

# Optimal delayed control for an overhead crane

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**Abstract:** This note studies a delayed control approach, open and closed loop controllers, in order to attenuate the oscillations of a three degrees of freedom (3DOF) overhead crane system. The proposed control schemes have the capability of attenuate the oscillations during the travel phase and eliminated them at the end point. Both control schemes give us simple expressions for the optimal control parameters. We compared our results with numerical simulations and performed experiments over a laboratory overhead crane.

**Keywords:** Control applications, Delay analysis, Open-loop control, Oscillation and Optimal control.

## 1. INTRODUCTION

The need of faster load transportation using cranes keeping oscillations of the load low is a conflicting objective, see Abdel-Rahman et al. (2003). To move fast the load requires high acceleration on the trolley which produces larger oscillations; these oscillations slow down the rate of operation and increase the operational cost. In the last years various control approaches have been applied to increase the operator's skills. These approaches fall into open loop and closed loop controllers.

Closed-loop control is known to be less sensitive to disturbances and parameter variations. Also it is commonly known in the control literature that the existence of a delay in oscillatory plants may produce a stabilization effect, see Kharitonov et al. (2005). Hence, it is attractive to include a delay in the feedback control law. Pyragas (1992), proposed a delayed feedback control law which stabilizes a particular unstable periodic orbit of a chaotic system. In the work of Masoud et al. (2005), they developed a delayed position-feedback control for a container crane. Later Erneux and Kalmár-Nagy (2007) gave conditions of stability for the control proposed by Pyragas applied to an overhead crane. In this note we present a methodology to obtain the optimal delay for the control law proposed by Pyragas.

On the other hand, open loop schemes do not require the measurement of the load angle, in this context input shaping techniques, see Singer and Seering (1990) and Smith (1957, 1958), are the most popular which has proven effective on cranes for reducing oscillations, see Park et al. (2000). Another open loop approach is optimal control, which calculates the motion trajectory off line based in the mathematical model of the system, see Chernousko (1975) and Dadone and Valandingham (2002). Commonly open loop techniques can not handle disturbances, and sometimes is preferable work in conjunction with some feedback control.

The present paper uses the approaches of delay closed-loop control and open-loop control schemes together, in order to reduce the load oscillations in the travel phase and eliminate them completely at the end point. Our scheme has the particularity that it is compatible with actual manual operators and is

designed to operate transparently to the crane operator, that is, the control works over the on-off command sent by the operator.

## 2. PROBLEM FORMULATION

In this section we present a brief description of the problem. Figure 1 shows the coordinate system of a 3DOF overhead crane; the system consists of a variable length pendulum whose suspension point is capable to move along the  $X$  - axis due to the  $f_x$  force, the force  $f_l$  is responsible for the length variation of the rope. The load is considered as a point mass and the mass and stiffness of the cable are neglected. The following notation is used:  $M$  is the trolley mass,  $m$  the load mass,  $l$  is the length of the hoisting rope,  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  are the friction coefficients associated with the  $x$ ,  $l$  and  $\theta$  motions, respectively.

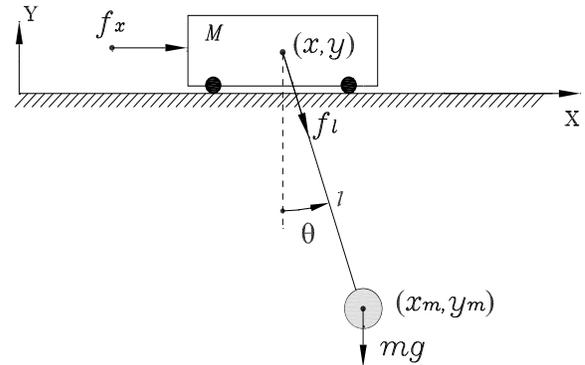


Fig. 1. Coordinate system.

