Robust Control Law Design for a Synchronous Motor Using Feedback Linearization Method

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Abstract: In this paper a combination of the feedback linearizing control technique and Glover-McFarlane control method is applied for the control of a synchronous motor. The use of feedback linearization requires the complete knowledge of the nonlinear system. In practice, there are many processes whose dynamics is very complex, highly nonlinear and usually incompletely known. To improve robustness, it may be necessary to modify the exact linearization controller. First, we apply the method of nonlinear control and state feedback linearization to synchronous motor model and we obtain a nonlinear control law. This law, aggregated with our nonlinear system, achieves input-output linearization and in the case of multivariable approach, the nonlinear control law achieves also decoupling. Then, Glover-McFarlane H_∞ design is used with the goal of increasing robustness of the existing controller. Finally, some simulation results are included to demonstrate the performance of these controllers and the results are compared with the classical dq vector control.

Keywords: synchronous machine, feedback linearization, robust control.

1. INTRODUCTION

A method largely used for the control of nonlinear systems is to calculate a linear controller for the linear approximation of the nonlinear system around an operating point. This kind of control works in a small neighbourhood of the operating point, and when the system is far from this point, the linear controller will not have the desired behaviour. Thus, the feedback linearization is a good technique because the nonlinear system is transformed into a linear system and only then the linear controller is applied. Therefore, a controller associating feedback linearization and linear control is working in any point, not only in a small neighbourhood of the operating point.

The robust feedback linearization is a new form of feedback linearization which gives a linearizing control law that transforms the nonlinear system into its linear approximation around an operating point, causing only a small transformation in the natural behaviour of the system which is desired in order to obtain robustness.

The control of synchronous motors has been widely investigated in a number of works under various points of view (Cerruto et al., 1995; Caravani et al., 1995; Di Gennaro et al., 1994). One of the most frequently used mathematical descriptions is expressed in the (d, q) frame and the coupling between the angular velocity and the electrical quantities results in a bilinear model with the angular velocity as a natural output to be controlled. If model parameters are perfectly known, non-linearities can be canceled by proper selection of state feedback controls via exact feedback linearization.

Several techniques from linear and nonlinear control theory have been applied to the problem of robust feedback linearization: Lyapunov redesign method, sliding modes, the H_∞ approach, Glover-McFarlane H_∞ design, etc.

Our goal is to combine the exact feedback linearization control technique and Glover-McFarlane design method in order to control a synchronous motor with nonlinearities and parameter uncertainties. The method of nonlinear control and state feedback linearization is applied to synchronous motor model and it is obtained a nonlinear control law. This law, together with our nonlinear system, achieves input-output linearization and in the case of multivariable approach, the nonlinear control law achieves also decoupling. Then the Glover-McFarlane H_∞ design is applied, with the goal of increasing robustness of existing controllers without significantly compromising performance.

The paper is organized as follows: in Section 2, the nonlinear mathematical model of the synchronous motor is presented together with two linearized models obtained by classical feedback linearization method and, respectively, robust feedback linearization method. In Section 3, first, the Glover-McFarlane H_∞ design method is applied in order to robustify the controller obtained by feedback linearization and pole placement method. Then, some simulation results are presented in order to compare the robustness and performances of the designed controllers with those obtained with the classical *dq* vector control. Some concluding remarks are presented in Section 4.

2. MATHEMATICAL MODELS OF SYNCHRONOUS MOTOR

2.1 Nonlinear Mathematical Model

The mathematical model of a permanent magnet synchronous motor, which is expressed in the so-called (*d*,*q*)-frame, and deduced from the application of the Park transformation, can be written as follow (Caravani et al., 1998):

$$\dot{x} = f(x) + \sum_{i=1}^{2} g_i(x) u_i \,, \tag{1}$$

$$y_j = h_j(x);\ j = 1{,}2 \,,$$

in which $f(x), g_1(x), g_2(x)$ are smooth vector fields

$$x^T = [i_d, i_q, \omega] \,,\quad u^T = [u_d, u_q]$$

$$f(x) = \begin{bmatrix} -\frac{R}{L} i_d + p\omega i_q \\ -\frac{R}{L} i_q - p\omega i_d - \frac{p\phi\omega}{L} \\ \frac{p\phi}{j} i_q - \frac{f}{j}\omega - \frac{m}{j} \end{bmatrix} \tag{2}$$

