Stopping Time for a Planar Link in Contact with a Granular Material

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Abstract: The impact of a free planar link with a granular material is studied in this research. The force of the granular material on the link is a linear superposition of a depth dependent resistance force and a velocity dependent frictional force. The granular material is defined as a conglomeration of discrete solids and both the solid like and fluid liquid characteristics are considered. The impact is examined from the contact moment until rest using various initial conditions. The results of the simulations are analyzed taking into consideration the initial impact conditions. The system under higher initial velocity comes to rest faster than the same system under lower initial velocity.

Keywords: impact; granular material; stopping time; initial velocity.

1. INTRODUCTION

An individual granule of a granular matter is solid and shares the physical properties of solid matter. According to circumstances, the granular material may act like a solid, fluid or gas. The first approach of the granular matter was based on the contact and the collision of solid particles (1; 2; 3; 4; 5). Other approaches were based on fluid dynamics and took the fluid-like characteristics of the materials into account (6; 7; 8; 9). Granular materials exhibits also unusual behaviors compared with solids, liquids, or gases. The contact forces exerted by granular materials constitute a network of forces at large scale (10; 11; 12; 13). The stereomechanics impact with a granular material is an interesting subject in the field of engineering. The impact with a granular material is not a simple problem because of contacts, collisions, and flows take place when a body impacts the material. Of course, the major phase state of the granular material is decided by the penetrating velocity of the impacting body. For high speed impact, the characteristic of the granular material in the vicinity of a rigid body is similar to a fluid. The granular material acts like a solid when the penetrating speed of the body is slow. However for the usual impact cases the behaviors of the granular materials exhibit a combination of these characteristics.

Recent studies focus their attention on relatively low speed regime and impact cratering. For low speed impact the following topics were studied: the horizontal resistance force exerted by granular materials (14), the jamming and the fluctuations of the resistance force (15; 16), the shape effects on the resistance force (17), and the vertical resistance force (18). The impact cratering, the size, the depth, and the form of the crater were also studied (19; 20; 21; 22).

A recent effort in the field is to develop a force law model for the granular impacts and to find a mathematical formula in order to measure the impact force (22). The resistance force models, linear to the depth of the penetrating body (21; 23), linear to the velocity (19), and linear to the square of velocity of the body have been studied and used to explain the motion in granular materials. Tsimring and Volfson analyzed the impact cratering of large projectiles into a dry granular material (20). In their study they proposed for the resistance force model a sum of a velocity dependent drag force and a depth dependent resistance force. The depth dependent resistance force (static resistance force) of a rigid body into a granular matter has been developed for the horizontal motion in (14; 15; 16; 17) and for the vertical motion in (18).

Hou, Peng, Liu, Lu, and Chan (21) analyzed the deceleration of projectiles impacting the medium and they decided the stopping time is not a linear function of initial impact velocity. The paper of Katsuragi and Durian has sparkled new interest because they introduced the resistance force model proposed by (20) to study the impact of a rigid sphere and verified the results with a line-scan digital CCD camera (22). They analyzed how rapidly a sphere impacting a granular material slows upon collision and they clarified the relation between the stopping time of a rigid body and the initial impact velocity for the vertical impact. Lee and Marghitu extended the study to the model of a rigid links obliquely impacting the material and reported the results for the oblique impact (24). Analysis shows that as the speed at which the spheres impact the material increases, the sooner it will come to rest for the vertical impact.

The impact of kinematic chains with granular matter is ubiquitous and can be applied to robotic locomotion, tracked vehicles, and heavy duty construction equipments because in general the operations take place in outfields with soil, sand, mud, or mixed environments. Developing or planning to develop multi-legged robots cannot avoid the continuous impact of the links with the granular materials. From this view point, the study of the impact with granular material can be applied to development including the design, the manufacture, and the optimization of kinematic chains.

In this study we focus on modeling and simulation of a free planar impacting a granular material using the resistance force model as the sum of a velocity dependent drag force and a depth dependent resistance force. We will analyze the relations among the initial impact velocities, the stopping time, and the penetrating depth based on simulation results.

