Rebound of a Robotic Kinematic

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Abstract: A model is presented for the impact of a robotic kinematic element in planar motion. The model consists of a system of nonlinear differential equations which considers the collisions as well as frictional effects at the contacting end, and allows one to predict the rigid and elastic body motion after the impact. The mode functions are selected such that the method can be made computationally as simple as possible, without compromising accuracy. To describe the impact between the kinematic link and the rigid surface the classical Hertzian contact theory and elasto-plastic indentation theory are used.

Keywords: impact, exible kinematic link, vibrational modes, nonlinear contact force.

1. INTRODUCTION

The propagation of impact-induced elastic waves has been the subject of many investigations. The latest development in the area has been the effect of elastic vibrations on the post-impact velocities of slender bars in motion. Gau and Shabana (1991; 1992), and Shabana and Gau (1993) examined the effect of the topological change in the propagation of longitudinal elastic waves in mechanical systems. The application of the analysis presented is demonstrated using a rotating rod that is subject to an axial impact by a rigid mass moving with a constant velocity. On the experimental front, Yigit et al. (1990a) veriffed the validity of using the algebraic generalized impulse momentum equations of a radially rotating beam, transversally impacting an external surface. Their conclusion was that the momentum balance method and an empirical coeffcient of restitution can be used with confidence in the impact of radially rotating beam. A spring-dashpot model for the dynamics of a radially rotating beam with impact is studied by the same authors (Yigit et al. 1990b). The impact that appeared to be single to the naked eye consists in reality of several collisions in rapid sequence. Existence of multiple collisions was recognized by Mason (1935), Goldsmith (1960), and experimentally by Yigit et al. (1990a; 1990b). Wang et al. (1995) studied the response of a right-angled bent cantilever beam subjected to an out-of-plane impulsive load applied to concentrated mass at its tip. A double-hinge mechanism is developed with a pure bending hinge in the first segment of the beam and a combined bending-torsion hinge in the second segment.

Stoianovici and Hurmuzlu (1996) used a discrete model for the dynamics of a exible bar during the collision process. They sectioned the bar in equal rigid segments, connected to one another by linear springs of uniform stiffness, and placed symmetrically at a specific transverse distance from the neutral axis. The stiffness constant was evaluated by using the axial displacement at the center of a vertically hanged bar caused by its weight. The transverse distance was determined by an equivalence with the rotation of a transversal section, at the center, of a horizontal cantilever bar acted upon by its weight. They consider Lagrange equations for the holonomic case with uniform slip during collision.

Marghitu and Hurmuzlu (1996) introduce a finite number of vibration modes to take into account the vibrational effect for the impact of a single straight beam. Effects such as multicollisions, slip reversal, local vortex vector, and different configurations are accommodated automatically.

The objective of the present article is to develop a procedure that can be followed to solve the collision with friction of robotic kinematic links using an analytical continuous model. The kinetic energy required has been derived using a generalized velocity field theory for elastic solids in rotation and translation. Using this continuous model, we can study the vibrations, the multiple collisions, and the rebound velocities.

2. KINEMATIC RELATIONS

The system to be analyzed, shown in Figure 1, consists of a planar flexible link with the length L. Consider the collision between the link of mass m and a fixed massive block that is bounded by a horizontal plane. When the link is undeformed, its coordinate system is denoted as **[i; j; k]**. The motion of the link in a Newtonian reference frame $[\mathbf{i}_0; \mathbf{j}_0; \mathbf{k}]$ is prescribed as a function of time. The link fixed reference system rotates and translates with the