$$g_1^T = \begin{bmatrix} \frac{1}{L} & 0 & 0 \end{bmatrix};\ g_2^T = \begin{bmatrix} 0 & \frac{1}{L} & 0 \end{bmatrix}$$

$$y_1 = h_1 = i_d$$

$$y_2 = h_2 = \omega$$

where R is the stator windings resistance, L the inductance, ϕ is the flux of the permanent magnets, i_d, i_q are the currents and u_d, u_q are the applied voltages, and p is the number of pole pairs; ω denotes rotor angular velocity, j is the rotor moment of inertia, f is the viscous friction coefficient and m is the load torque.

In angular velocity control problems typical outputs of interest are the current i_d and the angular velocity ω. In fact, the electromagnetic torque is generated by the i_q component of the current. Therefore, forcing the i_d component to zero tends to align the current vector along the q direction.

This optimizes the use of all the available current for torque producing purposes.

2.2 Feedback Linearizing Methods

The multivariable nonlinear system we consider is described in state space by equations of the following kind:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_j &= h_j(x) \qquad j = 1...m \end{aligned} \tag{3}$$

in which $f(x)$, $g_1(x)$, $g_2(x)$,...., $g_m(x)$ are smooth vector fields.

The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system (3) into a linear one using a state feedback and a coordinate transformation of the system's state.

We introduce now the Lie derivative of a function $h(x): R^n \to R$ along a vector field $f(x) = [f_1(x), \ldots f_n(x)]^T$

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \tag{4}$$

Definition. A multivariable nonlinear system of the form (3) has a relative degree $\{r_1, \ldots, r_m\}$ at a point x^0 if:

(i) $$L_{g_j} L_f^k h_i(x) = 0 \tag{5}$$

for all $1 \le j \le m$, for all $1 \le i \le m$ for all $k < r_i - 1$ and for x in a neighbourhood of x^0,

(ii) the $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_2(x) & . & . & L_{g_2} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & . & . & L_{g_2} L_f^{r_2-1} h_2(x) \\ . & . & . & . \\ L_{g_1} L_f^{r_m-1} h_m(x) & . & . & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{6}$$

is nonsingular at $x = x^0$.

Theorem. Let be the nonlinear system of the form (3). Suppose the matrix $g(x^0)$ has rank *m*. Then the State Space Exact Linearization Problem is solvable if and only if

(i) for each $0 \le i \le n-1$, the distribution G_i has constant dimension near x^0;

(ii) the distribution G_{n-1} has dimension n;

(iii) for each $0 \le i \le n-2$, the distribution G_i is involutive.

1) Classical Feedback Linearization: The classical feedback linearization is accomplished by using a linearizing control law of the form

$u_c(x,w)=\alpha_c(x)+\beta_c(x)w$, where w is a linear control, and a diffeomorphism $x_c=\phi_c(x)$, with

$$\alpha_c(x)=-A^{-1}(x)\left[L_f^{r_1}h_1(x)\cdots L_f^{r_m}h_m(x)\right]^T$$

$$\beta_c(x)=A^{-1}(x)$$

$$\phi_c^T(x)=\left[\phi_{c_1}^T(x)\cdots\phi_{c_m}^T(x)\right]$$

$$\phi_{c_i}^T(x)=\left[h_i(x)\ L_f h_i(x)\cdots L_f^{r_i-1}h_i(x)\right]$$

The linearized system is:

$$\dot{x}_c=A_c x_c+B_c w \qquad (7)$$

where A_c and B_c are the matrices of the Brunovski canonical form.

2) Robust Feedback Linearization: The main difference between the robust feedback linearization and the classical one is that the linearized system has the form

$$\dot{x}_r=A_r x_r+B_r v \qquad (8)$$

with $A_r=\partial_x f(0)$ and $B_r=g(0)$, which corresponds to the linear approximation of the nonlinear system (3).