To get the model of the overhead crane, we can apply the Euler-Lagrange equations, see Lee (1998), or the Newton laws, see Fliess et al. (1993). Because of the motor gear, responsible of the movement in the  $X$  - axis, we can consider that the movement of the payload does not affect the dynamic of the car. The model is the following:

$$M\ddot{x} + \delta_1\dot{x} = f_x \quad (1)$$

$$m\ddot{l} + \delta_3 \dot{l} = f_l \quad (2)$$

$$ml^2\ddot{\theta} + 2ml\dot{\theta} + mgl \sin \theta + ml\ddot{x} \cos \theta + \delta_2 \dot{\theta} = 0 \quad (3)$$

The control problem is to move the load from the origin point  $(x_0, y_0)$  to the final point  $(x_f, y_f)$ , keeping the oscillations of the load low, (oscillations over  $5^\circ$  are dangerous). We assume that changing the rope length is needed only to avoid obstacles in the path of the load; with these assumptions we can obtain a linearized model around the equilibrium point  $[x, \dot{x}, \theta, \dot{\theta}, l, \dot{l}]^T = [0, 0, 0, 0, L, 0]^T$  where  $L > 0$  is a constant, based on the Laboratory crane we selected  $L = 1.85m$ . Then we reduce the equations of motion to:

$$\ddot{x} = Lu_x \quad (4)$$

$$\dot{l} = u_l \quad (5)$$

$$\ddot{\theta} = -\frac{\delta_2}{mL^2}\dot{\theta} - \frac{g}{L}\theta - u_x \quad (6)$$

where  $u_x = \frac{1}{ML}[f_x - \delta_1 \dot{x}]$  and  $u_l = \frac{1}{m}[f_l - \delta_3 \dot{l}]$ . The parameters of the laboratory crane are:  $m = 1kg$ ,  $M = 0.6kg$ ,  $L = 1.85m$ ,  $\delta_1 = 1.3kg/s$ ,  $\delta_2 = 0.0087kg \text{ m}^2/s$  and  $\delta_3 = 4.1kg/s$ . These parameters are considered in the rest of the paper.

*Remark 1.* With the control law  $u_x$  we have  $f_x = MLu_x + \delta_1 \dot{x}$ .

### 3. DELAYED FEEDBACK CONTROL

The aim of this section is to study the feedback control law proposed by Pyragas, i.e.,

$$u_x = -k[\theta(t-h) - \theta(t)] \quad (7)$$

This control law consists of a constant that multiply the difference between the delayed angle  $\theta(t-h)$  and the angle  $\theta(t)$ , see Pyragas (1992). Figure 2 shows how will be implemented this control law.

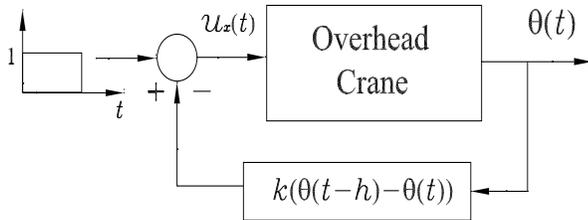


Fig. 2. Delayed feedback control.

Now substituting the control law (7) into equation (6) we have:

$$\ddot{\theta}(t) + 2\mu\omega_0\dot{\theta}(t) + \omega_0^2\theta(t) = k(\theta(t-h) - \theta(t)) \quad (8)$$

where  $\mu = \frac{\delta_2}{2mL^2\omega_0} = 0.00055$  dimensionless coefficient, and  $\omega_0^2 = \frac{g}{L} = 5.74rad^2/s^2$ .

#### 3.1 Stability Analysis

The characteristic equation of the system (8) is:

$$s^2 + 2\mu\omega_0 s + (\omega_0^2 + k) - ke^{-\tau s} = 0 \quad (9)$$

The system (8) is asymptotically stable iff  $\exists \varepsilon > 0$ : roots of equation (9) has real parts less than  $-\varepsilon$ , see Bellman and Cooke (1963). Due to the fact that there are infinitely countable many roots, to study the roots location is a complex task; then in order to study stability of equation (9) we applied the D-subdivision method (or "continuity argument"), see Kolmanovskii and Myshkis (1999). This method says that in the parameter space  $(k, h)$ , there are regions where the numbers of stable (or unstable) roots of (9) is fixed. At the boundaries separating these regions the corresponding  $(k, h)$  parameters generates at least one pair of purely imaginary roots or zero root, see Minorsky (1948).