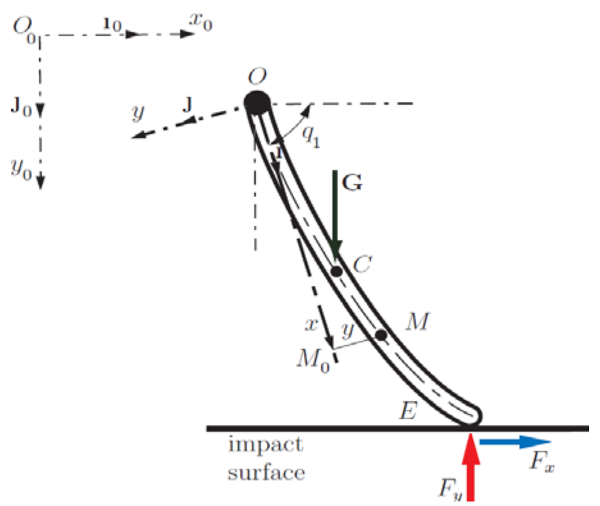

Fig. 1. A planar flexible link with the length

kinematic element as if the link was rigid. The problem is to calculate the general motion of the link (rigid body motion and exible deformations). Let x be the distance from O (the origin of **[i; j; k]**) to a generic cross section of M_0, when M_0 is undeformed. The incident angle of the bar at the impact moment is q_1 the angle between the unit vectors **i** and $\mathbf{i}_0$. Inspection of Figure 1 shows that the planar position vector of an arbitrary point on the deformable beam can be written as

$$\mathbf{R} = \mathbf{r}_O + \mathbf{r} + \mathbf{d} \tag{1}$$

where, $\mathbf{r}_O$ is the global position vector of the origin O, and $\mathbf{r}$ = x_1. For a link articulated at O the position vector $\mathbf{r}_O$ = 0 and the origins of the two reference frames are identical. The total elastic motion of the generic point can be expressed as

$$\mathbf{d}(x;\, n;\, t) = s\mathbf{i} + y\mathbf{j} \tag{2}$$

where $s(x; n; t)$ denotes the stretch in the link along the elastic axis

$$s(x,n,t) = \sum_{i=1}^{n} \Phi_{1i}(x)\cdot q_{1+i}(t) \tag{3}$$

where $q_{1+i}(t)$ are the generalized elastic coordinates, and $n \in N$ is the total number of vibrational modes (N is the set of natural numbers).

The transverse elastic displacement is

$$y(x,n,t) = \sum_{i=1}^{n} \Phi_{2i}(x)\cdot q_{1+i}(t) \tag{4}$$

Here the unrestricted spatial functions $\Phi_{ij}(x)$, $j=1,2$ can be chosen as the mode shapes of a articulated-free beam.

The relation between the axial and transverse displacements is given by

$$x + s(x,n,t) = \int_0^{\eta} \left\{ 1 + \left[\frac{\partial y(\sigma,n,t)}{\partial \sigma} \right]^2 \right\}^{1/2} d\sigma \tag{5}$$

where $\eta = x + s$.

The motion of the elastic link is characterized by the generalized reference speed $u_1 = \dot{q}_1$ and the generalized elastic speeds defined as

$$u_{1+i} = \dot{q}_{1+i}\ ,\ \mathrm{i}=1,2\ldots\mathrm{n} \tag{6}$$

Now, we introduce the generalized position and speed (reference and elastic) vectors as

$$\mathbf{q} = [q_1, q_2, \ldots\ldots q_{n+1}]^T ,$$
$$\mathbf{u} = [u_1, u_2, \ldots\ldots\ u_{n+1}]^T .$$

The impact event is initiated when the link contacts the massive surface at the contact point E. The generalized speeds u_i, $i = 1,2, \ldots\ n+1$ at the instant t_0 at which the linkk comes into contact with the surface are presumed to be known. We seek to determine the values u_i at the time t_f, the instant at which the ink completely loses contact with the surface (end of collision). The kinetic energy of the kinematic element is given by

$$K(q,u,n) = \frac{p}{2}\cdot \int_0^L v(q,u,x,n)\cdot v(q,u,x,n)\cdot dx \tag{7}$$

where $\rho = m/L$ is the constant mass per unit of length, and m is the mass of the bar.