The robust feedback linearization is accomplished by using a linearizing control law of the form $u(x,w)=\alpha(x)+\beta(x)v$, where v is a linear control, and a diffeomorphism $x_r=\phi(x)$, with

$$\alpha(x)=\alpha_c(x)+\beta_c(x)LT^{-1}\phi_c(x)$$

$$\beta(x)=\beta_c(x)R^{-1}$$

$$\phi(x)=T^{-1}\phi_c(x) \qquad (9)$$

$$L=-A(0)\partial_c\alpha_c(0)$$

$$T=\partial_x\phi_c(0),\ R=A^{-1}(0)$$

The functions $\alpha(x)$, $\beta(x)$, and $\phi(x)$ satisfy

$$\partial_x\alpha(0)=0,\ \beta(0)=I,\ \text{and}\ \partial_x\phi(0)=I \qquad (10)$$

2.3 Linearized Models of the Synchronous Motor

1) Classical Feedback Linearized Model: We consider as output variables

$$y_1=i_d$$

$$y_2=\omega$$

Easy calculus show that the matrix for mathematical model of the synchronous motor is nonsingular and the relative degree is $\{r_1,r_2\}=\{1,2\}$.

For the system given by (1), the decoupling matrix is

$$A(x)=\begin{bmatrix} L_{g1}L_f^0h_1(x) & L_{g2}L_f^0h_1(x)\\ L_{g1}L_f^1h_2(x) & L_{g2}L_f^1h_2(x)\end{bmatrix}=\begin{bmatrix}\frac{1}{L} & 0\\ 0 & \frac{p\phi}{Lj}\end{bmatrix} \qquad (11)$$

Now, the input-output system can be rewritten in the form:

$$\begin{bmatrix}\dot{y}_1\\ \ddot{y}_2\end{bmatrix}=\begin{bmatrix}L_f h_1(x)\\ L_f^2h_2(x)\end{bmatrix}+A(x)\begin{bmatrix}u_d\\ u_q\end{bmatrix} \qquad (12)$$

Because the decoupling matrix (11) is not singular, it is possible to design a nonlinear input

$$\begin{bmatrix}u_d\\ u_q\end{bmatrix}=A^{-1}(x)\begin{bmatrix}-L_f h_1(x)+w_1\\ -L^2{}_f h_2(x)+w_2\end{bmatrix} \qquad (13)$$

The functions for the classical linearizing feedback control law are

$$\alpha_c(x)=\begin{bmatrix} Ri_d-Lpi_q\omega\\ Ri_q+Lpi_d\omega+p\phi\omega+\dfrac{Lfi_q}{j}-\dfrac{Lf^2}{jp\phi}\end{bmatrix},$$

$$\beta_c(x)=\begin{bmatrix}L & 0\\ 0 & \dfrac{jL}{p\phi}\end{bmatrix},\ \phi_c(x)=\begin{bmatrix} i_d\\ \omega\\ \dfrac{p\phi i_q-f\omega}{j}\end{bmatrix}$$

In the new coordinate we have

$$\begin{aligned}\dot{y}_1&=w_1,\\ \ddot{y}_2&=w_2.\end{aligned} \qquad (14)$$

The state feedback (13) transforms this system into a system whose input-output behavior is identical to that of a linear system having transfer function matrix of the form

$$H(s)=\begin{bmatrix}\frac{1}{s} & 0\\ 0 & \frac{1}{s^2}\end{bmatrix} \qquad (15)$$

2) Robust Feedback Linearized Model: The functions for the robust linearizing control law are $\alpha(x)$, $\beta(x)$, and

$\phi(x)$ calculated using the functions $\alpha_c(x)$, $\beta_c(x)$, and $\phi_c(x)$ given before and the matrices

$$L = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{p\phi R}{jL} - \frac{p\phi f}{j^2} & \frac{f^2}{j^2} - \frac{p^2\phi^2}{jL} \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \frac{p\phi}{j} & -\frac{f}{j} \end{bmatrix},$$

$$R = \begin{bmatrix} L & 0 \\ 0 & \frac{jL}{p\phi} \end{bmatrix}.$$

3. ROBUST CONTROL DESIGN

Imposing on the system (15) an additional feedback of the form

$$v_1 = -c_{10}(y_1 - y_{1ref}) \qquad (16)$$

$$v_2 = -c_{20}(y_2 - y_{2ref}) - c_{21}\dot{y}_2$$

then, the obtained system has a linear input-output behavior, described by the following diagonal transfer function matrix

$$H(s) = \begin{bmatrix} \frac{c_{10}}{s + c_{10}} & 0 \\ 0 & \frac{c_{20}}{s^2 + c_{21}s + c_{20}} \end{bmatrix} \qquad (17)$$

3.1 Glover McFarlane Control Design

We consider the structure of the control system shown in Fig. 1, where are implemented the control laws (16) and K_{r1}, K_{r2} are the robustifying controllers (G_{s1}, G_{s2} are the nominal shaped plants).