2. GRANULAR MATERIAL FORCES

There are various models for the resistance forces on the penetrating body. The Bingham's model is defined by $F_R = F_0 + b\,v$, where v is velocity of the body relative to a material, and has been utilized for viscous drag. The model has originated for a viscoplastic material that behaves as a solid at low stress but flows as a viscous fluid at high stress. Bruyn and Walsh applied this model and calculated the penetration of spheres into loose granular material (19). The Poncelet's model $F_R = F_0 + c\,v^2$, where b and c are drag coefficients, was initiated on high speed impact. The Bingham's model has recently been recommended for granular impact, while the Poncelet's model has long been used for ballistics applications.

Allen, Mayfield, and Morrison (25; 26) suggested a combination of Bingham and Poncelet's models as $F_R = A\,v^2 + B\,v + C$ for the study of projectile penetrating the sand, where A is a classical fluid dynamic drag parameter, B is a deceleration parameter due to kinetic fiction on the surface of the projectile, and C is the deceleration force due to the inherent structural characteristics of the target material.

However, some recent research (14; 15; 16; 17; 18) indicate that the static resistance force, defined as a constant for the Bingham's and the Poncelet's models, depends on the depth of penetration linearly or non-linearly. Lohse, Rauhe, Bergmann, and van der Meer formulated the resistance force model made up of the only the static resistance force (23). They considered the resistance force as $F_R = k\,y$, where k is a constant and y is the penetrated depth. Tsimring and Volfson (20) proposed a resistance force model based on the Poncelet's model and the static resistance force model derived in (14) as a generalized Poncelet's model. The total resistance forces acting on a moving body into a granular matter can be generalized as the sum of the static resistance force characterized by a depth-dependent friction force and the dynamic frictional force characterized by a velocity-dependent drag force

$$F_R = F_s(z) + F_d(v), \tag{1}$$

where F_s is the static resistance force and F_d is the dynamic frictional force. With this formula the resistance force model can use the solid-like and fluid-like characteristics simultaneously.

2.1 Dynamic Frictional Force F_d

The dynamic frictional force F_d is conceptually the same as the "drag" force widely used in the field of fluid dynamics. Even though the state of a granular material is not a fluid but a solid, individual grains begin to loose the stationary state and the granular material begins to fluidize acting liquid-like when the external driving forces, including tilting and shaking, exceed the stationary condition. From this fluid-like behavior of the granular matter, the resistance force is assumed to be as a drag force impeding the body motion (27; 28; 29). Like the drag force model in fluid dynamics, the dynamic frictional force F_d also had been modeled as a linear drag force or as a quadratic drag force. However the research results using a dilute granular flow condition which is not affected by the static resistance force (27) and the experiments at low speed impact (20; 22) show that the quadratic drag force model is more suitable for the dynamic frictional force, F_d, than the linear drag force model, even at relatively low speed regime. The dynamic frictional force vector can be modeled as

$$\mathbf{F_d} = -\frac{\mathbf{v}}{|\mathbf{v}|}\,\eta_d\,\rho_g\,A_r\,\mathbf{v}\cdot\mathbf{v}, \tag{2}$$

where η_d is a drag coefficient determined from experimental results, ρ_g is the density of the granular material, and A_r is the reference area of the body. The term $-\mathbf{v}/|\mathbf{v}|$ is applied because the dynamic force is opposed to the velocity vector $\mathbf{v}$.

2.2 Static Resistance Force F_s

The static resistance force is an internal resisting force and appears when an external force is applied to a body. This resistance force does not depend on the velocity or the acceleration of a body and acts as an internal stress. Therefore this force acts as a main resistance force when a body penetrates a granular material at very low speed.

The theoretical modeling of the static force originated from the characteristics of the granular material in stationary state. Different research results explain the origin of this resistance force as the network of forces generated from contacts between granules within the material pile (10; 11; 12; 13). Due to this complicated process of transmitting the force, there are few general continuum theories completely describing the static resistance force (18). In this study, we considered horizontal and vertical static resistance forces developed by theoretical and empirical approaches.