3. IMPACT FORCES

The equations of impulsive motion are determined by combining the classical Hertzian contact theory (Goldsmith 1960) and elastic-plastic indentation theory (Johnson 1985). There is a linear relationship between the plastic deformation q_p and contact force F_y, as follows

$$q_p = (F_y - F_c) \tag{8}$$

In the above equation the coeffcient η has the following expression

$$\eta = \frac{1}{2\cdot\pi\cdot R\cdot H} \tag{9}$$

where H characterizes the plastic property of the material and can be pproximate with the Brinell hardness, and R is the radius of the impacting body. The critical value of the impact force F_c, can be expressed in terms of the yield stress σ_y

$$F_c = \frac{8 \cdot \pi^3 \cdot R^3 \cdot \sigma_y^3}{k_1^2} \quad (10)$$

Where

$$k_1 = \frac{2}{3\left(1-\nu^2\right)} \cdot e \cdot \sqrt{R} \quad (11)$$

The Poisson's ratio is ν. The elastic deformation qe as a function of the contact force is described by the Hertz's law

$$q_e = \left(\frac{F_y}{k_1}\right)^{2/3} \quad (12)$$

and the total normal deformation for the elasto-plastic impact is the sum of elastic and plastic deformation

$$q = q_e + q_p = \left(\frac{F_y}{k_1}\right)^{2/3} + \eta \cdot \left(F_y - F_c\right) \quad (13)$$

Figure 2 shows the representation of the force F_y with respect to the deformation q. The critical deformation qc corresponds to the force F_c, and the maximum deformation q_m appears when the maximum force F_m is applied.

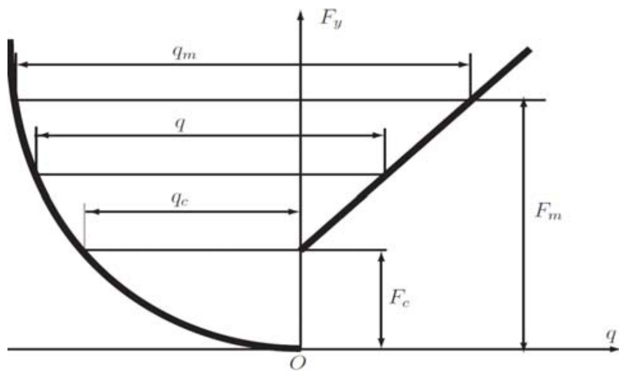

Fig. 2. Representation of the force F_y with respect to the deformation q

The impact force at the impact point is

$$\mathbf{F} = F_x \mathbf{i_0} + F_y \mathbf{j_0} \quad (14)$$

where F_x is the friction force.

Contact force was determined and examined. When the contact force becomes positive this represents the separation of the body. In this way we can capture the multiple impacts. When the bar has a nonzero tangential velocity at the onset of the collision $\nu_E^t \neq 0$, there will be a phase of slip.

Using Lagrange's method, the nonlinear impact equations of motion are of the form

$$M(q) \cdot \ddot{q} + f\left(q, \dot{q}, t\right) = 0 \quad (15)$$

where q is the generalized coordinates vector, M is the mass matrix, and f is a nonlinear vector, which contains the generalized vector and its derivative.

4 SIMULATION RESULTS

The objective of the work is to investigate the validity of the analytical frictional impact model presented and its applicability for rigid and elastic robotics links. Figure 3 represents the variation of the impact angle q_1 during impact.

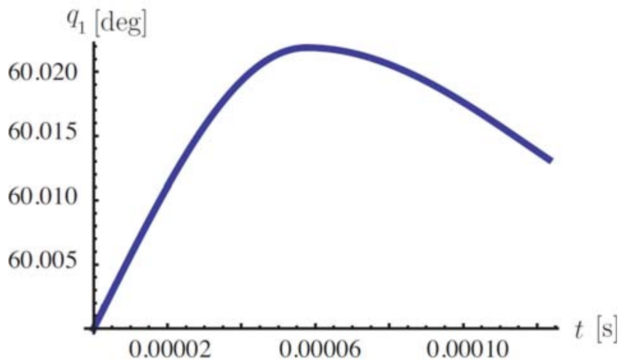

Fig. 3. The variation of the impact angle q_1 during impact

The initial impact angle is $\pi/3$, the initial angular velocity is ω_0 = 10 rad/s, and the coefficient of friction is μ = 0.3.

Figure 4 shows the elastic normal displacement of the impact point and Figure 5 depicts the impact force during the contact.