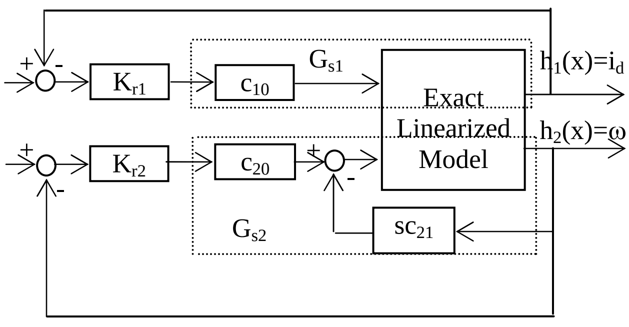

Fig. 1. The control loop.

In this design, the model uncertainties are included as perturbations to the nominal model, and robustness is guaranteed by ensuring that the stability specifications are satisfied for the *worst-case* uncertainty. Here, since the system is decoupled, we design separately the two robustifying controllers. The method described next [McFarlane and Glover 1992] is applied to the nominal shaped plants G_{s1}, G_{s2}, but the indices are neglected.

Let $G_s = N / M$ be the normalized coprime factorization of the nominal shaped plant.

The normalized coprime factor uncertainty characterization is given by

$$\left\{ \frac{N + \Delta_N}{M + \Delta_M} : \left\| [\Delta_N \Delta_M] \right\| \le \varepsilon \right. \quad (18)$$

The following steps yield the optimal controller that assumes a state-space (A,B,C) available for the transfer function G_s :

1) Obtain Z by solving the algebraic Riccati equation (ARE)

$$AZ + ZA - ZC^TCZ + BB^T = 0 \qquad (19)$$

2) Obtain X by solving the ARE

$$AX + XA - XBB^TX + C^TC = 0 \qquad (20)$$

3) Compute the maximum possible ε for the given nominal shaped plant

$$\varepsilon_{\max} = (1 + \rho(XZ))^{-1/2} \qquad (21)$$

where ρ denotes the spectral radius. Hence, in this design scheme there is no need for an explicit characterization of uncertainty. The method detects and solves for the worst-case scenario.

4) The robustness margin ε is chosen to be slightly less than $\varepsilon_{\max}$. Let $\gamma = 1/\varepsilon$.

5) The state-space realization of the robustifying controller K_r is given by

$$\begin{bmatrix} A + BF + \gamma^2 (L^T)^{-1} ZC^TC & \gamma^2 (L^T)^{-1} ZC^T \\ B^TX & 0 \end{bmatrix} \qquad (22)$$

where $F = -B^TX$ and $L = (1 - \gamma^2)I + XZ$.

An important feature of this algorithm consists in the similitude of the loop transfer functions, before and after robustification.

3.2 Simulation Results

The simulation was performed for a synchronous motor having the following parameters:

$R = 0,6\,(\Omega)\,;\, L = 1,2\times10^{-3}\,(\mathrm{H});\; f = 1,4\times10^{-3}\,(\mathrm{Nms});$

$j = 2,5\times10^{-3}\,(\mathrm{Kgm}^2);\, \phi = 0,12\,(\mathrm{Wb});\; p = 4$

We are testing the control performance for step changes in the reference. The simulation was done for the equation model (1) and the nonlinear control law (9), (12). The design parameters are computed using a pole-placement design technique.

For load torque m = 0, y_{1ref} = 0 and a series of step references for y_{2ref} (30 rad/sec at start, 70 rad/sec at 0.5 sec and 90 rad/sec at 1.5 sec), the evolution of variables u_d, u_q, i_d, i_q, ω and ω_{ref} is presented in Fig. 2. It can be seen that the i_d current tends to zero and the angular velocity ω achieves each time the reference after less than 0.1 sec.