The condition that will help us to define the boundary between stable and unstable solutions in the parameter space  $(k, h)$  is that the real and imaginary parts of (9) will be equal to zero:

$$-\omega^2 + \omega_0^2 + k - k \cos \omega h = 0 \quad (10)$$

$$k \sin \omega h + 2\mu\omega_0\omega = 0 \quad (11)$$

Combining these equations and defining  $\phi = \frac{h\omega}{2}$ , we obtain the following parametric equations:

$$k = -\frac{4\mu\omega_0\phi}{h \sin 2\phi} \quad (12)$$

$$h^2\omega_0^2 - 4\mu\omega_0\phi h \tan \phi - 4\phi^2 = 0 \quad (13)$$

equation (13) always admit a positive root:

$$h = \frac{2\mu\phi \tan \phi}{\omega_0} + \frac{2}{\omega_0} |\phi| \sqrt{\mu^2 \tan^2 \phi + 1} \quad (14)$$

With equations (12) and (14) we can plot the boundaries between stability and instability, assigning continuous values to  $\phi$  with a fixed step. Figure 3 shows the stable and unstable regions for the first period (grey region).

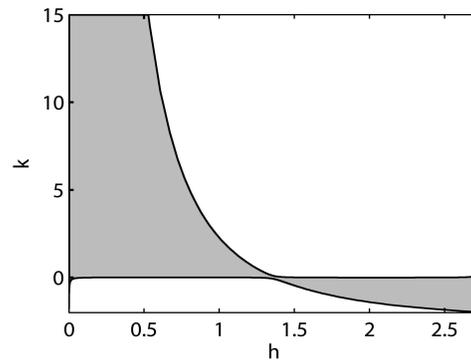


Fig. 3. Stability region (grey) for  $(0 < h < T = \frac{2\pi}{\omega_0\sqrt{1-\mu^2}})$ .

In the stability analysis we consider the maximum length available in the laboratory crane ( $L = 1.85m$ ), then it is important to know what happen with the stability region (3) if the cable length decrease; if  $L$  decrease the period  $T = \frac{2\pi\sqrt{L}}{\sqrt{g}\sqrt{1-\mu^2}}$  decrease, so the stability region decrease (see fig. 4); it is important consider this decrement when we select the control parameters  $(k, h)$ .

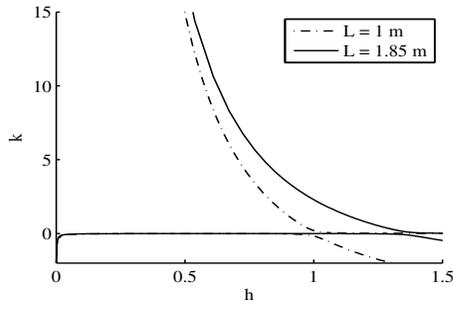


Fig. 4. Effect of  $L$  in the stability region.

Now the question is, Which parameters  $(k, h)$  are the better ones?, i.e., Which values of  $(k, h)$  give the greater damping to the system?; actually if we increase the value  $k$  in the stable region also the damping of the system increase, i.e., the greater  $k$ , the greater damping. However we cannot increase the value of  $k$  so much because for large values of  $k$ , saturation in the actuators appear and the dynamics of the system changes. With this precedent the problem to solve is the next one: given a small value of  $k$ , What is the delay  $h$  that provide the largest damping into the system?

### 3.2 Optimal delay

To solve the optimal delay problem we will apply the *averaging method*, see Minorsky (1948), in order to obtain the solution of the system (8) valid for small values of  $k$ . If we apply the change of variable  $\theta(t) = e^{-\mu\omega_0 t} z(t)$  to the equation (8), we obtain a system of the form:

$$\ddot{z}(t) + \omega_1^2 z(t) = kz(t-h) \quad (15)$$

where  $\omega_1^2 = \omega_0^2(1 - \mu^2) + k$ . Following the *averaging method* we will find a solution for the system (15) which has the next form:

$$z(t) = a(t) \sin(\omega_1 t + b(t)) \quad (16)$$

The amplitude  $a(t)$  and the phase  $b(t)$  are slowly time varying and the derivative of the solution should satisfy the following equation:

$$\dot{z}(t) = a(t)\omega_1 \cos(\omega_1 t + b(t)) \quad (17)$$

this condition establishes the next restriction:

$$\dot{a}(t) \sin(\omega_1 t + b(t)) + a(t)\dot{b}(t) \cos(\omega_1 t + b(t)) = 0 \quad (18)$$

in order to simplify, we will omit the dependence of time in the rest of the section. Substituting  $z$ ,  $\dot{z}$  and  $\ddot{z}$  into (15) and considering (18) we have:

$$\dot{b} = -ke^{-\mu\omega_0 h} \sin(\gamma - \sigma) \sin \gamma \quad (19)$$

$$\dot{a} = kae^{-\mu\omega_0 h} \sin(\gamma - \sigma) \cos \gamma \quad (20)$$

where  $\gamma = \omega_1 t + b$  and  $\sigma = \omega_1 h$ . Following the standard procedure of *averaging*, see Kryloff and Bogoliuboff (1943), we obtain the next equations:

$$\dot{b} = -\frac{ke^{-\mu\omega_0 h}}{2\pi\omega_1} \int_0^{2\pi} \sin(\gamma - \sigma) \sin \gamma d\gamma \quad (21)$$

$$\dot{a} = \frac{kae^{-\mu\omega_0 h}}{2\pi\omega_1} \int_0^{2\pi} \sin(\gamma - \sigma) \cos \gamma d\gamma \quad (22)$$

solving (21) and (22) we have:

$$b = c_1 - \frac{ke^{-\mu\omega_0 h}}{2\omega_1} \cos(\omega_1 ht) \quad (23)$$

$$a = c_2 e^{-\frac{k}{2\omega_1} e^{-\mu\omega_0 h} \sin(\omega_1 ht)t} \quad (24)$$

where  $c_1 = b(0)$  and  $c_2 = a(0)$  are the initial conditions. Finally the solution of system (8) for small values of  $k$  is:

$$\theta = c_2 e^{-\delta(h)t} \sin(\omega_1 t + c_1 - \frac{ke^{-\mu\omega_0 h}}{2\omega_1} \cos(\omega_1 ht)) \quad (25)$$

where  $\delta(h) = \mu\omega_0 + \frac{k}{2\omega_1} e^{-\mu\omega_0 h} \sin(\omega_1 h)$ . Figure 5 shows the function  $\delta(h)$  for different values of  $k$ , in the interval  $0 < h < 1.2$ , this range falls into the first region of stability. Now the natural question is: Given a small value of  $k$  from the stability region (fig. 3), What is the optimal delay  $h$ ?, i.e., What is the delay  $h$  that give the largest damping to the system?.

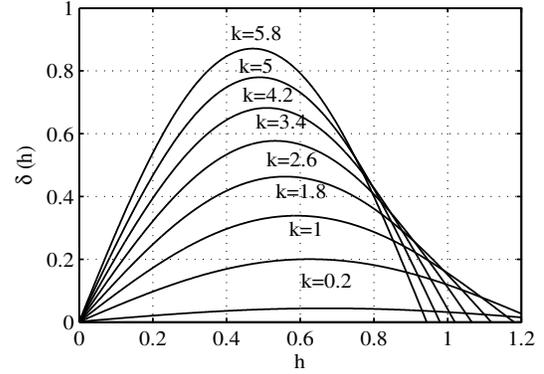


Fig. 5.  $\delta(h)$  for some values of  $k$ .

In order to obtain the value  $h$  that maximizes the function  $\delta(h)$ , for a given  $k$ , we used the first derivative criterion; finally we obtain the expression for the optimal delay:

$$h_{op} = \frac{1}{\omega_1} \arctan\left(\frac{1}{\mu\omega_0}\right) \quad (26)$$

### 3.3 Numerical approach

In this section, we present a numerical approach in order to obtain the optimal delay  $h_{op}$  for the system (8). A common way to study the damping in a linear system is to measure the rate of decay of oscillation, see Beards (1995); by definition, the logarithmic decrement,  $\Lambda$ , is the natural logarithm of the ratio of any two successive peaks in the same direction, so if we measure two successive peaks  $A_1$  and  $A_2$  as in fig. 6, we have

$$\Lambda = \ln\left(\frac{A_1}{A_2}\right) = \frac{2\pi\mu}{\sqrt{1 - \mu^2}} \quad (27)$$

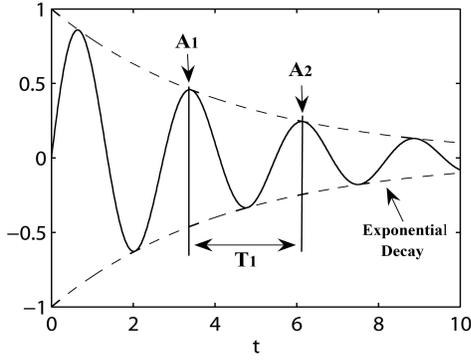


Fig. 6. Exponential decay.