3. EQUATIONS OF MOTION FOR A FREE LINK

Figure 1 shows a planar free rigid link impacting a granular material. The forces acting on the link are: the gravity force $\mathbf{G}$ at the mass center C and the resistance force, $\mathbf{F}_R$ including $\mathbf{F}_s$ and $\mathbf{F}_d$, acting at the point E. The point E is the centroid point of immersed part as shown in Fig. 1. The general equations of motion for the planar kinematic chain can be written in the following form

$$m\,\ddot{\mathbf{r}}_C = \Big(\mathbf{F}_s + \mathbf{F}_d\Big) \;\&\; I_C\,\ddot{\mathbf{q}} = \mathbf{r}_{CE} \times \Big(\mathbf{F}_s + \mathbf{F}_d\Big), \tag{3}$$

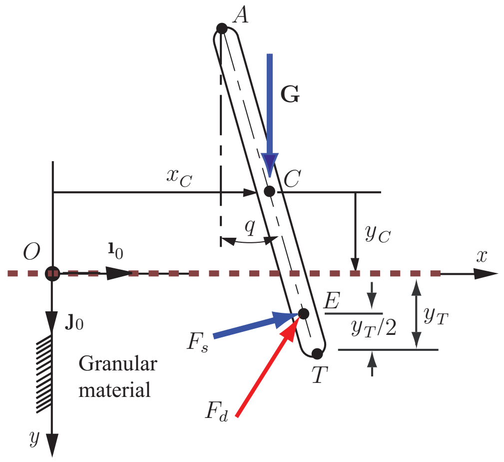

Fig. 1. Link in planar motion

where m_c is the mass of the link, $\mathbf{r}_C = x_C\,\imath_0 + y_C\,\mathbf{J}_0$ is the position vector of the mass center C of the link, $\ddot{\mathbf{q}} = \ddot{q}\,\mathbf{k}_0$ is the angular acceleration vector of the link, and $I_C = m\,L^2/12$ is the mass moment of inertia of the slender link about C. The position vector from the mass center C to the resistance force application point E is

$$\mathbf{r}_{CE} = L_{CE}\,\sin q\,\imath_0 + L_{CE}\,\cos q\,\mathbf{J}_0, \tag{4}$$

where L_{CE} is the length between the mass center C and the resistance force application point E and calculated as

$$L_{CE} = \frac{L}{2} - \frac{y_T}{2\cos q}, \tag{5}$$

where y_T is the immersed depth of the end T. The velocity vector of the resistance force acting point E, $\mathbf{v}_E$, the reference area of the penetrating bar A_r, and the moving angle q_m for calculating the dynamic frictional force are represented as

$$\mathbf{v}_E = \frac{\mathrm{d}\mathbf{r}_C}{\mathrm{d}t} + \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} \times \mathbf{r}_{CE}, \tag{6}$$

$$A_r = d_c\,\frac{y_T}{\cos q}\,|\,\sin\left(q - q_m\right)|, \tag{7}$$

$$q_m = \tan^{-1}\left(\frac{v_{Ex}}{v_{Ey}}\right). \tag{8}$$

The immersed volume V for calculating the static resistance force is calculated as

$$V = \frac{\pi\,{d_c}^2}{4}\,\frac{z_T}{\cos q}. \tag{9}$$

Hence, the horizontal and vertical static resistance forces, $\mathbf{F}_{sx}$ and $\mathbf{F}_{sy}$, are

$$\mathbf{F}_{sx} = -\mathrm{sign}\Big(\dot{x}_C + L_{CE}\,\dot{q}\cos q\Big)\,\eta_h\,g\,\rho_g\,y_T^2\,d_c\,\imath_0, \tag{10}$$

$$\mathbf{F}_{sy} = -\mathrm{sign}\Big(\dot{y}_C - L_{CE}\,\dot{q}\sin q\Big)\,\eta_v\,\left(\frac{y_T}{d_c}\right)^{\lambda}$$

$$g\,\rho_g\,\frac{\pi\,{d_c}^2}{4}\,\frac{y_T}{\cos q}\,\mathbf{J}_0. \tag{11}$$