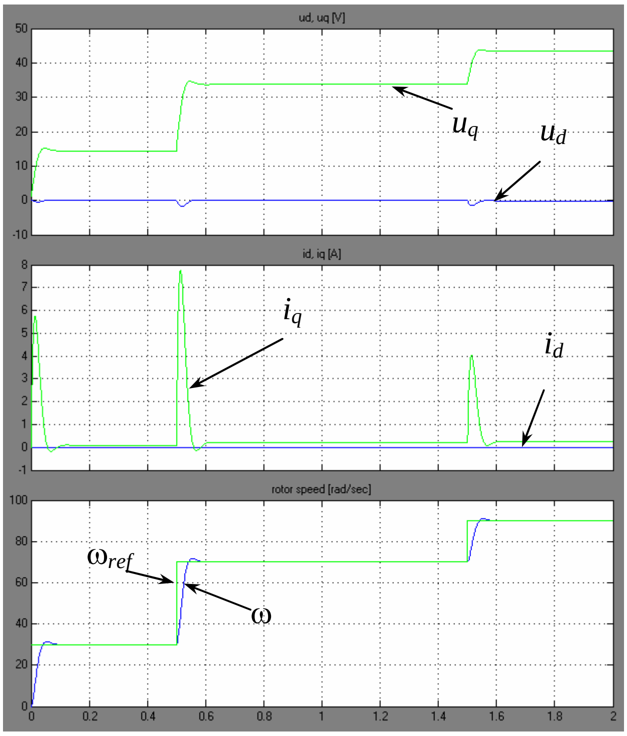

Fig. 2. The results of the simulation for the classical feedback linearized model.

The results of the simulation for the robust feedback linearized model are plotted in Fig. 3.

We notice the very good behavior of the control system. The two controlled variables are following the corresponding reference values with high accuracy. The speed has no override thanks to the proper regulation of the active current i_q. At its turn, this current is smoothly controlled by the corresponding voltage component, u_q. In the same delicate manner, the reactive current id is maintained rigorously at its null reference value.

The proposed control strategy is to be compared with the classical *dq* vector control of the PMSM.

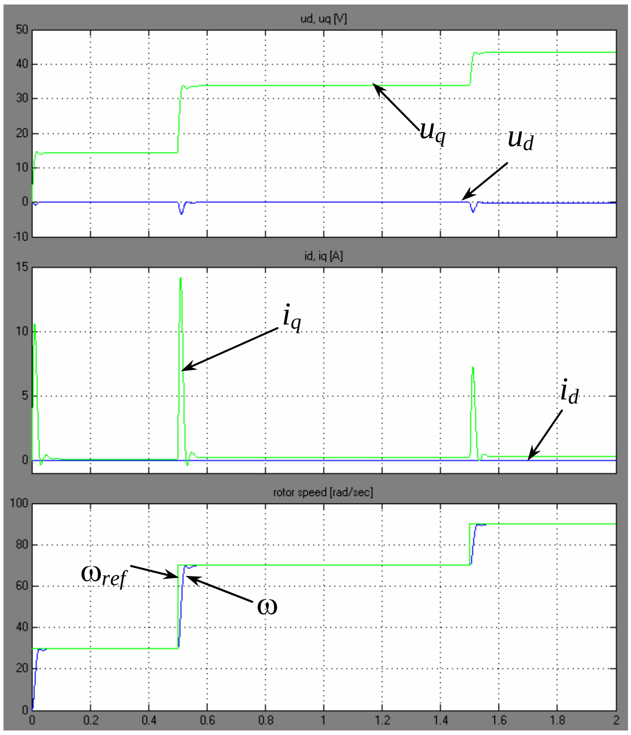

Fig. 3. The results of the simulation for the robust feedback linearized model.

The most basic such control imposes the *d*-axis component of the current to zero (Vas 1990, Ivanov 2008b). If a current source inverter is used for supplying the motor, the reference values of the two current components are transformed from the Park reference to the fix one, the resulted currents being the reference values for the inverter. When a voltage source inverter is used for supplying the PMSM, the reference values of the voltages result as functions by the necessary currents. If steady state operation is considered $\left(\dfrac{di_q}{dt} = 0\right)$, the two components result:

$$u_{dref} = -pL_q\Omega_m i_{qref}\ ,$$

$$u_{qref} = R_a i_{qref} + K_T\Omega_m$$

By using the models previously developed by the authors (Ivanov 2008a, b) and imposing the same profile for the reference speed, for the same motor, the resulted *dq* voltages, currents and rotor speed are the one plotted in Fig. 4.