If we measure  $\Lambda$  from the numeric solution we can get the damping factor  $\mu$  from (27). For small  $\mu$  is better to measure the amplitude of oscillations with many periods of separation so that an easily measurable difference exists, i.e.,

$$\Lambda = \frac{1}{N} \ln\left(\frac{A_1}{A_{N+1}}\right) \quad (28)$$

where  $N$  is the number of periods considered.

The algorithm to get the optimal delay is as follow:

- (1) The value of  $k$  is fixed.
- (2) The delay  $h$  will be varying with a fixed step (for example 0.01) between the interval  $(0 < h < T)$ .
- (3) With the integration method "dde23" of Matlab, the solution of (3) with Pyragas control is obtained for every value of  $h$  in the interval of interest, considering the initial condition  $\theta(0) = 0$  and  $\dot{\theta}(0) = 1$ .
- (4)  $\Lambda$  is measured from the numeric solution and consequently the value of  $\mu$  is obtained.
- (5) We made a comparison of the values of  $\mu$  for every value of  $h$  in the interval  $(0 < h < T)$ .
- (6) Finally the optimal delay, is the delay that generated the largest damping  $\mu$ .

The next table shows a comparison of the results obtained with the analytic solution (26) and the numerical approach.

$k$	$h_{op} = \frac{1}{\omega_1} \arctan\left(\frac{1}{\mu\omega_0}\right)$	$h_{op}$ obtained with the numeric method
.05	0.6784	0.69
0.1	0.6752	0.68
0.5	0.6516	0.66
1.5	0.6018	0.61
3	0.5447	0.55
5	0.4890	0.5

Numeric method validate the expression obtained with *Averaging* for the optimal delay.

#### 4. PREFILTER

In this section, the problem of load swing attenuation is studied in an open-loop scheme. It is known that the input shaping technique is able to suppress residual vibrations in linear time invariant systems, see Singer and Seering (1990), however

instead of shaping the input signal by a sequence of impulses we will apply a *prefilter* to the input signal like is shown in fig. 7, then it is only necessary to calculate the *prefilter* parameters  $A$  and  $\tau$ , where  $0 < A < 1$  for underdamped second order linear systems.

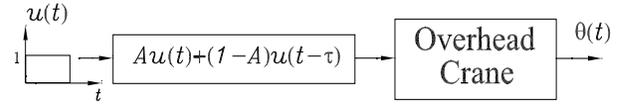


Fig. 7. Prefilter.

In the note we call *prefilter* to the input

$$u(t) = A1(t) + (1 - A)1(t - \tau) \quad (29)$$

where  $1(t - h)$  is a unit step starting at  $t = h$ .

#### 4.1 Transient Response Analysis

Substituting the signal control

$$f_x = -\omega_0^2 MLu + \delta_1 \dot{x} \quad (30)$$

in equation (6), we have

$$\ddot{\theta} + 2\mu\omega\dot{\theta}_0 + \omega_0^2\theta = \omega_0^2u$$

this is a typical second order linear differential equation, with transfer function  $g(s) = \frac{\omega_0^2}{s^2 + 2\mu\omega_0s + \omega_0^2}$ ; if we apply a unit step in  $t = 0$ , the system response is:

$$\theta(t) = 1 - e^{-\mu\omega_0 t} \cos(\omega_r t) - \frac{2\mu e^{-\mu\omega_0 t}}{\sqrt{1 - \mu^2}} \sin(\omega_r t)$$

where  $\omega_r = \omega_0 \sqrt{1 - \mu^2}$ ; now if we apply the input  $u(t) = A1(t) + (1 - A)1(t - \tau)$ , or in frequency domain  $u(s) = A\frac{1}{s} + (1 - A)\frac{e^{-s\tau}}{s}$ , the system response is

$$\begin{aligned} \theta(t) = & [A - Ae^{-\mu\omega_0 t} \cos(\omega_r t) - \frac{2A\mu e^{-\mu\omega_0 t}}{\sqrt{1 - \mu^2}} \sin(\omega_r t)]1(t) + \\ & [(1 - A) - (1 - A)e^{-\mu\omega_0(t-\tau)} \cos(\omega_r(t - \tau)) \\ & - \frac{2(1 - A)\mu e^{-\mu\omega_0(t-\tau)}}{\sqrt{1 - \mu^2}} \sin(\omega_r(t - \tau))]1(t - \tau) \end{aligned} \quad (31)$$

By definition  $\tau$  is the time required for the response to reach the first peak of the overshoot, which is given by

$$\tau = \frac{\pi}{\omega_0 \sqrt{1 - \mu^2}} \quad (32)$$

Equating the coefficients of  $\cos(\omega_r t)$  and  $\cos(\omega_r(t - \tau))$  of (31)

we have  $A = \frac{1}{1 + M_p}$ , where  $M_p = e^{\frac{\mu\pi}{\sqrt{1 - \mu^2}}}$ .