4. RESULTS

For the free link shown in Fig. 1, the following dimensions are given: the length $L = 0.2$ m and the diameter d_c

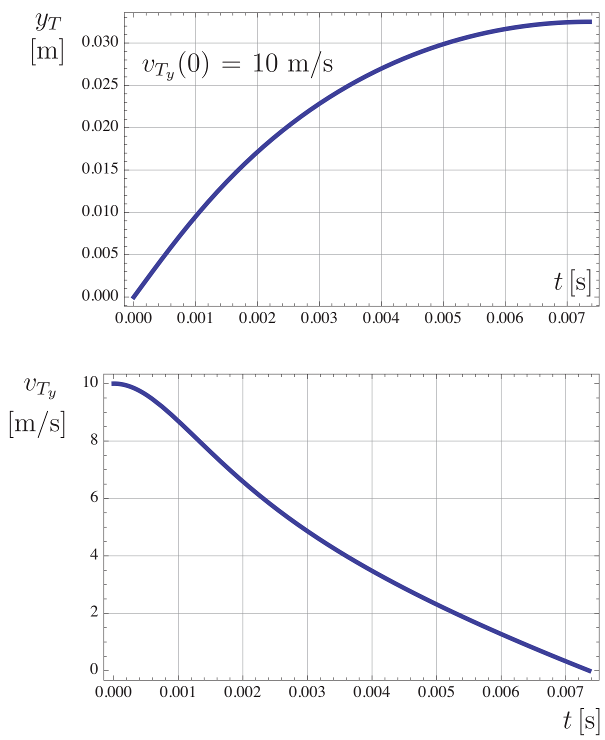

Fig. 2. Simulation results for the free link with the initial vertical impact velocity $v_{T_y}(0) = 10$ m/s

$= 0.01$ m. The density of the link is 7.7×10^3 kg/m^3. The granular material used is “Play sand” (Quikrete 1113-51). The density of the granular material is ρ_g, is 2.5×10^3 kg/m^3. The resistance force coefficients are: the dynamic frictional force coefficient $\eta_d = 6.5$, the horizontal static resistance force coefficient $\eta_h = 8$, and the vertical static resistance force coefficients $\eta_v = 22$ and $\lambda = 1.1$. The gravitational acceleration g is applied as 9.81 m/s^2. Figures 2-5 represent the simulation results for the impact of the free link with different initial vertical impact velocities, $\dot{y}_T(0) = v_{T_y}(0)$. The initial impact angle is $q(0) = \pi/4$, the initial angular velocity is $\dot{q}(0) = 0$, and the initial tangential velocity of the end T is $\dot{x}_T(0) = v_{T_x}(0) = 0$. Figure 2 depicts the results for $v_{T_y}(0) = 10$ m/s. The simulation results for the stopping time of the end T is $t = 0.007363$ s and the traveled distance is $y_T = 0.032508$ m. Figure 3 depicts the results for $v_{T_y}(0) = 5$ m/s and the simulation results for the stopping time of the end T is $t = 0.013261$ s and the traveled distance is $y_T = 0.030973$ m. Figure 4 depicts the results for $v_{T_y}(0) = 1$ m/s and the simulation results for the stopping time of the end T is $t = 0.032544$ s and the traveled distance is $y_T = 0.021550$ m. The results are presented from the impact moment until the vertical velocity of the link end T, v_{T_y}, becomes zero. The penetrating depth traveled by the end of the link T into the granular matter, y_T, is increasing with the initial velocity for all the cases as shown in Figs. 2, 3, and 4. However the stopping time into the granular material is decreasing when the initial impact velocity is increasing as shown in Fig. 5. In these simulation, the stopping time is defined as the time period starting with the moment of impact and ending when the vertical velocity of the end T into the granular material is zero. The faster the mass of the link