From Fig. 4 one can note the very good dynamical behavior, but the currents i_d and i_q are greater than in the Fig. 2 and 3.

The robustness of the proposed control law must be verified for alterations of the rated values of the motor parameters. An example is plotted in Fig. 5, where the stator resistance of the motor was increased by 20 %.

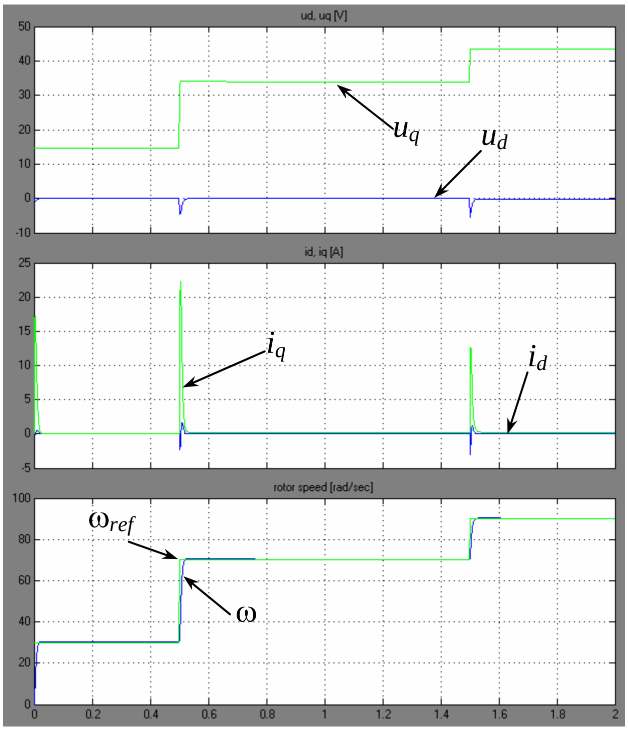

Fig. 4. The results of the simulation for the dq vector control.

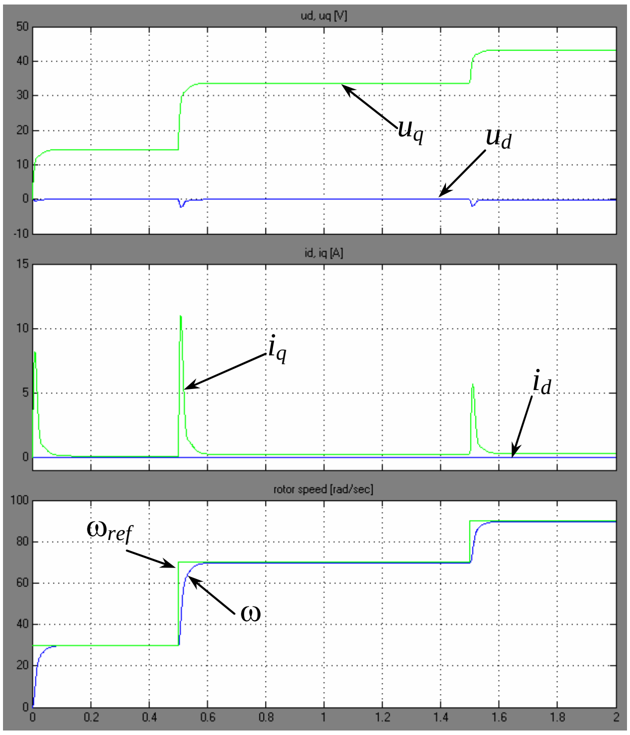

Fig. 5. The results of the simulation for the robust feedback linearized model with increased stator resistance.

Such alteration can occur naturally during operation, due to the increasing of the windings temperature.

One can notice that the behaviour of the system is still very good, due to the robust control loop.

4. CONCLUSIONS

It can be seen that, qualitatively, the results obtained with the proposed control strategy are close by the ones obtained with the classical dq control, which confirm the correctness of the proposed strategy. The results can be improved quantitatively by a more proper pole allocation. Further work will be made for testing the robustness properties, in real time, on an experimental bench.

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