*Remark 2.* The idea of this prefilter goes back to O. Smith, see Smith (1957) and Smith (1958).

## 5. CONTROL FOR CABLE LENGTH

As we mentioned in section 2, the control problem consists to move the load from the initial point  $(x_0, y_0) = (X_0, L_0)$  to the final point  $(x_f, y_f) = (X_3, L_0)$  (fig. 8). For this objective, a step of finite duration is applied to trolley in order to accelerate it and moved it to a desired position, this command emulated the signal send by the operator; moreover, in order to control the cable length, we propose an independent PI controller of the form (frequency domain):

$$u_l = k_p e(s) + k_i \frac{1}{s} e(s) \quad (33)$$

where  $e(s) = l_r(s) - l(s)$  and the reference ( $l_r$ ) is

$$l_r = \begin{cases} L_0 & \text{for } X_2 < x \leq X_0 \\ L_f & \text{for } x \leq X_2 \end{cases} \quad (34)$$

Then we need select the gains  $k_p$  and  $k_i$  and also fix the parameters  $X_0$  and  $X_2$  from fig. 8; for the reference, the considered parameters are:  $L_0 = 1.85m$ ,  $L_f = 1.55m$ ,  $X_0 = 0m$  and  $X_2 = 0.65m$ .

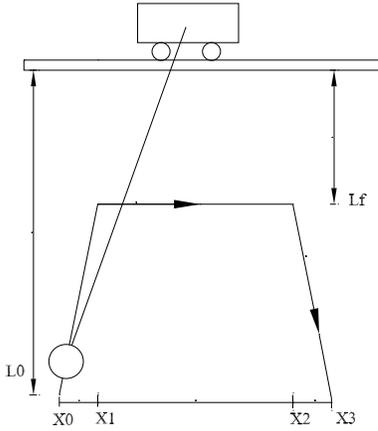


Fig. 8. Desirable path for the cable length.

## 6. SIMULATIONS

Simulations were performed in Matlab-Simulink, with the non-linear model of the crane (1)-(3) and every control scheme (delayed control (7) and prefilter (29)); the operator input is a step of finite duration; independently, the PI controller is the responsible of the cable length variation. We considered  $k = 5$  and  $h_{op} = 0.4885$  for the delayed feedback control, and  $A = 0.5$ ,  $\tau = 1.36$  for prefilter parameters; for the PI control we selected:  $k_p = 15$  and  $k_i = 1$ . For comparison purposes a proportional derivative (PD) control law  $u_x = k_1 \theta + k_2 \dot{\theta}$ , is implemented; we considered the parameters  $k_1 = 5$  and  $k_2 = 0.24$ , with these parameters we have the closed loop dynamic  $\ddot{\theta} + 2\delta\omega\dot{\theta} + \omega^2\theta = 0$ , where  $\delta = 0.5$  and  $\omega = 2.3$ . Note that the product  $\delta\omega = 1.15$ , which is equivalent to the value  $\delta(h)$ , is greater than the value considered for the delayed control with  $k = 5$ , that is  $\delta(h_{op}) = 0.79$ . The plots obtained are:

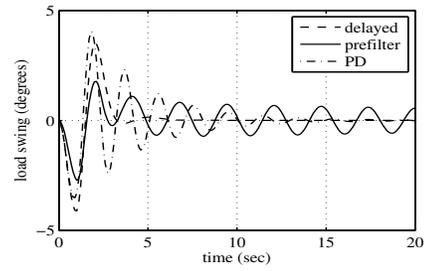


Fig. 9. Simulation results,  $\theta$  with the 3 schemes of control.

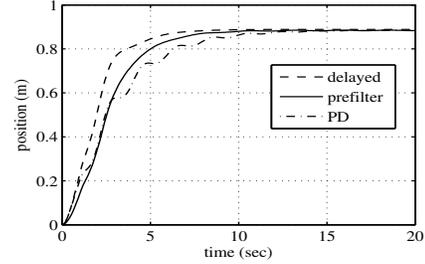


Fig. 10. Simulation results,  $x$  position with the 3 schemes of control.