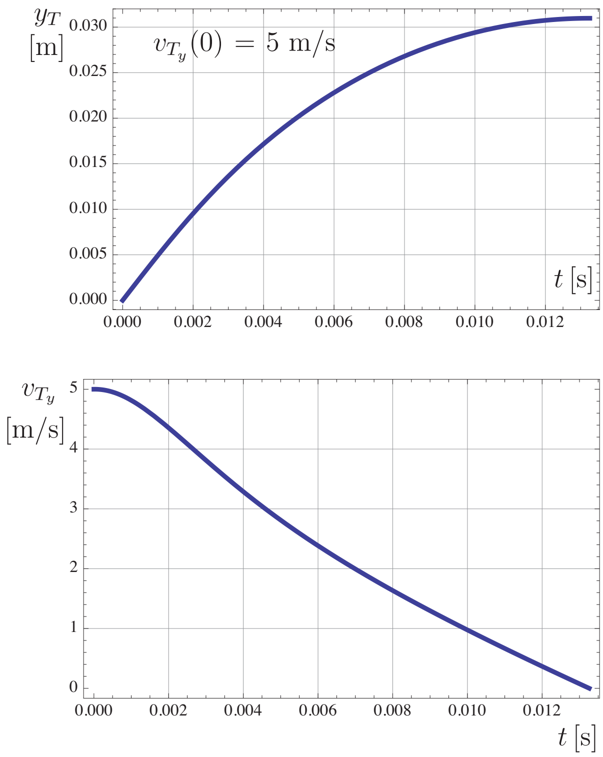

Fig. 3. Simulation results for the free link with the initial vertical impact velocity $v_{T_y}(0) = 5$ m/s

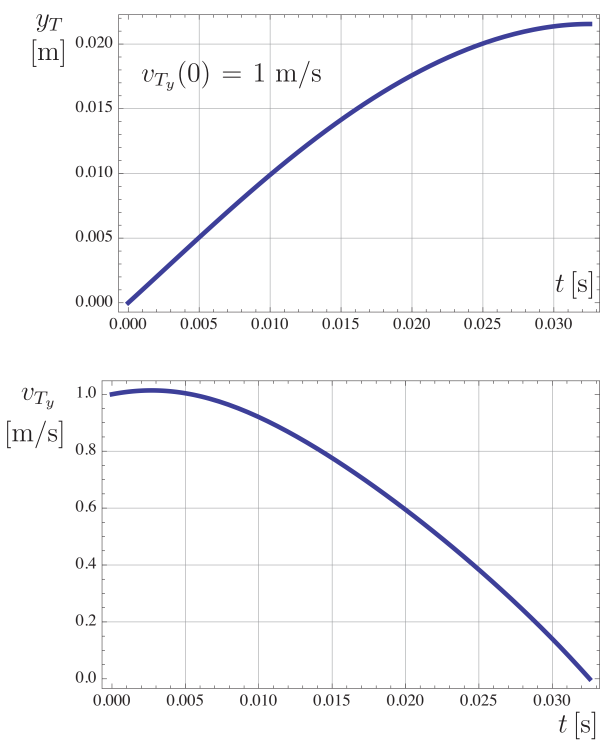

Fig. 4. Simulation results for the free link with the initial vertical impact velocity $v_{T_y}(0) = 1$ m/s

