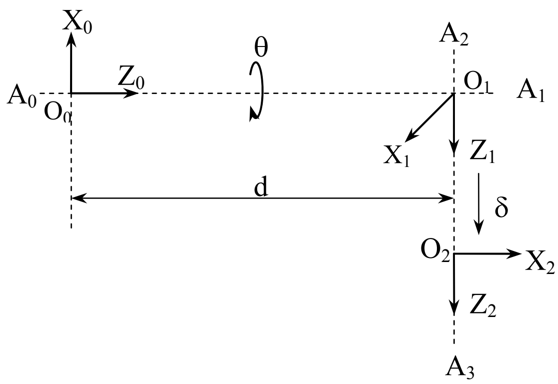


Fig. 5. Mapping of the coordinate frame

$$\begin{array}{lll} \delta_1 \leftarrow d & \delta_2 \leftarrow 0 & \delta_3 \leftarrow \delta \\ \theta_1 \leftarrow \theta & \theta_2 \leftarrow -\pi/2 & \theta_3 \leftarrow 0 \end{array} \tag{4}$$

Those parameters represent entries for the operator $\mathbf{T}_{\mathrm{CKS}}$ from (2). The expression of $\mathbf{T}_{\mathrm{CKS}}$ becomes (5):

$$\mathbf{T}_{\mathrm{CKS}} = \begin{bmatrix} 0 & -S\theta & -C\theta & -\delta C\theta \\ 0 & C\theta & -S\theta & -\delta S\theta \\ 1 & 0 & 0 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

By classical methods we obtain the same operator if we have the same position and orientation for the frames $\mathbf{R_0}$ (fix) and $\mathbf{R_2}$ (mobile).

### *5.2. Application 2*

In this second case we want to obtain the kinematical model of the robotic structure from Figure 6. This structure is a more complex structure and we need 3 CKSs for cover it because there are 3 joints which have the parallel axis.

The first CKS covers the first two joints, the second CKS covers the third joint and the third CKS covers the last two joint of the robot. In this case we need to attach 2 intermediary frames: first (**R’**) on link index 2 and the second (**R’’**) on link index 3.

For the first substructure (Figure 7) we identify the parameters (6) of the $\mathrm{CKS}_1$ and the operator $\mathbf{T^1}_{\mathrm{CKS}}$ (7) results applying (2), for the second structure (Figure 8) we identify the parameters (8) of the $\mathrm{CKS}_2$ and the operator $\mathbf{T^2}_{\mathrm{CKS}}$ (9) results applying (2) and for the last structure (Figure 9) we identify the parameters (10) of the $\mathrm{CKS}_3$ and the operator $\mathbf{T^3}_{\mathrm{CKS}}$ (11) results applying (2).

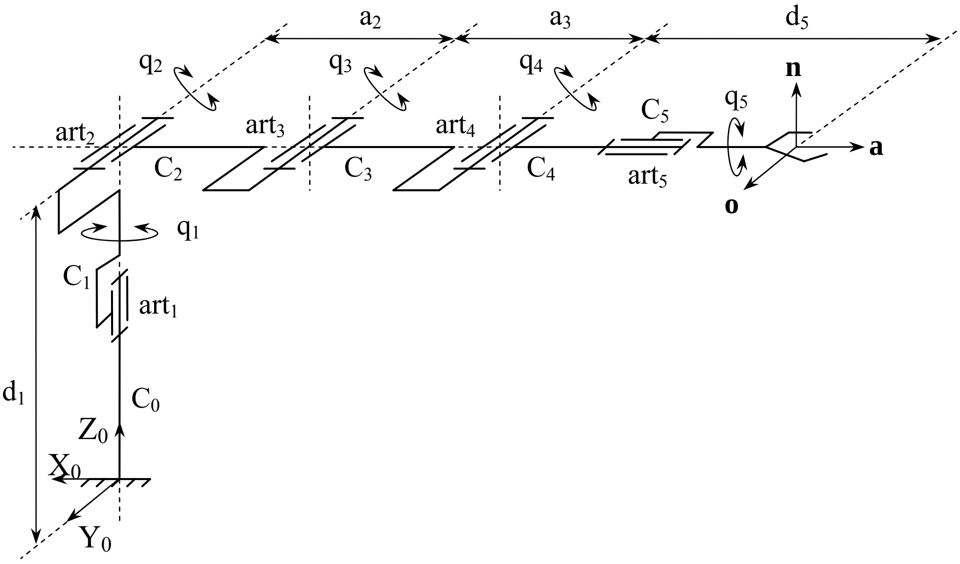


Fig. 6. Robotic structure for the Application 2

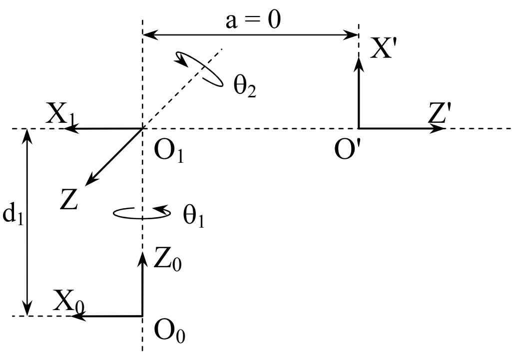


Fig. 7. First robot substructure

$$\begin{array}{lll} \delta_1 \leftarrow d_1 & \delta_2 \leftarrow 0 & \delta_3 \leftarrow 0 \\ \theta_1 \leftarrow \theta_1 & \theta_2 \leftarrow -\pi/2 + \theta_2 & \theta_3 \leftarrow 0 \end{array} \tag{6}$$