The coefficient of restitution was deffined as a kinematic constant that determines the ratio of the normal rebound velocity to the nornal approach velocity at the contact point. The accuracy of impact theory to model collisions depends obviously on the accuracy and behavior of this coefficient. The coefficient of restitution was believed to be a material properties and its more important role is in the representation of the elasto-plastic conduct due to deformation at the contact region. This coefficient depends not only on the normal velocity component, but also the tangential component as well (or alternatively, on the angle of incidence).

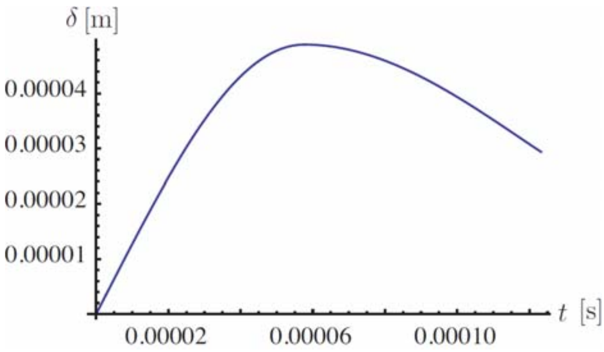

Fig. 4. The elastic normal displacement of the impact point

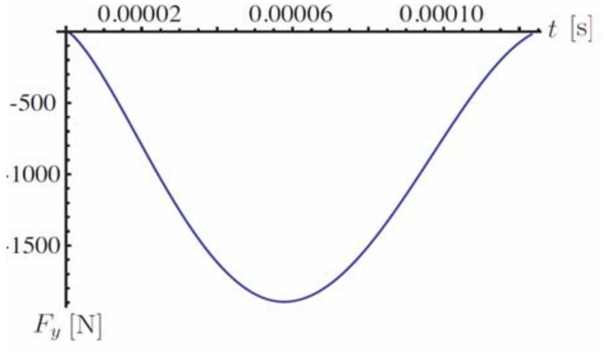

Fig. 5. The impact force during the contact

Figures 6 and 7 show how simulation values for *e*, for low-speed collisions, vary with the coefficient of friction, μ and the initial impact velocity ω_0.

5 CONCLUSIONS

We introduce a finite number of vibration modes to take into account the vibrational effect on the beam for different configurations. The impulse during impact is dependent on the coefficient of friction and the the initial impact velocity.

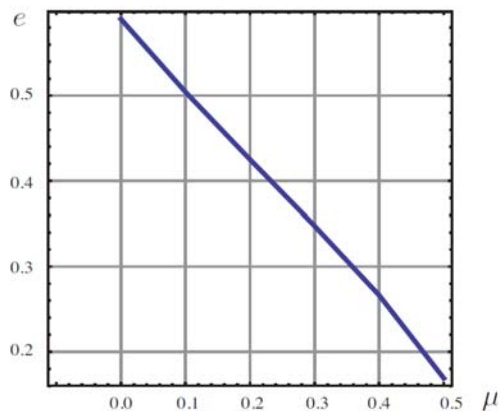

Fig. 6. The variation of *e*, for low-speed collisions, with the coefficient of friction μ

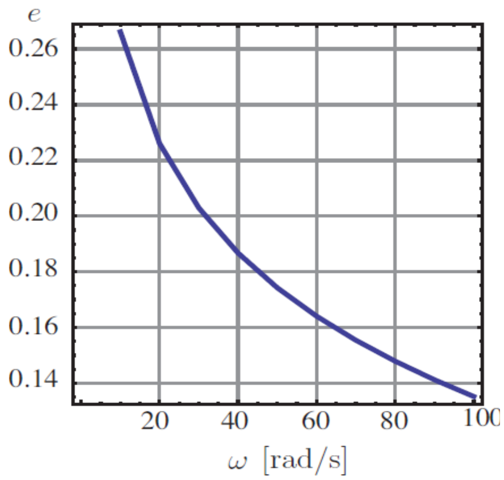

Fig. 7. The variation of *e*, for low-speed collisions, with the initial impact velocity ω_0.

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