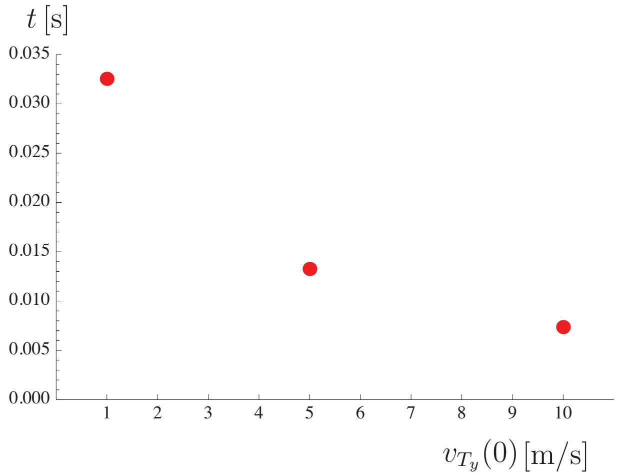

Fig. 5. Stopping time function of initial vertical impact velocity

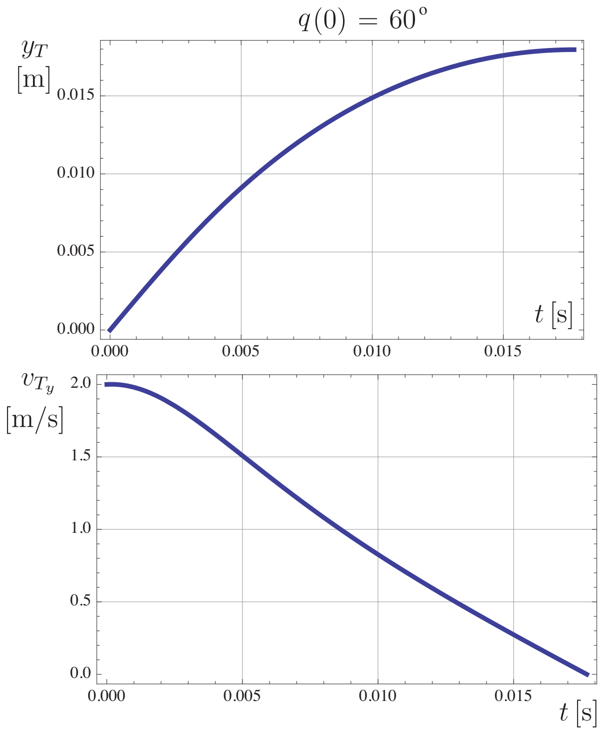

Fig. 6. Simulation results for the free link for the initial impact angle: $q(0) = 60°$

impacts the surface of the granular material, the sooner it will come to a stop. The increasing of the initial velocity causes the stopping time into the granular material to decrease. This an interesting phenomenon involving how rapidly a body strikes the granular material is slowing down upon contact. Figures 6-9 represents the simulation results for the impact of the same link for different initial impact angles $q(0)$. The initial vertical impact velocity is $\dot{y}_T(0) = v_{T_y}(0) = 2$ m/s, the initial tangential velocity of is $\dot{x}_T(0) = v_{T_x}(0) = 0$, and the initial angular velocity is $\dot{q}(0) = 0$. Figure 6 depicts the results for $q(0) = 60°$. The penetrating depth traveled by the end of the link T into the granular matter is $y_T = 0.017961$ m and the stopping time is $t = 0.017698$ s. Figure 7 depicts the results for $q(0) = 45°$. The penetrating depth traveled by the end of

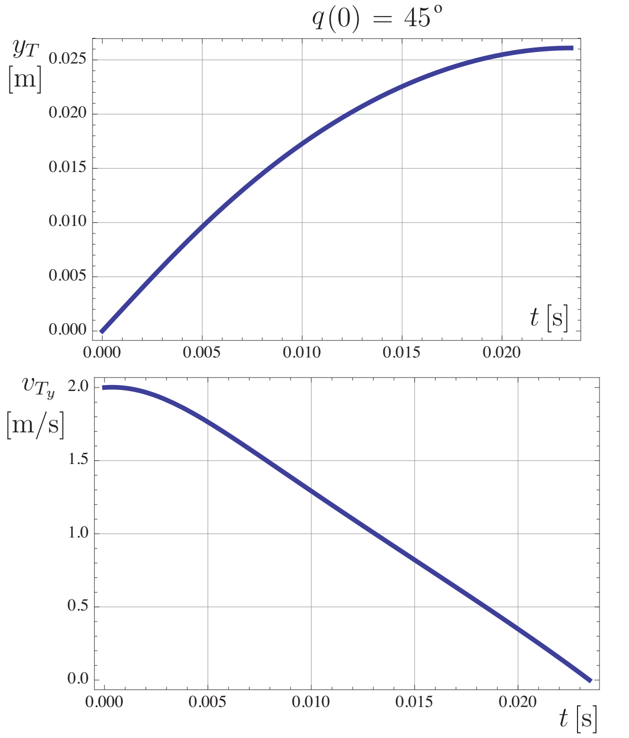

Fig. 7. Simulation results for the free link for the initial impact angle: $q(0) = 45°$