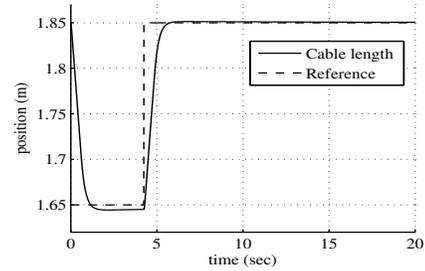


Fig. 11. Simulation results, cable length with the PI control law.

## 7. EXPERIMENTS

Experiments were performed in a laboratory crane (Inteco) using the feedback control law (7) and prefilter (30), with the parameters considered in simulations. The crane is driven by two DC servo motors, for traveling and hoisting respectively. There are two encoders measuring the state variables ( $x, \theta$ ); the derivatives ( $\dot{x}, \dot{\theta}$ ) are obtained with filters. The following results were obtained:

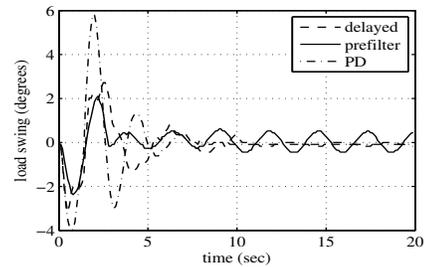


Fig. 12. Experimental results,  $\theta$  with the 3 schemes of control.

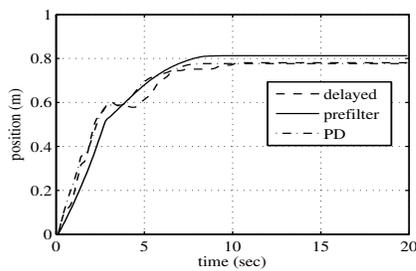


Fig. 13. Experimental results,  $x$  position with the 3 schemes of control.

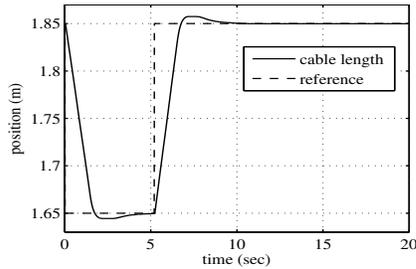


Fig. 14. Experimental results, cable length with the PI control law.

### 7.1 Prefilter compensation

In order to improve the control performance, the used of both approaches together (open-loop and closed-loop control) is proposed as is shown in fig. 15.

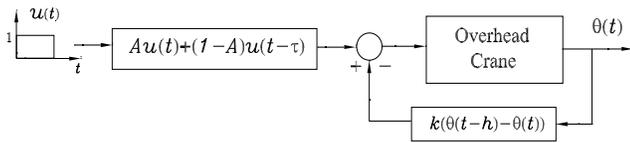


Fig. 15. Prefilter compensation.

Prefilter compensation reduce significantly the angle  $\theta$  in the first period and almost eliminated the residual oscillations. Besides, the prefilter can be combined with other approaches as PD control to increase the performance for the oscillation attenuation problem in an overhead crane.

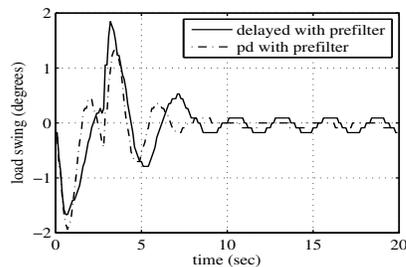


Fig. 16. Experimental results,  $\theta$  angle with prefilter compensation.

## 8. CONCLUSIONS

In this note a delayed closed loop control and open loop control schemes were studied in order to attenuate the oscillations

of a 3DOF overhead crane. The theoretical and experimental results show that the proposed control schemes guarantees both rapid damping of load oscillation and accurate control of rope length with excellent transient response for the practical case of simultaneous traveling and slow hoisting motion.

*Averaging technique* give us, in a simple expression, a very good approximation to obtain the optimal delay for Pyragas control; comparison with a numeric method validate the result.

In the industry, to measure the angle position of the load is a hard task, for this reason the prefilter is preferable over the closed-loop schemes. Extension to the 5DOF case and ship-mounted cranes are considered for future work.

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