$$\mathbf{T}^1_{\text{CKS}} = \begin{bmatrix} C_1S_2 & -S_1 & -C_1C_2 & 0 \\ S_1S_2 & C_1 & -S_1C_2 & 0 \\ C_2 & 0 & S_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\begin{array}{lll} \delta_1 \leftarrow a_2, & \delta_2 \leftarrow 0 & \delta_3 \leftarrow 0 \\ \theta_1 \leftarrow 0 & \theta_2 \leftarrow \theta_3 & \theta_3 \leftarrow 0 \end{array} \tag{8}$$

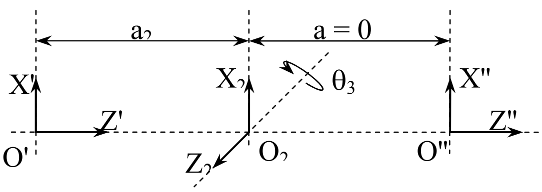


Fig. 8. Second robot substructure

$$\mathbf{T}^2_{\text{CKS}} = \begin{bmatrix} C_3 & 0 & S_3 & 0 \\ 0 & 1 & 0 & 0 \\ -S_3 & 0 & C_3 & a_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

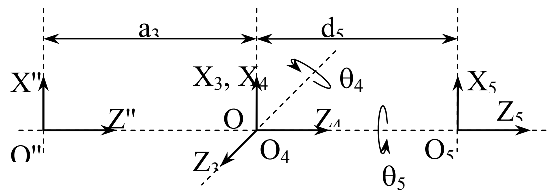


Fig. 9. Third robot substructure

$$\begin{array}{lll} \delta_1 \leftarrow a_3 & \delta_2 \leftarrow 0 & \delta_3 \leftarrow d_5 \\ \theta_1 \leftarrow 0 & \theta_2 \leftarrow \theta_4 & \theta_3 \leftarrow \theta_5 \end{array} \tag{10}$$

$$\mathbf{T}^3_{\text{CKS}} = \begin{bmatrix} C_4C_5 & -C_4S_5 & S_4 & d_5S_4 \\ S_5 & C_5 & 0 & 0 \\ -S_4C_5 & S_4S_5 & C_4 & a_3 + d_5C_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The operator between the end-effector coordinate frame and the reference coordinate frame is:

$\mathbf{T}(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5) =$

$\mathbf{T}^1_{\text{CKS}}(\theta_1, \theta_2) \cdot \mathbf{T}^2_{\text{CKS}}(\theta_3) \cdot \mathbf{T}^3_{\text{CKS}}(\theta_4, \theta_5) =$

$\mathbf{T}_{\mathbf{ARM}}(\theta_1, \theta_2, \theta_3) \cdot \mathbf{T}_{\mathbf{TERM}}(\theta_4, \theta_5) =$

$$
= \begin{bmatrix} C_1S_{234}C_5 - S_1S_5 & -C_1S_{234}S_5 - S_1C_5 & -C_1C_{234} & -C_1\left(a_2C_2 + a_3C_{23} + d_5C_{234}\right) \\ S_1S_{234}C_5 + C_1S_5 & -S_1S_{234}S_5 + C_1C_5 & -S_1C_{234} & -S_1\left(a_2C_2 + a_3C_{23} + d_5C_{234}\right) \\ C_{234}C_5 & -C_{234}S_5 & S_{234} & d_1 + a_2S_2 + a_3S_{23} + d_5S_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)
$$

Notations: $S_2 = \sin\theta_2$; $C_2 = \cos\theta_2$; $S_{23} = \sin(\theta_2 + \theta_3)$; $C_{23} = \cos(\theta_2 + \theta_3)$; $S_{234} = \sin(\theta_2 + \theta_3 + \theta_4)$; $C_{234} = \cos(\theta_2 + \theta_3 + \theta_4)$

## 6. CONCLUSIONS

The direct kinematics problem is reduced to find a transformation matrix that relates the body-attached coordinate frame to the reference coordinate frame using vector and matrix algebra. By Denavit-Hartemberg method a homogeneous transformation matrix results for every couple of two adjacent rigid mechanical links and all those matrices have to be multiplied.