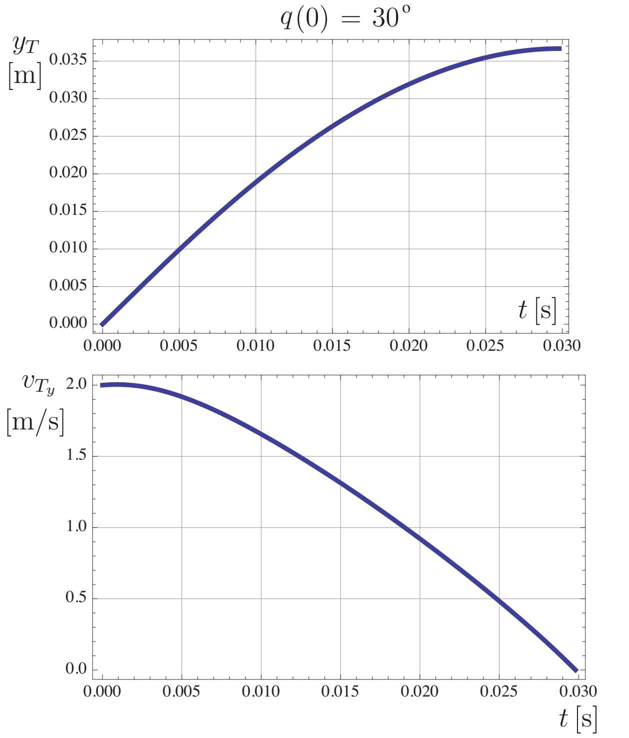

Fig. 8. Simulation results for the free link for the initial impact angle: $q(0) = 30°$

the link T into the granular matter is $y_T = 0.026098$ m and the stopping time is $t = 0.023452$ s. Figure 8 depicts the results for $q(0) = 30°$. The penetrating depth traveled by the end of the link T into the granular matter is $y_T = 0.036668$ m and the stopping time is $t = 0.029802$ s. The depth traveled by the end of the link T into the granular matter is y_T and the the stopping time, Fig. 9, are decreasing with the initial impact angle for all the cases as shown in Figs. 6, 7, and 8.

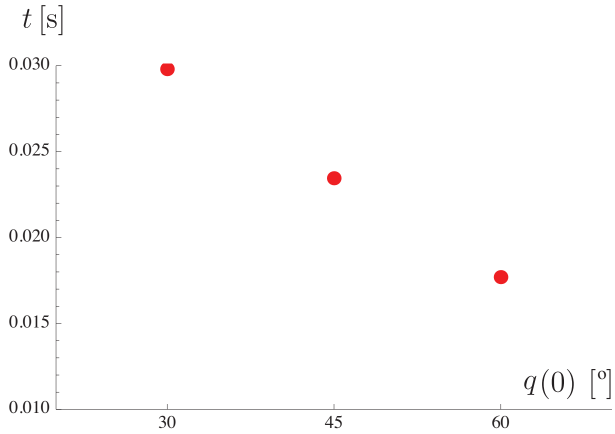

Fig. 9. Stopping time function of initial impact angle

5. CONCLUSIONS

A theoretical model for the collision of a free link with a granular material has been presented. This model was implemented so that the stopping time was calculated as a function of the initial angular velocity and the initial impact angle. The increasing of the initial velocity causes the stopping time into the granular material to decrease. This an interesting phenomenon involving how rapidly a body strikes the granular material is slowing down upon contact. The initial impact angle of the link has also an influence on the stopping time and the penetrating depth traveled by the end of the link into the granular matter. To validate the theoretical model experimental results are needed for different links and chains of elements.

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