This paper presents a faster method for robot modelling by direct kinematics using „Complete Kinematical Structure" (CKS) which is a structure with 3 revolute joints and 3 prismatic joints which can assure a complete positioning and orientation in space for a object attached to its terminal. Its homogeneous transformation operator between fix frame and mobile frame is determined and used in an algorithm for robot modelling. Finally, two applications of some robotic structures are presented.

Present method is based on DH approach but decreases drastically the number of the matrices and simplifies a lot of the calculations.

By classical methods we obtain the same operator if we have the same position and orientation for the frames $\mathbf{R_0}$ (fix) and $\mathbf{R_N}$ (mobile).

Using this procedure, a great part of tedious and time-consumer operations of matrix multiplication (present in classical method) is eliminated.

## REFERENCES


Asada, H. and J.-J.E. Slotine. (1986). *Robot Analysis and Control*, John Wiley & Sons, Inc., New York.

Bishop, R.H. (Editor-in-Chief). (2002). *The Mechatronics Handbook*. CRC Press, New York.

Coiffet, P. (1983). *Modelling and Control*. Rob. Tech., Hermes Publishing.

Craig, J.J. (1989). *Introduction to Robotics. Mechanics and Control*, Addison-Wesley Publishing Company, New York.

Davidoviciu, A., s.a. (1986). "*Modelarea, simularea şi comanda manipulatoarelor şi roboţilor industriali*". Editura Tehnică, Bucureşti.

Denavit, J. and R.S. Hartemberg. (1964). *Kinematic Synthesis of Linkages*. McGraw-Hill, New York.

Drimer, D., ş.a. (1985). *Roboţi industriali şi manipulatoare.* Editura Tehnică, Bucureşti.

Fu, S.K., Gonzales, R.C., Lee, C.S.G. (1987). *Robotics: Control, Sensing, Vision, and Intelligence*. McGraw-Hill Book Company, New York.

Groover, M., et al. (1986). *Industrial Robotics. Technology, Programming and Applications*. McGraw-Hill, Inc., New York.

Ispas, V. (1990). *Aplicaţiile cinematice în construcţia manipulatoarelor şi a roboţilor industriali.* Editura Academiei Române, Bucureşti.

Ivănescu, M. (1994). *Roboţi industriali-Algoritmi şi sisteme de conducere.* Editura Universitaria, Craiova.

Ivănescu, M. (2003). Sisteme avansate de conducere a robotilor. Editura Scrisul Romanesc, Craiova.

Klafter, R.D. (1989). *Robotic engineering: An Integrated Approach*. Prentice-Hall, New Jersey, USA.

Kovacs, Fr., Cojocaru, Gh. (1982). *Manipulatoare, roboţi şi aplicaţiile lor industriale*. Editura Facla, Timişoara.

Lamineur, P., Cornillie, O. (1984). *Industrial Robots*. Pergamon Press, New York, USA.

Murray, R.M., Li, Z., Sastry, S.S. (1994). *A Mathematical Introduction to Robotic Manipulation*. CRC Press Inc., London.

Paul, R.P., Shimano, B., Mayer, G. (1981). Kinematic Control Equations for Simple Manipulators. *IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-11, No. 6.*

Paul, R.P. (1981). *Robot Manipulators: Mathematics, programming and Control. The Computer Control of Robot Manipulators.* The MIT Press, Cambridge, Massachusetts and London, England.

Pelecudi, Chr. (1985). *Mecanisme*. Editura Didactica si Pedagogica, Bucuresti.

Pieper, D. (1968). *The Kinematics of Manipulators under Computer Control*. Ph.D. Thesis, Stanford University.

Ranky, P.G., Ho, C.Y. (1985). *Robot Modelling-Control and application with software*. Springer-Verlag, New York, USA.

Renaud, M. (1980). *Contribution a la modelisation et a la commande dynamique des robots manipulateurs*. These de Docteur d' Etat, Univ. Paul Sabatier, Toulouse, France.

Samson, C. (1991). *Robot Control*. Clarendon Press, Oxford.

Schilling, J.R. (1990). *Fundamentals of Robotics, Analysis and Control*. Prentice Hall, New York, USA.

Stadler, W. (1992). *Analytical Robotics and Mechatronics*. McGraw-Hill, Inc., New York, USA.

Stoian, V. (1994). Extension of the DH method for robotic structure kinematical model. *Proceedings of the SINTES 7 Symposium*, Craiova, Romania, may 20-21, vol. 1, pp. 252-260.

Stoian, V. (2003). *Roboti industriali. Aplicatii – Vol I.* Editura Universitaria, Craiova.

Stoian, V. (2006). *Roboti industriali. Aplicatii – Vol II.* Editura Universitaria, Craiova.

Tsai, L., Morgan, A. (1984). Solving the Kinematics of the Most General Six-and Five-degree-of-freedom Manipulators by Continuation Methods. *Paper 84-DET-20, ASME Mechanism Conference*, pp. 57-63. Boston, USA.